Description of hypernuclei in scalar
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Abstract

Hypernuclear spectra are calculated by using the scalar derivative coupling model of Zimanyi and Moszkowski. Although the fit is not unique, as in similar models, the experimental data are reproduced with accuracy. We discuss also how much information is contained in the spectroscopic data concerning the properties of the Λ in nuclear matter. In particular, we discuss the saturation density and the minimum of the chemical potential of the Λ particle.

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For many years, investigations of Λ -nuclear potential have attracted much attention [1]. Data from emulsion experiments, (K^- , π^-) and (π^+ , K^+) reactions yield necessary information about Λ binding energies in an appreciable range of mass number A of nuclear core. Moreover, (π^+ , K^+) data provide us in many cases with a whole set of bound and first unbound energy levels. A quite satisfactory description of the spectra has been achieved by using a variety of models, from the simple Woods-Saxon potential introduced in a Schrödinger equation [2,3] to the relativistic mean field theory [4-6]. In addition, attempts have been made to calculate Λ -nucleus potential starting from a free Λ - N interaction [7,8]. In this work, we used as a framework for hypernuclear calculations the relativistic mean field approach. Here baryons do not interact with each other directly but through common meson fields. As a consequence, once the coupling constants are chosen all baryon-baryon interactions are determined. This unification capability becomes more important when the system contains several species of baryons, as in the hypernuclear case. The relativistic formalism proved already many times its ability to describe various phenomena in (hyper)nuclear structure and dynamics [9,10]. However, no calculation of hypernuclei so far has been performed within the Dirac approach using the scalar derivative coupling model of Zimanyi and Moszkowski (ZM) [11].

The study of properties of homogeneous baryonic matter [12], as well as calculations of finite nuclei [13] revealed that the ZM model has features pretty similar to the non-linear $\sigma - \omega$ model [14]. Consequently its predictions of hypernuclear spectra are not expected to differ so much from predictions of more conventional mean field approaches. On the other hand, it may still be useful to check the ZM model in the hypernuclear sector as it offers an interesting alternative in respect to the non-linear $\sigma - \omega$ models. A recent work on the relativistic SU(6) model of the meson-baryon couplings suggests that the derivative scalar coupling has its origin in the constituent-quark picture of the nucleons [15]. Its relatively small spin orbit splitting, with which problems may arise in the nuclear case, is rather an advantage in the hypernuclei. Moreover, with these calculations we would like to take the opportunity to discuss how much information is contained in the hypernuclear spectroscopic data concerning properties of the Λ chemical potential μ_Λ in nuclear matter. In the absence of other experimental data, fitting the hypernuclear spectra is an obligatory step towards the description of more complex baryonic systems. The predictions depend obviously on the variation of the Λ chemical potential versus the density of baryonic matter. It is thus interesting to study the limits imposed by the Λ -hypernuclear spectroscopy. In particular we may wonder if the data fix the position of the μ_Λ minimum.

The paper is organized as follows. Section 2 is devoted to a short recall of the ZM model, and results for the hypernuclear spectra are presented. Conclusions are drawn in Section 3.

II. MODEL AND RESULTS

The formalism here is based on the Lagrangian density of the following form:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_N [i\gamma_\mu \partial^\mu - M_N^* - g_{\omega N} \gamma_\mu \omega^\mu - \frac{1}{2} g_{\rho N} \gamma_\mu \vec{b}^\mu \cdot \vec{\tau} - e\gamma_\mu \frac{1}{2} (1 + \tau_3) A^\mu] \psi_N \\ & + \bar{\psi}_\Lambda [i\gamma_\mu \partial^\mu - M_\Lambda^* - g_{\omega \Lambda} \gamma_\mu \omega^\mu + \frac{f_{\omega \Lambda}}{2M_\Lambda} \sigma^{\mu\nu} \partial_\mu \omega_\nu] \psi_\Lambda \\ & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\sigma^2 \omega_\mu \omega^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + \frac{1}{2} m_\rho^2 \vec{b}_\mu \cdot \vec{b}^\mu - \frac{1}{4} \vec{G}_{\mu\nu} \cdot \vec{G}^{\mu\nu} - \frac{1}{4} H_{\mu\nu} H^{\mu\nu}, \end{aligned} \quad (1)$$

where

$$\begin{aligned} F_{\mu\nu} & \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \\ \vec{G}_{\mu\nu} & \equiv \partial_\mu \vec{b}_\nu - \partial_\nu \vec{b}_\mu, \\ H_{\mu\nu} & \equiv \partial_\mu A_\nu - \partial_\nu A_\mu. \end{aligned} \quad (2)$$

and the scalar effective mass M_i^* of the i -baryon ($i = N, \Lambda$) is defined as

$$M_i^* = M_i (1 - \frac{g_{\sigma i}}{M_i} \sigma)^{-1}.$$

The model describes hypernucleus as a system of Dirac spinors (nucleons (N) and Λ hyperon(s) (Λ)) interacting via meson fields (σ, ω, ρ). In Eq.(1), m_σ, m_ω and m_ρ are the meson rest masses, whereas ψ_i ($i = N, \Lambda$), σ, ω^μ and $\vec{b}^\mu$ (for the ρ meson) are the corresponding field operators and A^μ is the electromagnetic field. For simplicity, hereafter we omit any explicit reference to this field, although it has been taken into account in the calculations.

It is to be noted that the Lagrangian $\mathcal{L}$ also includes the $\omega\Lambda\Lambda$ Lorentz-tensor coupling term. This additional term appeared to be crucial for explanation of the vanishing spin orbit interaction empirically observed in Λ hypernuclei [16–18]. Although the ZM model itself gives probably sufficiently small spin orbit force, we added the $\omega\Lambda\Lambda$ anomalous coupling term into the Lagrangian $\mathcal{L}$ for completeness. We have used $f_{\omega\Lambda} = -g_{\omega\Lambda}$ according to the constituent quark model predictions.

The system of coupled field equations for baryons and meson fields is obtained from the Lagrangian (1) using standard variational techniques. In order to write down the Dirac equation for baryons one expands the baryon fields $\psi_i(x)$ on a complete set of stationary single-particle Dirac spinors $\{f_{i\alpha}(\vec{x})e^{-iE_{i\alpha}t}\}$. Here, we take the static limit of the meson fields and restrict ourselves to the mean field approximation [19]. The resulting Dirac equation in a spherical symmetric case can be written as

$$\left[-i\vec{\alpha} \cdot \vec{\nabla} + \beta M_N^* + g_{\omega N} \omega_0 + \frac{1}{2} g_{\rho N} b_{03} \tau_3 \right] \varphi_{N\alpha}(\vec{r}) = E_{N\alpha} \varphi_{N\alpha}(\vec{r}) \quad \text{for nucleon,} \quad (3)$$

and

$$\left[-i\vec{\alpha} \cdot \vec{\nabla} + \beta M_\Lambda^* + g_{\omega \Lambda} \omega_0 - \frac{f_{\omega \Lambda}}{2M_\Lambda} i\vec{\gamma} \cdot \vec{\nabla} \omega_0 \right] \varphi_{\Lambda\alpha}(\vec{r}) = E_{\Lambda\alpha} \varphi_{\Lambda\alpha}(\vec{r}) \quad \text{for } \Lambda \text{ hyperon.} \quad (4)$$

The Klein-Gordon equations for meson fields read

$$(\Delta - m_\sigma^2) \sigma = \sum_{i=N,\Lambda} g_{\sigma i} \frac{M_i^*}{M_i^2} \rho_{Si}, \quad (5)$$

$$(\Delta - m_\omega^2) \omega_0 = - \sum_{i=N,\Lambda} g_{\omega i} \rho_i - \frac{f_{\omega \Lambda}}{2M_\Lambda} \rho_T, \quad (6)$$

$$(\Delta - m_\rho^2) b_{03} = - \frac{1}{2} g_{\rho N} \rho_3, \quad (7)$$

where

$$\rho_{Si} = \sum_\alpha \bar{\varphi}_{i\alpha} \varphi_{i\alpha}, \quad (8)$$

$$\rho_i = \sum_\alpha \bar{\varphi}_{i\alpha} \gamma_0 \varphi_{i\alpha}, \quad (9)$$

$$\rho_T = \vec{\nabla} \cdot \sum_\alpha \varphi_{\Lambda\alpha} i\vec{\alpha} \varphi_{\Lambda\alpha}, \quad (10)$$

$$\rho_3 = \rho_p - \rho_n = \sum_\alpha \bar{\varphi}_{N\alpha} \gamma_0 \tau_3 \varphi_{N\alpha}. \quad (11)$$

and α in summations runs over occupied particle states.

The nucleon effective coupling constants g_{iN} have been determined by a fit to nuclear matter and finite nuclei properties [11]. The parametrization of nucleonic sector is presented in Table 1a.

As far as the hyperonic sector is concerned, the two coupling constants $g_{\sigma\Lambda}$ and $g_{\omega\Lambda}$ (note that $g_{\rho\Lambda} = 0$) have to be fixed. This requires imposing two additional physical constraints.

In a homogeneous system, the Λ binding energy B_Λ depends only on the baryonic density ρ_B . B_Λ is equivalent to the Λ chemical potential $\mu_\Lambda = \frac{\partial \varepsilon}{\partial \rho_\Lambda}$ in the limit $\rho_\Lambda \rightarrow 0$, ε being the baryonic matter energy density. Fixing the position and the value of the minimum of the chemical potential μ_Λ , namely ρ_B^0 and $\mu_\Lambda(\rho_B^0)$, determines the two coupling constants. It seems natural to start by setting the minimum of μ_Λ to -28 MeV. This value comes from phenomenological extrapolations of the Λ binding energies in hypernuclei [3,4,6]. For ρ_B^0 , the nuclear matter saturation density might serve as a first reasonable estimate. However, we instead set the minimum of μ_Λ at different values of ρ_B^0 , namely $\rho_B^0 = 0.08, 0.12, 0.16$ and 0.20 fm^{-3} . This allowed us to check how sensitive are the spectroscopic data to the position of $\mu_\Lambda(\rho_B^0)$. (Note that the lowest value, $\rho_B^0 = 0.08 \text{ fm}^{-3}$, corresponds to a Λ essentially bound at the nuclear surface.) The resulting sets of coupling constants, denoted ZM_i , $i=1,4$, are listed in Table 1b. The corresponding curves of μ_Λ as functions of ρ_B are displayed in Fig. 1. Note that in the ZM model with only two parameters, fixing the binding energy and the saturation density constrains the compressibility, which vary from 340 MeV at $\rho_B^0 = 0.08 \text{ fm}^{-3}$ to 290 MeV at $\rho_B^0 = 0.20 \text{ fm}^{-3}$.

The four sets of coupling constants have been used to calculate hypernuclear levels in ${}^A\Lambda O$ and ${}^A\Lambda Y$. The results are displayed in Figs. 2 and 3, and compared to experimental data. The position of the 1s level is quite sensitive to the saturation density, especially in the low density range. The 1s-1p splitting is, however, more

important, and an accurate determination of this splitting would be helpful in fixing the coupling constants. We have verified that the binding energy of the 1s state can always be fitted to its experimental value by increasing B_Λ , even at $\rho_B^0 = 0.08 \text{ fm}^{-3}$. Moreover, the 1s-1p energy difference remains about the same, almost independent on the 1s energy. The other states behave in a way similar to the 1s and 1p levels, with a lower saturation density dependence as ℓ increases. Note that the level density is increasing with compressibility, and is thus higher at $\rho_B^0 = 0.08 \text{ fm}^{-3}$. The level density is governed by the shape of the potential seen by the Λ , which is peaked in the nuclear surface for $\rho_B^0 = 0.08 \text{ fm}^{-3}$, and is consequently sharper around its minimum than the other cases which follow more or less the nuclear one-body density. In such a geometry, the states of low orbital angular momentum are pushed up.

The results also indicate that the Λ is mainly bound inside the nuclear volume. Saturation densities lower than the normal nuclear matter density yield too small 1s-1p splitting. This is in agreement with the earlier findings by Ruffa et al [4], and Mares and Zofka [6]. In particular, the coupling constants for $\rho_B^0 = 0.08 \text{ fm}^{-3}$ lie outside the range determined by their fits to spectroscopic data, for $\rho_B^0 = 0.12 \text{ fm}^{-3}$ they stand at the border line.

Up to this point, the results we have obtained provide us with a qualitative picture. In order to check the ZM model on a more quantitative level, we have searched for fits to experimental hypernuclear data. We did not attempt to explore the whole domain of coupling constants compatible with experimental data in a systematic way. Instead we selected two particular cases which yield a satisfactory agreement. They clearly illustrate the situation.

In order to obtain a reasonable fit to spectroscopic data [20], looking at the previous results we found necessary to increase slightly B_Λ at normal nuclear matter density and at the same time to push the minimum of μ_Λ towards higher densities. This is required in order to match the actual single particle energy spreading, particularly the 1s-1p splitting. Two values for the minimum of μ_Λ have been chosen: $\rho_B^0 = 0.24$ and 0.34 fm^{-3} . The coupling constants leading to the best fits and the values of $\mu_\Lambda(\rho_B)$ are displayed in Table 2. These two parametrizations are referred to ZM_5 and ZM_6 .

The hypernuclear spectra have been calculated for ^{17}O , ^{41}Ca and ^{90}Y . The results are displayed in Table 3 and compared to the experimental values. Both parametrizations yield reasonable fit to the data. They are of the same quality as other approaches [3-6]. For completeness we present predictions of the Λ spectra in ^{140}La and ^{208}Pb , as well. They have been measured recently at KEK [21] and the data are going to be published soon [22].

In order to get some more insight into the problem, we have plotted in Fig. 4 the Λ -chemical potential μ_Λ against ρ_B for the three sets of coupling constants corresponding to ZM_3 , ZM_5 and ZM_6 . This figure clearly shows that in order to deepen the 1s level while keeping the single particle spreading it is necessary to increase both ρ_B^0 and the value of μ_Λ at its minimum, which steepens the low density part of μ_Λ . This is directly linked to the fact that a two-parameter model, like ZM, generates a restricted functional space, which does not necessarily reproduce the actual density dependence of the chemical potential. In this respect, we would like to add a remark. Since the nuclear matter density barely overcomes $\rho_B = 0.17 \text{ fm}^{-3}$, only the low-density part of μ_Λ is relevant for hypernuclear spectra. The differences be-

tween the models are not so large in this region. On the contrary, the three curves of Fig. 4 show quite different patterns in the high density part. This indicates that predicting the properties of strange baryonic matter beyond the nuclear matter density could be hazardous at the present stage. It is to be stressed here that there are no theoretical arguments or experimental evidence for rejecting the above models as unrealistic on the ground that ρ_B^0 is much larger than the ordinary nuclear matter saturation density. Figure 4 shows that although the spectroscopic data do not allow us to determine with precision the minimum of μ_Λ , they fix its value to $\simeq -28 \text{ MeV}$ for $\rho_B \simeq 0.16 \text{ fm}^{-3}$.

Finally, Fig. 4 suggests that the minima of μ_Λ that fit the hypernuclear data lie roughly on a straight line when plotted as a function of ρ_B . This reminds us the linear relationship observed between the pairs of coupling constants leading to acceptable fits in previous works [4,6]. Indeed, the coupling constants of ZM_3 , ZM_5 and ZM_6 build also a straight line in the $(\alpha_s, \alpha_\omega)$ plane. This can be inferred from the expression of the baryonic matter energy density $\epsilon(\rho_B)$ [12], which is linear in $g_\omega \Lambda$ and $g_\omega \Lambda$ for $\rho_\Lambda \rightarrow 0$. Consequently, fixing (μ_Λ, ρ_B^0) is biunivocal to fixing $(\alpha_s, \alpha_\omega)$.

III. CONCLUSIONS

In this work we have applied the scalar derivative coupling model of Zimanyi and Moszkowski to the calculation of hypernuclear spectra. The strange sector is driven by two coupling constants. (We did not discuss the value of the tensor coupling constant $f_{\omega\Lambda}$ which affects the size of the spin orbit splitting.) Fixing more or less the binding energy of a single Λ in nuclear matter but varying the saturation density ρ_B^0 , i.e., the baryonic density at which the Λ chemical potential has its minimum, allowed us to study the sensitivity of the spectroscopic data to the saturation density. At a qualitative level, we found that the Λ is merely bound by the interior of the nuclear medium. The key quantity that should be measured with accuracy is the 1s-1p energy splitting. Such a conclusion could be inferred from the Bertmann-Martin inequalities [23], which have been found recently to bring useful information in the case of hypernuclei [24]. In the local, non-relativistic limit, the first inequality connects the radius of the 1s orbital the 1s-1p energy difference. It reads

$$< r^2 >_{1s} \leq \frac{3\hbar^2}{2m_\Lambda(E_{1p} - E_{1s})}. \quad (12)$$

Inserting the free mass of the Λ and the measured splitting, the rms radius of the 1s orbital is found to increase like $1.6A^{0.125}$, which is significantly slower than the rms radius of the nucleus.

The binding energy of the Λ in nuclear matter, around $\rho_B^0 = 0.16\text{--}0.17 \text{ fm}^{-3}$, is found between 29-30 MeV, i.e. in the same range as other models predictions. The increase of B_Λ at higher densities, which is necessary to account for both the 1s binding energy and the single particle spreading, may be misleading. It arises from specific restrictions imposed by the ZM model, which contains only two free parameters, on the functional space describing the density dependence of μ_Λ . Thus it is dangerous to draw firm conclusions about the properties of strange matter at higher densities on the basis of fits to hypernuclear spectra.

At a quantitative level, we have determined pairs of coupling constants reproducing hypernuclear data. We made calculations of the Λ spectra in ^{140}La and ^{208}Pb which were measured recently at KEK [21]. The data are going to be published soon [22]. The fit is not unique but it shows the level of accuracy which can be obtained within the ZM model.

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TABLES

Table 1a: The parametrization of the nucleonic sector. The masses are given in (MeV).

M_N	m_σ	m_ω	m_ρ	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$
938.	420.0	783.	763.	5.824	6.417	5.49

Table 1b: The coupling ratios used for hyperonic sector. ZM_i denotes the parametrizations for B_Λ minimum at $\rho_B^0=0.08, 0.12, 0.16$, and 0.20 fm^{-3} for $i=1,4$, respectively.

model	ZM_1	ZM_2	ZM_3	ZM_4
$\alpha_\sigma = g_{\sigma\Lambda}/g_{\sigma N}$	1.283	0.792	0.591	0.464
$\alpha_\omega = g_{\omega\Lambda}/g_{\omega N}$	2.0	1.1	0.75	0.54
$\alpha_T = f_{\omega\Lambda}/g_{\omega\Lambda}$	-1.0	-1.0	-1.0	-1.0

Table 2: Couplings constants and data of the parametrizations ZM_5 and ZM_6 .

	ρ_B^0	$g_{\sigma\Lambda}/g_\sigma$	$g_{\omega\Lambda}/g_\omega$	B_Λ at $\rho_B=0.16 \text{ fm}^{-3}$	μ_Λ at its minimum
ZM_5	0.24 fm^{-3}	0.417	0.44	28.6 MeV	30.8 MeV
ZM_6	0.34 fm^{-3}	0.34	0.3	28.9 MeV	35.1 MeV

Table 3: Λ single particle energies for ZM_5 and ZM_6 parametrizations. The experimental values [20] are presented for comparison.

		$1s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$	$1d_{5/2}$	$1d_{3/2}$	$2s_{1/2}$	$1f_{7/2}$	$1f_{5/2}$	$2p_{3/2}$	$2p_{1/2}$	$1g_{9/2}$
$^{17}_\Lambda\text{O}$	ZM_5	12.81	2.76	2.68								
	ZM_6	13.15	2.52	2.45								
$^{16}_\Lambda\text{O}$	exp.	13.	2.5									
$^{41}_\Lambda\text{Ca}$	ZM_5	18.69	10.38	10.33	2.16	2.01	2.08					
	ZM_6	19.19	10.46	10.40	1.92	1.92	1.84					
$^{40}_\Lambda\text{Ca}$	exp.	19.	11.									
$^{90}_\Lambda\text{Y}$	ZM_5	22.38	16.47	16.45	9.95	9.89	8.58	3.11	3.03	2.00	1.98	
	ZM_6	22.72	16.65	16.62	9.96	9.91	8.59	2.95	2.87	1.83	1.81	
$^{89}_\Lambda\text{Y}$	exp.	22.7	16.		9.9			2.9				
$^{140}_\Lambda\text{La}$	ZM_6	23.70	19.14	19.13	13.84	13.80	12.19	7.95	7.90	5.79	5.77	1.72
	ZM_6	24.85	21.06	21.05	16.67	16.65	15.39	11.75	11.71	9.76	9.74	6.42

FIGURE CAPTIONS

Fig.1: Chemical potential μ_Λ as function of ρ_B . The four curves correspond to the parametrizations ZM_1 - ZM_4 of Table 1b.

Fig.2: Single particle energies of the $1s$ and $1p$ levels of $^{17}_\Lambda\text{O}$ as function of the saturation density ρ_B^0 . The values correspond to the parametrizations ZM_1 - ZM_4 of table 1b. The solid lines indicate the experimental values.

Fig.3: Idem Fig. 2 for the $1s$, $1p$, $1d$ and $1f$ levels of $^{90}_\Lambda\text{Y}$.

Fig.4: Chemical potential μ_Λ as function of ρ_B for the three parametrizations ZM_3 (solid line), ZM_5 (dashed line) and ZM_6 (dot-dashed line). The straight line joining the minima of μ_Λ has been drawn to guide the eyes.

Fig. 1

Fig. 2

Fig. 3

Fig. 4