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Effective field theory description of Green and vertex functions in the infrared domain

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Abstract

This article discusses applications of a path-integral approach to fermionic Green and vertex functions which has a systematic expansion in terms of contours (particle “world lines”), thereby enabling the evaluation of all-order results in the coupling constant. Our approach provides an *effective* field theoretical framework for studying the low-energy regime of the underlying dynamics. In particular, the infrared behavior of the full fermion Green functions in QED can be analyzed without imposing assumptions on the asymptotic states of the charged sector, and keeping the γ -matrix structure of spinorial fields intact. Analytic solutions are derived for one and n fermions and the renormalization constants and associated anomalous dimensions are determined nonperturbatively; they comply with previously obtained results from exactly solvable models (e.g., Bloch-Nordsieck QED). Within an Abelian infrared setup, a universal vertex function has been calculated which embeds the soft interactions into a nontrivial contour with an infinitesimal self-intersecting loop. Adjusting the involved scales to those of the heavy quark effective theory, the result emulates the Isgur-Wise form factor and complies with the recent CLEO II data.

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I. INTRODUCTION

In a companion paper by two of us [1], a novel approach to the description of fermionic systems was presented with particular attention being focused to the derivation of effective actions which include topological terms in two, three, and four spacetime dimensions. This approach is characterized by three major conceptual features: (i) The conversion of fermionic *field* systems in d -dimensional Euclidean spacetime into a *particle-based* path-integral description in the sense of Polyakov [2]. (ii) The incorporation of a *built-in* resolution scale in the initial casting of the field system. (iii) The possibility of using the paths' *geometry* rather than the expansion in powers of the coupling constant as a guiding principle to identify the leading contributions. Expanding in terms of contours makes it possible to obtain all-order expressions in the coupling constant, emphasizing the nonperturbative character of the approach.

The emerging mathematical structure of the underlying space is $\mathbb{R} \otimes \mathbb{R}^d$, where $\mathbb{R}^d$ represents Euclidean spacetime and $\mathbb{R}$ accomodates a “proper time” parameter τ , which serves to continuously describe dynamical development of the system along a particle “world line”. An important advantage of this treatment is that the spacetime resolution scale, which serves to define quantities like mass and spin, and the field theoretical cutoff, which controls the high-frequency fluctuations of the spin-1/2 fields, are clearly separated. Furthermore, since the regularized casting of the field system takes place within a *continuous* spacetime background – meaning that the resolution scale can be sent to zero at every stage of the calculations – typical problems of lattice formulations, like fermion doubling, etc., are not encountered.

It appears attractive to extend this method to the calculation of Green and vertex functions in a region of momenta, where perturbative techniques are not reliable. First results in this direction have been reported in earlier works [3,4]. It is our goal in this paper to provide further understanding of the method, emphasizing, in particular, its physical sig-

nificance for phenomenological applications. Such a program requires a series of theoretical concepts and technical steps concerning the *geometric simulation* of soft interactions and considerable effort will be devoted towards this end. An introduction to the basic ideas and directions of this program has been given in [5].

There has recently been a resurgence of interest in the Bloch-Nordsieck (BN) model [6] in connection with the heavy quark effective theory (HQET). [For a recent review, see, eg., [7].] As we have discussed in [3], it is possible to relate the one-fermion Green function of the BN model [8] to that calculated within our scheme by selecting in the particle path integral the IR-dominant contours. In both approaches, the effect of the soft photons is governed by the same anomalous dimension $\gamma_F(\alpha, \lambda) = -(\alpha/2\pi)(3 - \lambda)$, (where λ is the gauge parameter and $\alpha = e^2/4\pi$ with $\hbar = c = 1$), which smears the pole structure of the fermion propagator and gives rise to the “infraparticle” phenomenon [9].

In a subsequent publication [4], we have devised a three-point function and evaluated the particle path integral along a contour with an infinitesimal self-intersecting loop. The latter serves to mimic the overlap of soft degrees of freedom of heavy mesons undergoing a weak transition. The upshot of this calculation is a *universal* function describing the dynamical rearrangement of the configuration of the light degrees of freedom of an infraparticle which suddenly changes its four-velocity. Since that function depends only on the invariant four-velocity change and is independent of the infraparticle mass, it bears physical characteristics akin to the Isgur-Wise form factor in HQET [10].

It is the intent of the present publication to outline these results and to show that our geometrical analysis provides a systematic method for describing the low-energy region of gauge theories in a nonperturbative way. The method is to be understood as “effective”, i.e., its dynamical properties are as though soft degrees of freedom were being exchanged. While Feynman diagrams are by far the predominant tool in perturbative calculations, they are of little help in describing nonperturbative phenomena because, actually, one has to deal with an infinite number of them. Hence we believe that our scheme, which enables the evaluation

of the path integral without resort to series expansion in the coupling constant, may lead to further physical insight – especially for measurable quantities.

The focus of our attention will be on the computation and multiplicative renormalization of fermionic Green functions (propagators) and a properly constructed vertex function (form factor) pertinent to the weak decay of one heavy meson into another in the momentum range in which our effective framework is applicable. A key feature of these considerations concerns the proper incorporation into the effective action of gauge-field fluctuations which have to be computed according to our specifications of the IR regime. Although the use of nontrivial contours in the evaluation of the path integral complicates the calculation, our results show that the renormalizability of the theory is not prohibited. Of particular importance in this respect is the employment of a point-splitting regularization technique, which replaces pointlike “world lines” by ribbons [11], thus nicely fitting into our particle-based scheme.

The remainder of the paper is organized as follows. In Sec. II we study the full one-fermion Green function of QED in the context of our particle-based representation. This section provides the background for Sec. III in which the IR structure of the one-fermion Green function is determined. Section III is quite central for the later developments. Because of the explicit spacetime dependence of the particle “world lines”, the calculations are carried out in configuration space and ultraviolet (UV) divergences emerging from the gauge sector are treated within the ribbon regularization. We describe in Sec. IV how our IR considerations can be generalized to many-fermion Green functions. In Sec. V we discuss the calculation of a three-point function (vertex function) in terms of four-velocity states. It seems comforting that inclusion of a contour with an infinitesimal self-intersecting loop can mimic within our scheme the overlap of the light degrees of freedom in the weak transition of a heavy meson. Indeed, in Sec. VI we will be able to make a concrete connection with the results obtained within the HQET, by *de facto* identifying the calculated vertex function with the Isgur-Wise form factor. In this section we will also show that our theoretical calculation is sufficient to reproduce the data on semileptonic heavy meson decays of several

collaborations. Concluding comments are formulated in Sec. VII, where also a summary of our results is given. Important technical issues are carried out in three appendices.

II. PARTICLE PATH-INTEGRAL REPRESENTATION OF THE FERMION PROPAGATOR IN QED

We will consider a field theoretical system describing fermions coupled to an external Abelian gauge field within d -dimensional Euclidean spacetime. The corresponding action with a built-in resolution scale (spacetime cutoff) α has the generic form

$$S^D = - \sum_{\mathbf{n}=-\infty}^{\infty} \alpha^d \bar{\psi}(n\alpha) (\not{D}^D + m^D) \psi(n\alpha) . \quad (1)$$

The particular scheme adopted in Ref. [1] is specified by

$$(\not{D}^D + m^D) \psi(n\alpha) = - \int d^d y [(\not{\partial} - m) f(|y|)] U(L_{n\alpha, n\alpha+y}) \psi(n\alpha + y) , \quad (2)$$

where m is a bare mass parameter and $f(|y|)$ is a function of fast decrease dropping practically to zero within distances of order α , i.e., $f(|y|) \xrightarrow{\alpha \rightarrow 0} \delta(y)$, which is, e.g., satisfied by a Gaussian distribution. The function $f(|y|)$ provides a weight that relates the discretized copies of the field system along the open contour $L_{x, x+y}$ which connects the points x and $x + y$ in $\mathbb{R}^d$. Gauge invariance is manifest due to the non-integrable phase factor

$$U(L_{x, x+y}) \equiv \mathcal{P} \exp \left[-ig \int_{L_{x, x+y}} A_\mu(z) dz_\mu \right] . \quad (3)$$

We assume that over the range determined by $f(|y|)$ the contour $L_{x, x+y}$ is simple (i.e., non-cusped and/or self-intersecting) and smooth (i.e., differentiable). Although ultimately the gauge field will be quantized, the presence of a Maxwell term in the action will be postponed for a while.

To obtain the full one-fermion Green function, we introduce spinorial sources η and $\bar{\eta}$ and consider an appropriately defined generating functional

$$Z^D[\bar{\eta}, \eta|A] = \int \prod_n [d\bar{\psi}(n\alpha)] [d\psi(n\alpha)] \exp \left\{ \alpha^d \sum_n [\bar{\psi} (\not{D}^D + m^D) \psi + \bar{\eta} \psi + \bar{\psi} \eta] \right\} , \quad (4)$$

where the arguments of sources and fields are suppressed for brevity. The well-defined Graßmann integrations entering (4) can be carried out with the result

$$Z^D[\bar{\eta}, \eta|A] = \det (\not{D}^D + m^D) \exp \left\{ -\alpha^d \sum_n [\bar{\eta} (\not{D}^D + m^D)^{-1} \eta] \right\} . \quad (5)$$

Functional differentiation with respect to the sources η and $\bar{\eta}$ and functional integration over gauge field configurations yields

$$G(x, y) = \int [dA_\mu(x)] \exp \left\{ -\frac{1}{2} \int d^d z A_\mu(z) \mathcal{D}_{\mu\nu}^{-1}(\lambda) A_\nu(z) \right\} \det [G_D^{-1}(x, y|A)] G^D(x, y|A) , \quad (6)$$

where a comprehensive continuous spacetime notation has been used and a covariant gauge restriction has been imposed on the integration over A_μ , denoted by the gauge parameter λ . Accordingly, $\mathcal{D}_{\mu\nu}(\lambda)$ represents the configuration-space gauge-field propagator in an arbitrary covariant gauge. The Green function $G^D(x, y|A)$ describes the first-quantized motion of spinor particles in the presence of an external gauge potential A_μ and is actually defined in discretized form by

$$G^D(n\alpha, m\alpha|A) = \left\langle n\alpha \left| (\not{D}^D + m^D)^{-1} \right| m\alpha \right\rangle . \quad (7)$$

The discrete set of states $\{|n\alpha\rangle\}$ employed in (7) obeys completeness and orthogonality conditions given by

$$\sum_n \alpha^d |n\alpha\rangle \langle n\alpha| = 1, \quad \langle n\alpha|m\alpha\rangle = \frac{\delta_{mn}}{\alpha^d} . \quad (8)$$

Using Schwinger's "proper-time" formalism [12], the matrix element defined by (7) can be expressed in the form

$$\left\langle n\alpha \left| (\not{D}^D + m^D)^{-1} \right| m\alpha \right\rangle = \lim_{\epsilon \rightarrow 0} \int_\epsilon^\infty dT \left\langle n\alpha \left| e^{-(\not{D}^D + m^D)T} \right| m\alpha \right\rangle . \quad (9)$$

Following the procedure described in Ref. [1], (9) can be converted into a path-integral expression of the form

$$G(x, y|A) = \lim_{c \rightarrow 0} \int_c^\infty dT \int_{\substack{x(0)=x \\ x(T)=y}} [dx(\tau)] [dp(\tau)] \exp \left[i \int_0^T d\tau p(\tau) \cdot \dot{x}(\tau) \right] \\ \times \mathcal{P} \exp \left\{ - \int_0^T d\tau [i\gamma \cdot p(\tau) + m] \right\} \exp \left\{ ig \int_0^T d\tau \dot{x}(\tau) \cdot A[x(\tau)] \right\} . \quad (10)$$

Here T is a parameter associated with the length of the path and c serves as an UV cutoff for matter field fluctuations. For ease of notation, we shall henceforth suppress the $\lim_{c \rightarrow 0}$. The above expression explicitly displays the particle-based mode of description that pervades our approach to fermionic systems. It is very similar to a corresponding expression obtained in Ref. [8] within the Bloch-Nordsieck (BN) model of QED, except that here the spin degrees of freedom are well represented, since the γ -structure of the fermions is completely retained. The particular utility of such a description, which is based on a phase-space path integral, will become more apparent below.

The full one-fermion Green function is furnished by the path-integral expression

$$G(x, y) = \int_c^\infty dT \int_{\substack{x(0)=x \\ x(T)=y}} [dx(\tau)] [dp(\tau)] \exp \left[i \int_0^T d\tau p(\tau) \cdot \dot{x}(\tau) \right] \\ \times \mathcal{P} \exp \left\{ - \int_0^T d\tau [i\gamma \cdot p(\tau) + m] \right\} \left\langle \exp \left\{ ig \int_0^T d\tau \dot{x}(\tau) \cdot A[x(\tau)] \right\} \right\rangle_A , \quad (11)$$

where

$$\left\langle \exp \left\{ ig \int_0^T d\tau \dot{x}(\tau) \cdot A[x(\tau)] \right\} \right\rangle_A = \int [dA_\mu(x)] \det [G^{-1}(x, y|A)] \\ \times \exp \left\{ ig \int_0^T d\tau \dot{x}(\tau) \cdot A[x(\tau)] \right\} \\ \times \exp \left\{ -(1/2) \int d^d z A_\mu(z) \mathcal{D}_{\mu\nu}^{-1}(\lambda) A_\nu(z) \right\} . \quad (12)$$

Let us now check the consistency of the above result by verifying its validity in certain well-known cases. Suppose, for example, that the gauge field is turned off. Setting $\langle \rangle_A = 1$ in (11), we determine by rearranging the term $\int_0^T d\tau p(\tau) \cdot \dot{x}(\tau)$ via partial integration

$$\int_{\substack{x(0)=x \\ x(T)=y}} [dx(\tau)] [dp(\tau)] \exp \left[i \int_0^T d\tau p(\tau) \cdot \dot{x}(\tau) \right] = e^{ip(T)y - ip(0)x} \delta[\dot{p}(\tau)] . \quad (13)$$

From this result we infer that propagation of the spin-1/2 particle occurs with “proper time”-independent, i.e., constant, momentum. Then the functional integration over $[dp(\tau)]$ becomes trivial and leads to

$$G(x, y) = \int \frac{d^d p}{(2\pi)^d} e^{-ip \cdot (x-y)} \int_c^\infty dT e^{-T(i\gamma \cdot p + m)} . \quad (14)$$

We immediately recognize the free (Euclidean) Dirac propagator in parametric representation. This result can be continued back to Minkowski space by standard means, so that one readily recovers the Feynman propagator $G_F = (\not{p} - m + i\epsilon)^{-1}$.

Consider next the situation that arises by making the replacement $\gamma_\mu \rightarrow u_\mu$ in (11), where u_μ is some fixed four-velocity vector. This approximation constitutes the essence of the well-known BN model which aims at an effective nonperturbative description of QED in the long-distance region. Enforcement of the BN approximation expedites the computation of the momentum functional integral to yield

$$\int [dp(\tau)] \exp \left\{ i \int d\tau p(\tau) \cdot [u - \dot{x}(\tau)] \right\} = \delta[u - \dot{x}(\tau)] . \quad (15)$$

This result reflects the characteristic BN property according to which propagation of the matter field quanta occurs asymptotically (in the long-distance) limit along straight-line paths with constant four-velocity. Proceeding along this line of arguments, we finally arrive at a fermion propagator which near the mass shell behaves like

$$G(p) \sim (p^2 - m^2 + i\epsilon)^{-(1+a)} , \quad (16)$$

rather than having a simple pole (see, e.g., Ref. [8] and earlier references cited therein). The soft-photon cloud encoded in $e^{-a \ln |p^2 - m^2|}$ and governed by the IR anomalous dimension $\gamma_F(\alpha, \lambda) = -a = -(\alpha/2\pi)(3 - \lambda)$ [8] turns the charged particle pole into a logarithmic cut starting at $p^2 = m^2$.

According to (15), four-velocity constitutes a “good” quantum number in the BN “effective field theory” description of QED and scattering processes can be naturally characterized

by the particle's momentum $p^\mu = mv^\mu$ – amounting to a velocity superselection rule [13,14]. Correspondingly, charged states defined this way naturally incorporate the BN coherence factor $p^\mu/(p \cdot k)$. [A detailed study of these issues within a gauge-invariant approach has been given in Ref. [15].]

The BN removal of the γ -matrices in favor of the components of a four-velocity vector is tantamount to suppressing the spin attribute of the propagating particle; the spin components decouple from each other. It is a particular feature of the present approach that such a simplification must *not* be introduced and this marks an important difference from usual formulations. Indeed, (11) makes it apparent that the full spin-1/2 entity is always kept intact. The long-distance structure of the one-fermion Green function will be extracted from (11) by evaluating the particle path integral along the IR-dominant contours. These considerations will become more transparent in the next section.

III. INFRARED STRUCTURE OF THE FERMION PROPAGATOR IN QED

In this section we investigate the IR structure of the fermion propagator in QED on the basis of the representation for the full Green function given in (11). Previous approaches to this problem have placed primary focus on formulating asymptotic versions of the theory [16,17]. In our approach, the characterization of the IR domain of QED involves two requirements [3]: The first one refers to gauge field fluctuations. We adopt the quenched approximation according to which vacuum polarization effects are ignored. Physically, this means that the feedback of matter fields on the gauge fields is neglected, corresponding to a point-like charged entity propagating in an environment without violent field effects. This implies that there is no pair creation and hence the fermion determinant is set equal to 1. That is to say, the photon propagator of our *low-energy effective theory* is reasonably represented by the free one. This is considered justifiable for this kinematic domain since the active modes of the gauge field have energies much lower than the electron mass m . The sec-

ond requirement refers to the determination of the dominant contours entering the particle path integral in (11). Asymptotically, the charged fermion is almost on-shell and its momentum fluctuates around the mass-shell by an amount of order $k_0 \rightarrow 0$. The corresponding change in its four-velocity vanishes as $k_0/m \rightarrow 0$ so that the fermion acquires a “heaviness” which makes it reasonable to assume that its motion is associated with reasonably smooth contours, meaning that if $x(\tau) \approx x(\tau')$, then $\tau \approx \tau'$, as illustrated in Fig. 1.

We are now in a position to proceed with our concrete calculations. To this end, let us first determine the contribution from the gauge field sector, entering the calculation of $\left\langle \exp \left\{ ig \int_0^T d\tau \dot{x}_\mu(\tau) A_\mu[x(\tau)] \right\} \right\rangle_A$. The evaluation of this expectation value is eventually encumbered by UV divergences coming from the high-frequency gauge field fluctuations. The calculational remedy we use for handling them employs a point-splitting technique (“ribbon regularization”) [11] which turns out to be particularly suitable for our purposes. Utilizing our first IR requirement, we write

$$\left\langle \exp \left\{ ig \int_0^T d\tau \dot{x}_\mu(\tau) A_\mu[x(\tau)] \right\} \right\rangle_A = \exp \left\{ -g^2 \int_0^T d\tau \int_\tau^T d\tau' \dot{x}_\mu(\tau) \dot{x}_\nu(\tau') \times \langle 0 | A_\mu[x(\tau)] A_\nu[x(\tau')] | 0 \rangle \right\}. \quad (17)$$

Introducing the Fourier representation according to

$$A_\mu[x(\tau)] = \int \frac{d^d k}{(2\pi)^d} e^{-ik \cdot x(\tau)} A_\mu(k), \quad (18)$$

the vacuum expectation value in (17) can be covariantly decomposed to read

$$\langle 0 | A_\mu[x(\tau)] A_\nu[x(\tau')] | 0 \rangle = A[x(\tau), x(\tau')] \delta_{\mu\nu} - (1 - \lambda) B_{\mu\nu}[x(\tau), x(\tau')], \quad (19)$$

where for $d = 4$

$$\begin{aligned} A[x(\tau), x(\tau')] &= \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} e^{ik \cdot [x(\tau') - x(\tau)]} \\ &= \frac{1}{4\pi^2} \frac{1}{|x(\tau') - x(\tau)|^2} \end{aligned} \quad (20)$$

and

$$\begin{aligned}
B_{\mu\nu}[x(\tau), x(\tau')] &= \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu k_\nu}{k^4} e^{ik \cdot [x(\tau') - x(\tau)]} \\
&= \frac{1}{8\pi^2} \frac{\delta_{\mu\nu}}{|x(\tau') - x(\tau)|^2} - \frac{1}{4\pi^2} \frac{[x_\mu(\tau') - x_\mu(\tau)][x_\nu(\tau') - x_\nu(\tau)]}{|x(\tau') - x(\tau)|^4}. \quad (21)
\end{aligned}$$

It becomes obvious from (20) and (21) that the remaining integrals contain short-distance infinities owing to their denominators, which become singular when the contours $x(\tau)$ and $x(\tau')$ approach each other. To render the integrals finite, we employ the “ribbon regularization” technique which naturally complies with the smoothness requirement to be imposed on contours in the IR region. To be specific, consider two distinct contours, parametrized by $x(\tau)$ and $x(\tau')$, which run parallel to each other, and imagine that they comprise corresponding edges of an untwisted and unwrinkled ribbon of infinitesimal width. Then the dominant contribution to the particle path integral stems from contiguous points on opposite edges of the ribbon. Thus the width of the ribbon splits the coinciding points rendering the integrals regular. Here the requirement of smooth contours is instrumental in helping us envision a stretched-out ribbon which does not involve topological obstructions. This is strictly an IR feature and reflects the quasi-free propagation naturally emerging within our setup.

To achieve a uniform parametrization of contours along the ribbon, we make the replacement

$$x_\mu(\tau) \rightarrow x_\mu(\tau) + b n_\mu(\tau), \quad (22)$$

where $n_\mu(\tau)$ is a unit vector normal to the contour direction and lying on the ribbon’s surface, whereas b represents the width of the ribbon. Substituting (22) into (20) and (21), we obtain the following regularized expressions:

$$A^{\text{reg}} = \frac{1}{4\pi^2} \frac{1}{|x(\tau') - x(\tau) - b n(\tau)|^2} \quad (23)$$

and

$$B_{\mu\nu}^{\text{reg}} = \frac{1}{8\pi^2} \frac{\delta_{\mu\nu}}{|x(\tau') - x(\tau) - b n(\tau)|^2}$$

$$- \frac{1}{4\pi^2} \frac{[x_\mu(\tau') - x_\mu(\tau) - b n_\mu(\tau)][x_\nu(\tau') - x_\nu(\tau) - b n_\nu(\tau)]}{|x(\tau') - x(\tau) - b n(\tau)|^4} . \quad (24)$$

Note that τ' serves to denote the independent propagation along one of the ribbon edges with respect to the other; it does not, however, amount to a different parametrization of the contour.

We encounter now the evaluation of the integrals

$$I_1^{(b)} = g^2 \int_0^T d\tau \int_\tau^T d\tau' (\dot{x}(\tau) \cdot \dot{x}(\tau')) A^{\text{reg}}[x(\tau), x(\tau')] \quad (25)$$

and

$$I_2^{(b)} = g^2(1 - \lambda) \int_0^T d\tau \int_0^\tau d\tau' \dot{x}_\mu(\tau) \dot{x}_\nu(\tau') B_{\mu\nu}^{\text{reg}}[x(\tau), x(\tau')] . \quad (26)$$

Because the main contribution results for $x(\tau) \approx x(\tau')$, i.e., from oppositely placed points on the ribbon, we employ a Taylor expansion to write

$$x_\mu(\tau') \simeq x_\mu(\tau) + (\tau' - \tau) \dot{x}_\mu(\tau) . \quad (27)$$

Recalling that $|\dot{x}| = 1$, expressions (25) and (26) become for $T \gg b$

$$\begin{aligned} I_1^{(b)} &\simeq \frac{g^2}{4\pi^2} \int_0^T d\tau \int_\tau^T d\tau' \frac{1}{(\tau' - \tau)^2 + b^2} \\ &= \frac{g^2}{4\pi^2} \left(\frac{\pi}{2} \frac{T}{b} - \ln \frac{T}{b} + \text{finite terms} \right) \end{aligned} \quad (28)$$

and

$$\begin{aligned} I_2^{(b)} &\simeq \frac{g^2}{8\pi^2} (1 - \lambda) \left\{ \int_0^T d\tau \int_\tau^T d\tau' \frac{1}{(\tau' - \tau)^2 + b^2} - 2 \int_0^T d\tau \int_\tau^T d\tau' \frac{(\tau' - \tau)^2}{[(\tau' - \tau)^2 + b^2]^2} \right\} \\ &= \frac{g^2}{8\pi^2} (1 - \lambda) \left[\left(\frac{\pi}{2} \frac{T}{b} - \ln \frac{T}{b} + \text{finite terms} \right) - \left(\frac{\pi}{2} \frac{T}{b} - 2 \ln \frac{T}{b} + \text{finite terms} \right) \right] , \end{aligned} \quad (29)$$

respectively. To leading order in b , the final result for the second term is

$$I_2^{(b)} = \frac{g^2}{8\pi^2} (1 - \lambda) \ln \frac{T}{b} . \quad (30)$$

Collecting together (28) and (30), the final result for the gauge-field factor is

$$\left\langle \exp \left\{ ig \int_0^T d\tau \dot{x}_\mu(\tau) A_\mu[x(\tau)] \right\} \right\rangle_A^{\text{reg}} = \exp \left(-\alpha \frac{T}{2b} + a \ln \frac{T}{b} + \text{finite terms} \right) . \quad (31)$$

Similar results have been obtained in calculations of the expectation value of a Wilson loop operator in a pure (non-Abelian) gauge system [18–20]. Two notable differences should, however, be recorded: First, (31) applies to open, rather than to closed contours – a natural consequence of the fact that we are focusing on Green functions. Second, we are *not* dealing with infinitely massive matter field quanta “propagating” along Wilson lines. The only requirement we have imposed is that fermions are “heavy” enough, as outlined above, so that our IR suppositions are applicable. We reiterate that this treatment goes beyond standard semiclassical approaches because fermions keep their full spin-1/2 character through the calculations [cf. (11)] and because the particle path integral sweeps through the whole perturbation series.

Introducing a bare-mass dependence on the width of the ribbon, the physical mass m_r is read off the relation

$$m_r \stackrel{b \rightarrow 0}{=} m(b) + \frac{\alpha}{2b} . \quad (32)$$

Accordingly, the renormalized version of (11) should contain the factor $e^{-m_r T}$ instead of e^{-mT} . In the real world, virtual particle creation which requires the inclusion of internal fermion loops – removed by the quenched approximation – can shift the particle mass with the consequence that (32) should be replaced by a more generalized relation of the form $m_r \stackrel{b \rightarrow 0}{=} (1/b) f[\alpha(b)]$, where $f[\alpha(b)]$ is a regularization-scheme–dependent function of the coupling constant. Of course, the implication is that $m(b) \stackrel{b \rightarrow 0}{\simeq} 1/b$.

The logarithmic term entering the exponent in (31) furnishes a wave function renormalization constant Z_2 , which gives rise to an anomalous dimension γ_F for the fermion field [8]. To bring it out explicitly, we introduce a renormalization mass scale μ and write

$$\ln \frac{T}{b} = \ln(T\mu) - \ln(b\mu) , \quad (33)$$

from which we deduce

$$Z_2 = e^{-a \ln(\mu b)} . \quad (34)$$

Gathering all our findings together, we determine the following expression for the full, renormalized one-fermion Green function in the IR regime:

$$\begin{aligned} G_{\text{ren}}(x, y) \sim \mu^a \int_{0+}^{\infty} dT T^a e^{-m_r T} \int_{\substack{x(0)=x \\ x(T)=y}} [dx(\tau)][dp(\tau)] \exp \left[i \int_0^T d\tau p(\tau) \cdot \dot{x}(\tau) \right] \\ \times \mathcal{P} \exp \left\{ -i \int_0^T d\tau [m + i \not{p}(\tau)] \right\} . \end{aligned} \quad (35)$$

In Appendix A we connect to our previous [3] calculation, obtained within dimensional regularization. In the same appendix, we also establish the interchangeability between point-splitting and dimensional regularization techniques.

The important content of (35) is that the path integral over $[dx(\tau)]$ has been reduced to a form identical to that entering free-particle propagation, [cf. (13)]. Proceeding in exactly the same way as before, we arrive at the following result

$$\begin{aligned} G_{\text{ren}}(x, y) \sim \mu^a \int_{0+}^{\infty} dT T^a e^{-m_r T} \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} e^{-ip \cdot \gamma} \\ = \mu^a \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \frac{\Gamma(1+a)}{(i \not{p} + m_r)^{1+a}} , \end{aligned} \quad (36)$$

which can be analytically continued to Minkowski space. This nonperturbative result expresses the fact that under our IR constraints the propagating matter field excitation is assigned a (gauge-dependent) *fractal* dimension $1+a$ in Polyakov's language [2]. This quantity, which naturally translates into an IR anomalous dimension

$$\gamma_F = \frac{\mu}{Z_2} \frac{\partial Z_2}{\partial \mu} = -a \quad (37)$$

for the charged particle [8], endows the fermion with an infraparticle-structure by smearing the asymptotic form of its propagator [9].

Closing this section, we would like to offer the following perspective on the preceeding analysis, the full significance of which will become more transparent through an application we will consider in Sec. V. Without invoking an asymptotic version of QED, we have been

able to determine its IR content at the Green function level, by carrying out the complete renormalization program. In essence, we have arrived at an *effective* field theory description of QED, applicable to an energy scale much smaller than the mass of the electron. The emergence of such an effective description from a given, more fundamental, field theory, as one moves down the energy scale, entails the following two occurrences. On one hand, there is a reduction of “active” degrees of freedom and, on the other hand, there is a clear factorization, i.e., separation of short- and long-distance effects [14]. In the present case, the smoothness condition on the contours in the path integral, serving as “world lines” for the fermions, amounts to vesting them with a “heaviness” property. In an implicit manner we have given up their association with *bona fide* quantum field degrees of freedom in favor of particle states labeled by four-velocity, states which more appropriately characterize “heavy” particles [14]. Short-distance effects have, at the same time, been accumulated into a fractal dimension associated with the wave function renormalization constant Z_2 .

Our results are *all-order* expressions in the coupling constant and conform in the IR domain, in which the quenched approximation works well, to those obtained for exactly solvable models (like the Bloch-Nordsieck model) via calculations done without resort to series expansion [8]. The theoretical link between the present geometrical approach and the operator product expansion, underlying renormalization group treatments, is a challenging subject under current investigation.

IV. THE n-FERMION GREEN FUNCTION IN THE INFRARED REGION

In this section we pass on to the more general problem of determining the IR structure of the n-fermion Green function $G^{(n)}(x_1, \dots, x_n, y_1, \dots, y_n)$. This function is obtained from (5) by differentiating with respect to $\prod_{i=1}^n \bar{\eta}^i(x_i)\eta^i(y_i)$ and subsequently setting the sources equal to zero. Invoking our IR requirements, the n-fermion Green function is succinctly represented by

$$\begin{aligned}
G^{(n)}(\underline{x}, \underline{y}) = & \left(\prod_{i=1}^n \int_c^\infty dT_i \right) \int_{\substack{x^i(0)=x^i \\ x^i(T_i)=y^i}} \prod_{i=1}^n [dx^i(\tau)] \int \prod_{i=1}^n [dp^i(\tau)] \prod_{i=1}^n \exp \left[i \int_0^{T_i} d\tau p^i(\tau) \cdot \dot{x}^i(\tau) \right] \\
& \times \prod_{i=1}^n \mathcal{P} \exp \left\{ - \int_0^{T_i} d\tau \left[i\gamma \cdot p^i(\tau) + m^i \right] \right\} \\
& \times \left\langle \prod_{i=1}^n \exp \left\{ ig_i \int_0^{T_i} d\tau \dot{x}^i(\tau) \cdot A[x^i(\tau)] \right\} \right\rangle_A
\end{aligned} \tag{38}$$

with $\underline{x} = (x_1, \dots, x_n)$, $\underline{y} = (y_1, \dots, y_n)$ and where

$$\begin{aligned}
\left\langle \prod_{i=1}^n \exp \left\{ ig_i \int_0^{T_i} d\tau \dot{x}^i(\tau) \cdot A[x^i(\tau)] \right\} \right\rangle_A \approx \\
\prod_{i=1}^n \exp \left\{ -g_i^2 \int_0^{T_i} d\tau \int_\tau^{T_i} d\tau' \dot{x}_\mu^i(\tau) \dot{x}_\nu^i(\tau') \langle 0 | A_\mu[x^i(\tau)] A_\nu[x^i(\tau')] | 0 \rangle \right\} \\
\prod_{i \neq j} \exp \left\{ -g_i g_j \int_0^{T_i} d\tau \int_0^{T_j} d\tau' \dot{x}_\mu^i(\tau) \dot{x}_\nu^j(\tau') \langle 0 | A_\mu[x^i(\tau)] A_\nu[x^j(\tau')] | 0 \rangle \right\}.
\end{aligned} \tag{39}$$

We shall refer to the first factor in (39) as the “diagonal” and to the second one as the “non-diagonal” particle term, denoted by E_{diag} and $E_{\text{non-diag}}$, respectively. In the foregoing expressions, τ and τ' are ordering parameters along the same contour in E_{diag} , whereas in $E_{\text{non-diag}}$ they parametrize distinct contours. Thus, in the first case, a ribbon regularization is anticipated to avoid singularities appearing when τ and τ' coincide. The computation is a direct descendant of that performed in the previous section for one fermion. We obtain

$$\begin{aligned}
E_{\text{diag}} = & \prod_{i=1}^n \exp \left\{ -g_i^2 \frac{1}{8\pi^2} \int_0^{T_i} d\tau \int_\tau^{T_i} d\tau' \dot{x}_\mu^i(\tau) \dot{x}_\nu^i(\tau') \right. \\
& \times \left[\frac{(1+\lambda)\delta_{\mu\nu}}{|\dot{x}^i(\tau') - \dot{x}^i(\tau)|^2} + \frac{(1-\lambda)[x_\mu^i(\tau') - x_\mu^i(\tau)][x_\nu^i(\tau') - x_\nu^i(\tau)]}{|\dot{x}^i(\tau') - \dot{x}^i(\tau)|^4} \right] \left. \right\},
\end{aligned} \tag{40}$$

which, by virtue of the ribbon regularization, takes the form

$$E_{\text{diag}}^{\text{reg}} = \prod_i \exp \left(-\alpha_i \frac{T_i}{2b} + a_i \ln \frac{T_i}{b} + \text{finite terms} \right) \tag{41}$$

with $\alpha_i = g_i^2/(4\pi)$ and $a_i = (\alpha_i/2\pi)(3-\lambda)$.

The non-diagonal term requires a somewhat different treatment. Here we encounter expressions of the form

$$E_{\text{non-diag}}^{ij} = \exp \left\{ -g_i g_j \frac{1}{8\pi^2} \int_0^{T_i} d\tau \int_0^{T_j} d\tau' \dot{x}_\mu^i(\tau) \dot{x}_\nu^j(\tau') \left[\frac{2\delta_{\mu\nu}}{|x^j(\tau') - x^i(\tau)|^2} - (1-\lambda) \left(\frac{\delta_{\mu\nu}}{|x^j(\tau') - x^i(\tau)|^2} - \frac{2[(x_\mu^j(\tau') - x_\mu^i(\tau)][(x_\nu^j(\tau') - x_\nu^i(\tau))]}{|x^j(\tau') - x^i(\tau)|^4} \right) \right] \right\}. \quad (42)$$

Although we do not need to invoke point-splitting regularization to render the denominators finite, we shall make the assumption that any nontrivial contribution to the n -fermion Green function stems from those contours that come close together within a distance bounded by the ribbon width. Otherwise, the n -fermion Green function amounts to nothing more than a trivial superposition of independent, i.e., “noncommunicating” (in the language of Georgi [21]) infrapropagators. Under these working conditions the nontrivial contribution to $E_{\text{non-diag}}^{ij}$ becomes

$$E_{\text{nondiag}}^{ij} \sim \exp \left\{ -g_i g_j \frac{1}{8\pi^2} \int_0^{T_i} d\tau \int_0^{T_j} d\tau' \left[\frac{2}{(\tau' - \tau)^2 + b^2} - (1-\lambda) \left(\frac{1}{(\tau' - \tau)^2 + b^2} - \frac{2(\tau' - \tau)^2}{[(\tau' - \tau)^2 + b^2]^2} \right) \right] \right\}. \quad (43)$$

In deriving this expression we have made the substitution $x^j(\tau') \approx x^i(\tau') + b n(\tau')$, which harmonizes the two parameters, and subsequently set $x^i(\tau') \approx x^i(\tau) + (\tau' - \tau) \dot{x}^i(\tau)$.

Upon performing the integrations in (43), we get

$$E_{\text{non-diag}}^{ij} \approx \exp \left\{ -g_i g_j \frac{1}{8\pi^2} \left[\frac{\pi(T_i + T_j)}{b} - \ln \frac{T_i}{b} - \ln \frac{T_j}{b} + \ln \frac{|T_i - T_j|}{b} \right] \right\}. \quad (44)$$

Combining (41) and (44), and setting $g_i = g$, $\forall i$, we arrive, after mass renormalization, at the following result

$$\left\langle \prod_{i=1}^n \exp \left\{ i g \int_0^{T_i} d\tau \dot{x}^i(\tau) \cdot A[\dot{x}^i(\tau)] \right\}_A \right\rangle \sim \exp \left\{ -\frac{n(n+1)}{2} a \ln(\mu b) + \ln \left[\mu^{n(n+1)/2} \frac{\prod_{i=1}^n T_i^n}{\prod_{i \neq j}^n |T_i - T_j|^{1/2}} \right] \right\}. \quad (45)$$

On the basis of (45), we can immediately identify the wave function renormalization constant attributed to gauge field fluctuation effects:

$$Z_2^{(n)} = \exp \left[-\frac{n(n+1)}{2} a \ln(\mu b) \right], \quad (46)$$

from which we extract the anomalous dimension

$$\gamma_F^{(n)} = -\frac{n(n+1)}{2} \frac{\alpha}{2\pi} (3 - \lambda) . \quad (47)$$

The renormalized n-fermion Green function in momentum space can now be determined following the same procedure which leads from (28) to (33). We find

$$G_{\text{ren}}^{(n)} \sim \mu^{n(n+1)a/2} \prod_{i=1}^n \int_{0^+}^{\infty} dT_i e^{-T_i(m_i + i\gamma \cdot p_i)} \left[\frac{\prod_{k=1}^n T_k^n}{\prod_{k \neq j}^n |T_k - T_j|^{1/2}} \right]^a . \quad (48)$$

The above results agree with those computed previously in Ref. [8] within the BN version of QED. The interested reader may consult this paper for further considerations related to (48) and for a detailed analysis of the mass-shell behavior of the two-fermion Green function [22].

As a final remark it is worth to be mentioned that allowing for completely *distinguishable* contours, i.e., going to the other extreme case of infraparticles propagating disjointly from each other, the n-particle Green function reduces to the product of n one-particle Green functions, each being renormalized by a renormalization constant $Z_2 = e^{-a \ln(\mu b)}$.

V. EFFECTIVE FIELD THEORY CALCULATION OF A UNIVERSAL VERTEX FUNCTION

The techniques developed in the preceding sections will now be generalized with the basic aim of deriving a universal vertex function of infraparticles. We will show that this vertex function is capable of emulating the physical content of the Isgur-Wise form factor in the weak decays of heavy mesons within HQET [see Fig.(2)].

We propose to describe the following process, illustrated in Fig. 3: An infraparticle on route along a straight-line path from x to y with four-velocity u_1 is suddenly derailed at the spacetime point z and then proceeds to propagate, still as an infraparticle, on a different straight-line path characterized by the four-velocity u_2 . As we shall see below, such a process can be unambiguously described by an appropriate three-point function which accounts for

the four-velocity discontinuity at z and which effectively describes the dynamics of the “clouds” of soft quanta – the concomitants of charged particles – in terms of a nontrivial contour with an infinitesimal self-intersecting loop.

In the context of HQET, infraparticles will represent heavy on-shell modes [23], while the complicated hadronic soft dynamics, governing the overlap of the light degrees of freedom (light quarks and gluons), will be collectively simulated by the contribution owing to the self-intersecting loop. The claim is that since the light degrees of freedom in the weak transition are “blind” to the flavor (mass) and spin orientation of the heavy quark and “feel” only its color field with wavelengths much larger than the Compton wavelength of the heavy quark – whereas the typical momenta exchanged between the heavy quark and the light constituents are of order $\Lambda_{QCD} \ll m_Q$ – this *effective* infraparticle description provides a realistic assessment of the physical picture.

Our efforts now will consist in mimicking the complicated non-Abelian soft dynamics inside two heavy mesons undergoing an electroweak transition by a single universal vertex function which depends only on the invariant four-velocity change $w = u_1 \cdot u_2$, where u_1 and u_2 are the four-velocities of the infraparticles. Although we continue to work within an Abelian field theory framework, we pretend to describe processes involving quarks, rather than electrons. How such a treatment can be realized will become clear in the course of our analysis.

Following our purposes, we define the following three-point function

$$\Gamma(x, y, z) \stackrel{c \rightarrow 0}{=} \int_c^\infty dT \int_0^T ds \int [dp(\tau)] \text{tr} \mathcal{P} \exp \left\{ \int_0^T d\tau [ip(\tau) \cdot \gamma + m] \right\} G(x, y, z), \quad (49)$$

where

$$G(x, y, z) = \int_{\substack{x(0)=x \\ x(s)=z \\ x(T)=y}}^{x(0)=x} [dx(\tau)] \exp \left[i \int_0^T d\tau p(\tau) \cdot \dot{x}(\tau) \right] \left\langle \exp \left\{ ig \int_0^T d\tau \dot{x}(\tau) \cdot A[x(\tau)] \right\} \right\rangle_A. \quad (50)$$

In the field theory context, one may think of Eq. (49) as having resulted via functional differentiation with respect to the source of the emitted current at the vertex point z . In the

particle mode of description, the same differentiation entails the factor $\int_0^T ds \delta[x(s) - z]$, modulo extra contributions pertaining to the nature of the current.

Our next step is to split the interaction point according to $x(s) \rightarrow [x(s_1), x(s_2)]$. In this way we generate an entrance and an exit point at the vertex z , the distance between them being of the order of the ribbon width b . At the same time we compensate for the doubling induced on the integration measure, i.e., $d^4x(s) \rightarrow d^4x(s_1)d^4x(s_2)$, via the insertion of the delta function $\delta[x(s_1) - x(s_2)]$. The net effect of the integration constraints, implemented through the product of delta functions $\delta[x(0) - x] \delta[x(T) - y] \delta[x(s_1) - x(s_2)] \delta[x(s_1) - z]$, is to rearrange the contour involved in the evaluation of the three-point function into three disjoint branches, one of which forms a self-intersecting loop at z .

As a result, the gauge-field fluctuation term acquires the form

$$\left\langle \exp \left\{ ig \int_0^T d\tau \dot{x}(\tau) \cdot A[x(\tau)] \right\} \right\rangle_A \approx \left\langle \exp \left[ig \int_{L_{x,y}^z} dw_\mu A_\mu(w) \right] \right\rangle_A \times \left\langle \exp \left\{ ig \int_{s_1}^{s_2} d\tau \dot{x}(\tau) \cdot A[x(\tau)] \right\} \right\rangle_A, \quad (51)$$

where we have combined the two disjoint infraparticle paths into a single composite path, denoted $L_{x,y}^z$, which is the broken contour from x to $z - 0^+$ and from $z + 0^+$ to y (see Fig. 4a). Note that the crossing point z , along with infinitesimal prongs to the left and to the right, has been *excised* from $L_{x,y}^z$, in order to account for the jump in the four-velocity. In a literal sense, the sharp bend at point z causes a break in the initial infraparticle path, so that the three contour-branches are disconnected. This event may be viewed as being the result of a dynamical “pinch” – a sudden impulse (or recoil) stroke – induced by an external source and taking place in the close vicinity of the point z .

We now introduce a coordinate rendering of $L_{x,y}^z$, which adapts itself to a parametrization in the interval $[0, 1]$ with the interaction point z removed. Specifically, we define the position variable $x^L(t)$ by

$$x^L(t) = \begin{cases} x[t(T - \tilde{\tau})], & 0 \leq t < \tilde{t} \\ x[t(T - \tilde{\tau}) + \tilde{\tau}] & \tilde{t} < t \leq T, \end{cases} \quad (52)$$

where $\tilde{t} = s_1 / (T - \tilde{\tau}) = (s_2 - \tilde{\tau}) / (T - \tilde{\tau})$ with $\tilde{\tau} = s_2 - s_1$ and $x^L(\tilde{t}) = z$. [Note that the interim point $\tilde{t}$ is excised from the interval $[0, 1]$.] The remaining branch of the three-point process has the geometric characteristics of a closed contour, designated C_{zz} (see Fig. 4b). The explicit contribution of this branch to $G(x, y, z)$ depends on the topology of C_{zz} . Recalling Eq. (51), the three-point function factorizes

$$G^{(i)}(x, y, z) = G(x, y|z) G^{(i)}(z, z), \quad (53)$$

where $G^{(i)}(z, z)$ represents the contribution from loop configurations and

$$G(x, y|z) = \int_{\substack{x^L(0)=x \\ x^L(1)=y}} [dx^L(t)] \delta[x^L(\tilde{t}) - z] \exp \left[-ig \mathcal{P}_{\tilde{t}} \int_0^1 dt \dot{x}^L(t) \cdot p^L(t) \right] \\ \times \left\langle \exp \left\{ ig \mathcal{P}_{\tilde{t}} \int_0^1 dt \dot{x}^L(t) \cdot A[x^L(t)] \right\} \right\rangle_A \quad (54)$$

is a purely kinematical factor describing coexisting in and out four-velocity states. In (54), $\mathcal{P}_{\tilde{t}}$ symbolizes the principal value of the integral according to

$$\mathcal{P}_{\tilde{t}} \int_0^1 dt \cdot = \int_0^{\tilde{t}-0^+} dt \cdot + \int_{\tilde{t}+0^+}^1 dt \cdot. \quad (55)$$

In the absence of the factor $G^i(z, z)$, $G(x, y|z)$ reduces to the product of two infrapropagators, separated at the junction point z by at least the resolution scale b (alias the ribbon width). According to our analysis in Sec. IV, we expect a totally disjoint infraparticle behavior, each fermion propagator being independently renormalized by a renormalization constant $Z_2 = e^{-a \ln \mu b}$. Indeed, we find

$$\Gamma_{\text{ren}}^{(0)}(x, y, z) \xrightarrow{c \rightarrow 0} \text{tr} \int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4} e^{-ip_1 \cdot (x-z) - ip_2 \cdot (z-y)} \int_c^\infty dT \int_0^1 T^{2a} \tilde{t}^a (1 + \tilde{t})^a \\ \times \exp \left[-T \tilde{t} (i\gamma \cdot p_1 + m) \right] \exp \left[-T (1 - \tilde{t}) (i\gamma \cdot p_2 + m) \right], \quad (56)$$

which, after integrating over all intermediate values of $\tilde{t}$, yields the anticipated structure of two factorized infrapropagators [4].

We now turn our attention to the factor of true interest, i.e., $G^{(i)}(z, z)$, which in our simulation scheme can be thought of as a “dynamical pinch” derailing the infraparticle and

causing the contour from x to y to develop a discontinuity in the four-velocity at z . The zeroth order term

$$G^{(0)}(x, y, z) = G(x, y|z)G^{(0)}(z, z) = G(x, y|z) \quad (57)$$

constitutes an overall kinematical factor, since $G^{(i)}(z, z) \rightarrow G^{(0)}(z, z) = 1$ corresponds to a loop that shrinks to zero upon the identification $s_1 = s_2$ (see Fig. 4a).

Next, we consider the nontrivial contribution coming from the configuration shown in Fig. 4b, termed $G^{(1)}(z, z)$. This time the self-crossing loop generates a topological obstruction in the contour, characterized by a deficiency angle complementary to the one formed by the in and out four-velocities. We remark that there is a notable difference between our case and calculations with cusped Wilson loops in pure gauge field theories [24]. Here, the self-crossing loop represents a piece of a particle's "worldline", to be entered and exited *smoothly* with corresponding four-velocities u_1 and u_2 , as illustrated in Fig. 5. The important point, physically, is that in our approach the self-intersecting loop in the contour accounts in a geometric sense for the effect of the "soft-cloud" dynamics of the process. Since, as already indicated, the cloud of soft quanta is transported intact through the self-intersection point z and around the loop, the three-point function $G^{(i)}(z, z)$ describes an *exclusive* process. It is exactly for this reason that $G^{(i)}(z, z)$ can be conceived of as mimicking the overlap of the light degrees of freedom in a heavy meson transition process. In contrast, a cusped contour at z without self-crossing corresponds rather to an *inclusive* process and may be brought in connection to the Bremsstrahlung effect [25]. The calculation of the closed-loop contribution that follows is based on the rationale just described.

Having explained our arguments and prescriptions, we proceed now with the explicit calculation of the contribution from gauge-field fluctuations. A uniform parametrization of the loop integral $\oint_{C_{zz}^{(1)}} dx_\mu A_\mu(x)$ is provided by the redefinition

$$\tau \rightarrow t = \tau - \frac{s_1 + s_2}{2} \quad (58)$$

accompanied by the coordinate readjustment

$$x(\tau) \rightarrow x^c(t) = x\left(t + \frac{s_1 + s_2}{2}\right), \quad (59)$$

where t is restricted in the interval $[-\frac{\tilde{\tau}}{2}, \frac{\tilde{\tau}}{2}]$ and $\tilde{\tau} = s_2 - s_1$. An immediate consequence of the closed contour integration according to our specifications is that contributions from terms containing the gauge parameter λ , which are proportional to $\oint dw_\mu \oint dz_\nu B_{\mu\nu}(w, z)$ [cf. (21) and (26)], vanish identically. This can be readily verified by appreciating that $B_{\mu\nu}(w, z)$ is a total derivative. Under this proviso

$$\frac{\partial}{\partial w_\nu} \frac{\partial}{\partial z_\nu} \ln |w - z| = \frac{\delta_{\mu\nu}}{|w - z|^2} - 2 \frac{(w_\mu - z_\mu)(w_\nu - z_\nu)}{|w - z|^4}, \quad (60)$$

the double-loop integral is obliged to vanish.

The I_1 term, whose regularized version is given by (25) in conjunction with (23) reads (in unregularized form)

$$I = - \frac{g^2}{8\pi^2} \int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt \int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt' \frac{\dot{x}^c(t') \cdot \dot{x}^c(t)}{|x^c(t') - x^c(t)|^2} \Big|_{\substack{\dot{x}^c(-\tilde{\tau}/2)=u_1 \\ \dot{x}^c(\tilde{\tau}/2)=u_2}}. \quad (61)$$

Invoking the picture of a sudden impulse process, which demands a minuscule loop, we are led to express the entrance segment of the loop at z by

$$x^c(t) = z_- + \left(t + \frac{\tilde{\tau}}{2}\right) u_1 \quad (62)$$

and the exit segment by

$$x^c(t') = z_+ + \left(t' - \frac{\tilde{\tau}}{2}\right) u_2. \quad (63)$$

Obviously, this parametrization describes propagation along a contour in the vicinity of the distortion point z , once entering and once exiting, with the rest of the loop making negligible contribution on account of its infinitesimal size.

Inserting parametrizations (62) and (63) into (61), and regularizing via the point-splitting method of the ribbon, we obtain

$$I(\theta) \sim - \frac{g^2}{8\pi^2} \theta \int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt \int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt' \frac{1}{t^2 + t'^2 - 2t t' \theta + b^2}, \quad (64)$$

where $\theta = u_1 \cdot u_2$. It is understood that the length scale in the above relation is of the order of the diminishing loop size (due to the sudden impulse approximation) and serves as an IR regulator.

It is established in Appendix B that

$$I(\theta) \simeq -\frac{g^2}{4\pi^2} \ln\left(\frac{\tilde{\tau}}{2b}\right) \frac{\pi\theta}{\sqrt{1-\theta^2}}. \quad (65)$$

In order to disentangle IR from UV effects, we make a scale readjustment according to which the coupling constant is tuned to values that are relevant to our effective description. Thus we write

$$g^2(b^2) \ln\left(\frac{\tilde{\tau}}{2b}\right) = g^2(\mu^2) \ln(\tilde{\tau}\mu) = g^2(\mu^2) \ln k. \quad (66)$$

We may view k as the parameter which relates the scales characterizing long- and short-distance regimes. In the case of QED, used throughout this work for the purposes of illustration, the soft photon cloud surrounding the charged particle is treated explicitly, while hard photons are taken care of by renormalization. Hence for the loop contribution to the vertex function we obtain, after renormalization,

$$G_{\text{ren}}^{(1)}(z, z) = \exp\left[-\frac{g^2}{4\pi} \frac{\theta}{\sqrt{1-\theta^2}} \ln k\right] \int_{x^c(-\frac{\tilde{\tau}}{2})=x^c(\frac{\tilde{\tau}}{2})=z} [dx^c(t)] \exp\left[-i \int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt p^c(t) \cdot x^c(t)\right]. \quad (67)$$

Inserting Eqs. (53), (67) into (49), we find

$$\Gamma_{\text{ren}}^{(1)}(x, y, z) = -\exp\left(-\frac{g^2}{4\pi} \frac{\theta}{\sqrt{1-\theta^2}} \ln k\right) \Gamma_{\text{ren}}^{(0)}(x, y, z) \quad (68)$$

with the minus sign cropping up because of the closed-contour contribution (recall the discussion in Sec. III of Ref. [1]).

To isolate the nonperturbative part of the vertex function, we must remove the kinematics. To this end, we define

$$F(x, y, z) = \frac{\Gamma_{\text{ren}}^{(0)}(x, y, z) + \Gamma_{\text{ren}}^{(1)}(x, y, z)}{\Gamma_{\text{ren}}^{(0)}(x, y, z)}. \quad (69)$$

Gathering together our findings, we determine

$$F(x, y, z) = F(\theta) = 1 - \exp\left(-\frac{g^2}{4\pi} \frac{\theta}{\sqrt{1-\theta^2}} \ln k\right), \quad (70)$$

which can be analytically continued to Minkowski space by replacing

$$\theta \rightarrow w \equiv \tilde{u}_\mu^1 \tilde{u}^{2\mu} = \frac{1}{\theta}, \quad (71)$$

where $\tilde{u}^1$ and $\tilde{u}^2$ are the Minkowski counterparts of u^1 and u^2 , respectively. (This is shown in Appendix C.)

The absence of an explicit dependence on the fermion (i.e., infraparticle) mass in (70) emphasizes the universal character of the calculated vertex function. It is a monotonically decreasing function of the invariant four-velocity change which at zero recoil, i.e., for $w = 1$ is normalized to unity: $F(0) = 1$. The physical significance of this function will be discussed in the next section.

VI. PHENOMENOLOGICAL APPLICATION TO WEAK DECAYS OF HEAVY MESONS

In the preceding section we have derived a three-point function which involves infraparticle dynamics within an Abelian formulation of particle path integrals. Now we will effect how the obtained result can be interpreted in the context of HQET. The claim is that it can *de facto* simulate the Isgur-Wise function [10]. This function [traditionally termed $\xi(w)$] represents the nonperturbative QCD contribution in the hadronic matrix elements between states of B and D mesons containing a heavy quark. Since it represents the nontrivial part of the transition process, it plays a central role in the HQET. It incorporates the large-distance dynamics of the color interactions of light degrees of freedom participating in transition processes as the one depicted in Fig. 2. A heavy quark moving from x to y via z undergoes a flavor transition at the vertex z by emitting a weak current. The light antiquark which is not participating in this transition is also displayed. In effect, the Isgur-Wise function provides

a measure for the overlap of the wave functions of the light (soft) degrees of freedom and, as such, cannot be calculated *ab initio*. Much like hadron distribution amplitudes in exclusive reactions (for a recent review, see, e.g., [26]), $\xi(w)$ is a fundamental nonperturbative quantity of QCD entering different weak decay processes in a universal way. Its calculation calls for nonperturbative methods, like QCD sum rules (see, e.g., [7]), lattice simulations (see, e.g., [27]), and bag-model calculations (see, e.g., [28]).

Let us now proceed to connect expression (70) to the (leading order) Isgur-Wise form factor. To this end, we rename $F(\theta)$ into $\xi(w)$ and recast (70) in the form

$$\xi(w) = 1 - \exp\left(-\frac{4}{3}\alpha_s \frac{1}{\sqrt{w^2-1}} \ln k\right), \quad (72)$$

where we have replaced e^2 by $(4/3)g_s^2$ and have introduced the strong-coupling constant $\alpha_s = g_s^2/4\pi$ to pass from QED to our simulated version of “QCD”. Next we adapt the scales involved to those of heavy quark decays by setting

$$k = \frac{m_Q}{\bar{\Lambda}}, \quad (73)$$

where m_Q is the heavy-quark mass, and $\bar{\Lambda}$ characterizes the energy scale of the light degrees of freedom. Using rather typical values for these parameters (see, e.g., [7]) and $\alpha_s \simeq 0.21$ [29], we find $k \approx 8$. With these numbers we finally obtain a vertex function which, within the accuracy limits of our framework, has the model form

$$\xi(w) = 1 - \exp\left(-\frac{0.582}{\sqrt{w^2-1}}\right). \quad (74)$$

This function is plotted in Fig. 6 (solid line) in comparison with other parametrizations for the Isgur-Wise form factor [7,30,31], and against recent data of different collaborations [32,33]. In the kinematic region accessible to semileptonic decays ($1 \leq w \leq 1.6$), it almost coincides with the pole-type ansatz of Neubert [7]. In addition, there is good agreement of our model with the data – in particular, with those of CLEO II [32]. At zero recoil the obtained result equals one and decreases monotonically at large recoil, as expected

on general grounds. On the other hand, one infers from (74) that the slope of our curve at zero recoil, i.e., for $w = 1$, vanishes, indicating a violation of the Bjorken bound [34] $\rho^2 > 0.25$; ρ^2 being the negative of the slope of ξ at $w = 1$. Although this parameter is very difficult to be measured, one expects on theoretical grounds that ρ^2 lies well above the Bjorken bound. Its violation by (74) clearly signals the limitations of our method which, albeit a first-principle approach, cannot be the substitute for a full QCD calculation. A first explanation of this failure is that in the limit of zero recoil, the four-velocities of the two infraparticles become equal and thus the corresponding Wilson lines – previously linked by the self-crossing loop – can now be simultaneously trivialized by choosing the gauge $u \cdot A = 0$. In technical terms this means that the self-crossing loop shrinks exactly to zero and as a result there is no exchange of soft quanta between the initial and the final state. Related questions will be addressed in detail in a forthcoming paper [25], in connection with cusped contours.

VII. SUMMARY AND CONCLUSIONS

In this work we have attempted to calculate the long-distance (IR) behavior of fermion propagators as well as a universal vertex function of infraparticles which *effectively simulates* the nontrivial QCD strong-interaction dynamics of the light degrees of freedom in heavy meson decays. These issues were treated hierarchically, starting in Sec. II from an all-encompassing expression for the one-fermion Green function in an Abelian framework (for which QED served mainly as an illustration) and ending with a specific application relevant to heavy-meson transitions in Secs. V and VI.

The IR dynamics of the fermionic Green functions was extracted upon imposing appropriate conditions on the IR-dominant contours contributing to the particle path integrals involved in the formalism. Short-distance contributions due to quantum fluctuations of the gauge field have been calculated nonperturbatively and their effects have been absorbed into

fermion anomalous dimensions which are *all-order* expressions in the fine-structure constant α (Secs. III and IV). The method we have used to control the UV singularities employs a point-splitting (“ribbon”) regularization technique, which, though mathematically equivalent to dimensional regularization (see Appendix A), is physically more suitable to treat subtleties associated with obstructions of particle contours. In the present investigation an infinitesimal self-intersecting loop was used in the evaluation of the particle path integral for the purpose of describing the dynamics of soft clouds, permanently accompanying charged particles. It turns out, providentially, that such a procedure amounts to a universal vertex function which mimics the physical characteristics of the Isgur-Wise form factor (Sec. V). In fact, in the kinematic region accessible in semileptonic decays, our model curve is correctly normalized and independent of the heavy quark mass, and complies with different sets of recent data [32,33].

On the basis of the above-mentioned observations, we were able to focus on the universal features of transitions between four-velocity particle states without relying on spin symmetry and without invoking specific forms of an asymptotic dynamics. The resulting physical picture is clearly supported by previous studies showing that heavy quark propagators can be described by Wilson lines [35,36] (see also [37]). Recalling Eq. (50), and the detailed arguments given in Sec. V, it is worth noting that due to the different velocities with which the inraparticle enters and exits the loop at the vertex z (see Fig. 4), choosing the gauge $u_{1(2)} \cdot A = 0$ for one branch of the contour cannot trivialize the dynamics. This agrees, in particular, with the observations made in Ref. [38].

An important aspect of the proposed theoretical scheme is the formulation of fermionic field systems in terms of a particle path integral with a built-in resolution scale, while completely preserving the full γ -structure of the theory [1]. Thus the spin-1/2 matter quanta enter the theory as active degrees of freedom, albeit with IR-related labels (four-velocities). Keeping large- and short-distance scales apart, our approach allows an intuitive inraparticle interpretation of fermions in the low-energy regime in the sense of *effective* degrees of

freedom. The method does not rely on perturbation expansion in the coupling constant and treats the soft degrees of freedom *collectively*, rather than as a truncated diagrammatic expansion. The applications considered in the present work may serve as an incentive to pursue further this course of investigations.

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Appendix A

In this appendix we discuss briefly the interplay between dimensional and point-splitting (ribbon) regularization of the Wilson-line exponential subject to our IR specifications. The dimensionally regularized expressions for the quantities $A[x(\tau), x(\tau')]$ and $B_{\mu\nu}[x(\tau), x(\tau')]$ entering (20) and (21) are, respectively,

$$\begin{aligned} A^{(d)} &\equiv \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2} e^{ik \cdot [x(\tau') - x(\tau)]} \\ &= \frac{\mu^{4-d}}{4\pi^{d/2}} \Gamma\left(\frac{d}{2} - 1\right) \frac{1}{|x(\tau') - x(\tau)|^{d-2}} \end{aligned} \quad (\text{A1})$$

and

$$\begin{aligned} B_{\mu\nu}^{(d)} &\equiv \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu}{k^4} e^{ik \cdot [x(\tau') - x(\tau)]} \\ &= \frac{\mu^{4-d}}{8\pi^{d/2}} \Gamma\left(\frac{d}{2} - 1\right) \left\{ \frac{\delta_{\mu\nu}}{|x(\tau') - x(\tau)|^{d-2}} \right. \\ &\quad \left. - (d-2) \frac{[x_\mu(\tau') - x_\mu(\tau)][x_\nu(\tau') - x_\nu(\tau)]}{|x(\tau') - x(\tau)|^d} \right\}. \end{aligned} \quad (\text{A2})$$

The corresponding regularized expression for the expectation value of the Wilson-line operator turns out to be

$$\begin{aligned} \left\langle \left\{ ig \int_0^T d\tau \dot{x}_\mu(\tau) A_\mu[x(\tau)] \right\} \right\rangle_A^{(d)} &\approx \exp \left\{ -\frac{g^2}{4\pi^{d/2}} \mu^{4-d} \Gamma\left(\frac{d}{2} - 1\right) [(d-1) - (d-3)\lambda] \right. \\ &\quad \left. \times \int_0^T d\tau \int_\tau^T d\tau' \frac{1}{(\tau' - \tau)^{d-2}} \right\}, \end{aligned} \quad (\text{A3})$$

where we have Taylor expanded around $x(\tau') \approx x(\tau) + (\tau' - \tau)\dot{x}(\tau)$ and used the property that $|\dot{x}(\tau)| = 1$. Then, performing the double integral, we obtain

$$\left\langle \left\{ ig \int_0^T d\tau \dot{x}_\mu(\tau) A_\mu[x(\tau)] \right\} \right\rangle_A^{(d)} \approx \exp \left\{ \frac{g^2}{8\pi^{d/2}} \Gamma\left(\frac{d}{2} - 1\right) \frac{[(d-1) - (d-3)\lambda]}{(4-d)(d-3)} (\mu T)^{4-d} \right\}. \quad (\text{A4})$$

This relation is analogous to the point-splitting result given by Eq. (31). The pole at $d = 3$ corresponds to the linear term and the pole at $d = 4$ to the logarithmic one. By working around $d = 4$, the linear term is washed out and we are left with

$$\left\langle \left\{ ig \int_0^T d\tau \dot{x}_\mu(\tau) A_\mu[x(\tau)] \right\} \right\rangle_A^{(d)} = \exp \left[\frac{\alpha}{\epsilon} + a \ln(\mu T) + \text{finite terms} \right] , \quad (\text{A5})$$

where $\epsilon = 4 - d$, and the parameters α, a are the same as in (31).

In order to explore the connection between ribbon and dimensional regularization in more detail, let us consider the quantity

$$I \equiv \int_0^T d\tau \int_\tau^T d\tau' \frac{1}{[(\tau' - \tau)^2 + b^2]^{(d-2)/2}} , \quad (\text{A6})$$

which for $d = 4$ corresponds to the ribbon regularized quantity encountered in the text, while for $b = 0$ it coincides with the dimensionally regularized expression contained in (A3).

As the first step we write

$$\frac{1}{[(\tau' - \tau)^2 + b^2]^{(d-2)/2}} = \frac{1}{\Gamma\left(\frac{d}{2} - 1\right)} \int_0^\infty d\xi \xi^{\frac{d}{2}-2} e^{-\xi[(\tau' - \tau)^2 + b^2]} , \quad (\text{A7})$$

whereupon it follows

$$I = \frac{1}{\Gamma\left(\frac{d}{2} - 1\right)} \int_0^T d\tau \int_0^\infty d\xi \xi^{\frac{d}{2}-2} e^{-\xi b^2} \int_0^T dx e^{-\xi x^2} . \quad (\text{A8})$$

Evaluating first the x -integration followed by the τ -integration, we find

$$I = \frac{1}{2\Gamma\left(\frac{d}{2} - 1\right)} \int_0^\infty d\xi \xi^{\frac{d}{2}-\frac{5}{2}} e^{-\xi b^2} \left[T\gamma\left(\frac{1}{2}, \xi T^2\right) - \xi^{-\frac{1}{2}}\gamma\left(1, \xi T^2\right) \right] , \quad (\text{A9})$$

where $\gamma(a, b)$ is the incomplete gamma function. Finally, carrying out the remaining ξ -integration leads to a result which involves hypergeometric functions:

$$I = \frac{\tau^2}{(\tau^2 + b^2)^{\frac{d}{2}-1}} \left[F\left(1, \frac{d}{2} - 1; \frac{3}{2}; \frac{\tau^2}{\tau^2 + b^2}\right) - \frac{1}{2} F\left(1, \frac{d}{2} - 1; 2; \frac{\tau^2}{\tau^2 + b^2}\right) \right] . \quad (\text{A10})$$

Consider first the case $b \rightarrow 0$. This gives

$$I = - \frac{T^{4-d}}{d-3} - \frac{T^{4-d}}{4-d} = - \frac{T^{4-d}}{(d-3)(4-d)} , \quad (\text{A11})$$

which is precisely the outcome of straight use of the dimensional regularization [cf. (A4)].

Alternatively, setting $d = 4$ in (A10), we get

$$F\left(1, 1; \frac{3}{2}; \frac{T^2}{T^2 + b^2}\right) = \frac{T^2 + b^2}{Tb} \arctan\left(\frac{T}{b}\right), \quad (\text{A12})$$

$$F\left(1, 1; 2; \frac{T^2}{T^2 + b^2}\right) = \frac{T^2 + b^2}{T^2} \ln\left(1 + \frac{T^2}{b^2}\right)$$

from which the result quoted in Sec. III can be readily deduced:

$$I = \frac{T}{b} \arctan\left(\frac{T}{b}\right) - \frac{1}{2} \ln\left(1 + \frac{T^2}{b^2}\right) \xrightarrow{b \rightarrow 0} \frac{\pi T}{2b} - \ln\left(\frac{T}{b}\right) - 1 + \mathcal{O}(b). \quad (\text{A13})$$

Appendix B

Here we shall derive (65), starting from the integral

$$I(\theta) = \theta \int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt \int_{-\tilde{\tau}/2}^{\tilde{\tau}/2} dt' \frac{1}{t^2 + t'^2 - 2tt'\theta + b^2}, \quad (\text{B1})$$

which is the same as (64) apart from an irrelevant multiplicative constant. Reparametrizing variables by $t \rightarrow \frac{2t}{\tilde{\tau}}$, $t' \rightarrow \frac{2t'}{\tilde{\tau}}$, and introducing an element of dimensional regularization methodology, let us consider the quantity

$$I^{(\epsilon)}(\theta) \equiv \theta \int_{-1}^1 dt \int_{-1}^1 dt' \frac{1}{(t^2 + t'^2 - 2tt'\theta + c^2)^{1-\epsilon/2}}, \quad (\text{B2})$$

where $c \equiv 2b/\tilde{\tau}$. It is understood that in the end the limit $\epsilon \rightarrow 0$ will be taken. Employing the identity

$$\frac{1}{A^{1-\epsilon/2}} = \frac{1}{\Gamma\left(1 - \frac{\epsilon}{2}\right)} \int_0^\infty d\xi \xi^{-\epsilon/2} e^{-\xi A}, \quad (\text{B3})$$

we get

$$\begin{aligned} I^{(\epsilon)}(\theta) &= \frac{\theta}{\Gamma\left(1 - \frac{\epsilon}{2}\right)} \int_0^\infty d\xi \xi^{-\epsilon/2} e^{-\xi c^2} \int_{-1}^1 dt e^{-\xi t^2(1-\theta^2)} \int_{-(1+t\theta)}^{1-t\theta} dx e^{-\xi x^2} \\ &\equiv I_1^{(\epsilon)}(\theta) - I_2^{(\epsilon)}(\theta), \end{aligned} \quad (\text{B4})$$

where

$$I_1^{(\epsilon)}(\theta) = \frac{\sqrt{\pi}\theta}{\Gamma\left(1 - \frac{\epsilon}{2}\right)} \int_0^\infty d\xi \xi^{-\frac{1}{2} - \frac{\epsilon}{2}} e^{-\xi c^2} \int_{-1}^1 dt e^{-\xi t^2(1-\theta^2)} \quad (\text{B5})$$

and

$$I_2^{(\epsilon)}(\theta) = \frac{\theta}{2\Gamma\left(1 - \frac{\epsilon}{2}\right)} \int_0^\infty d\xi \xi^{-\frac{1}{2} - \frac{\epsilon}{2}} e^{-\xi c^2} \int_{-1}^1 dt e^{-\xi t^2(1-\theta^2)} \left[\Gamma\left(\frac{1}{2}, \xi(1+t\theta)^2\right) + \Gamma\left(\frac{1}{2}, \xi(1-t\theta)^2\right) \right]. \quad (\text{B6})$$

Upon performing all integrations, the first term becomes

$$I_1^{(\epsilon)}(\theta) = 2\theta\sqrt{\pi} \frac{\Gamma\left(\frac{1}{2} - \frac{\epsilon}{2}\right)}{\Gamma\left(1 - \frac{\epsilon}{2}\right)} (1 - \theta^2 + c^2)^{\frac{\epsilon}{2} - \frac{1}{2}} F\left(1, \frac{1}{2} - \frac{\epsilon}{2}; \frac{3}{2}; \frac{1 - \theta^2}{1 - \theta^2 + c^2}\right). \quad (\text{B7})$$

For $\theta \neq 1$ we find

$$I_1^{(\epsilon)}(\theta) = 2\theta\sqrt{\pi} \frac{1}{\epsilon} \frac{\Gamma\left(\frac{1}{2} - \frac{\epsilon}{2}\right)}{\Gamma\left(1 - \frac{\epsilon}{2}\right)} (1 - \theta^2 + c^2)^{\frac{\epsilon}{2} - \frac{1}{2}} F\left(1, \frac{1}{2} - \frac{\epsilon}{2}; 1 - \frac{\epsilon}{2}; \frac{c^2}{1 - \theta^2 + c^2}\right) - 2\theta\pi \frac{1}{\epsilon} c^\epsilon (1 - \theta^2 + c^2)^{-\frac{1}{2}} F\left(\frac{1}{2}, 1 + \frac{\epsilon}{2}; 1 + \frac{\epsilon}{2}; \frac{c^2}{1 - \theta^2 + c^2}\right) \quad (\text{B8})$$

which, for $c^2 \ll 1 - \theta^2$, yields

$$I_1^{(\epsilon)}(\theta) \simeq 2\theta\sqrt{\pi} \frac{1}{\epsilon} \frac{\Gamma\left(\frac{1}{2} - \frac{\epsilon}{2}\right)}{\Gamma\left(1 - \frac{\epsilon}{2}\right)} (1 - \theta^2)^{\frac{\epsilon}{2} - \frac{1}{2}} [1 + \mathcal{O}(c^2)] - 2\theta\pi \frac{1}{\epsilon} (1 - \theta^2)^{\frac{\epsilon}{2} - \frac{1}{2}} [1 + \mathcal{O}(c^2)]. \quad (\text{B9})$$

Finally, for $\epsilon \rightarrow 0$, we obtain

$$\begin{aligned} I_1(\theta) &\simeq 2\theta\pi \frac{1}{\epsilon} (1 - \theta^2)^{-\frac{1}{2}} - 2\theta\pi \frac{1}{\epsilon} c^\epsilon (1 - \theta^2)^{-\frac{1}{2}} + \mathcal{O}(c^2) \\ &= 2\pi \frac{\theta}{\sqrt{1 - \theta^2}} \ln\left(\frac{1}{c}\right) + \mathcal{O}(c^2) \\ &= 2\pi \frac{\theta}{\sqrt{1 - \theta^2}} \ln\left(\frac{\tilde{\tau}}{2b}\right) + \mathcal{O}\left(\frac{4b^2}{\tilde{\tau}^2}\right). \end{aligned} \quad (\text{B10})$$

Proceeding along similar lines, the final result for the second term reads

$$I_2(\theta) = \text{const.} + \mathcal{O}\left(\frac{4b^2}{\tilde{\tau}^2}\right). \quad (\text{B11})$$

Consequently,

$$I(\theta) \stackrel[b \ll \tilde{\tau}]{\theta \neq 1} \simeq 2\pi \frac{\theta}{\sqrt{1-\theta^2}} \ln\left(\frac{\tilde{\tau}}{2b}\right) + \text{finite terms} \quad (\text{B12})$$

which establishes the validity of (65).

Appendix C

This appendix gives more details how the invariant inner product of the four-velocities u_1 and u_2 in Euclidean space is continued back to Minkowski space while preserving unit normalization. To effect this, let us denote by $\tilde{u}_1$ and $\tilde{u}_2$ their Minkowski versions and recall that they move on the unit hyperboloid. From the Euclidean viewpoint, such a situation calls, in general, for a rescaling of the four-vectors in order to bring their norm to unity. A consistent treatment of this transformation is achieved if $\tilde{u}_1$ and $\tilde{u}_2$ are symmetrically placed on the hyperboloid with $\tilde{u}^2 = 1$ (see Fig. 7).

Suppose that $\tilde{u}_1$ and $\tilde{u}_2$ have been specified in an arbitrary way. Then it is clear that one must find a proper reference frame within which a symmetrical placement can be realized. To this end, we represent them by

$$\tilde{u}_1 = (\cosh\theta, \sinh\theta, 0, 0), \quad \tilde{u}_2 = (\cosh\phi, \sinh\phi, 0, 0), \quad (\text{C1})$$

without any loss of generality. One notices that $\tilde{u}_1^2 = \tilde{u}_2^2 = 1$.

Consider now the Lorentz boost specified by the matrix

$$\mathbf{M} = \begin{pmatrix} \cosh\omega & \sinh\omega & 0 & 0 \\ \sinh\omega & \cosh\omega & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (\text{C2})$$

with $\omega = -(\theta + \phi)/2$. In the boosted frame, one has

$$\tilde{u}'_1 = \left(\cosh \frac{\theta - \phi}{2}, \sinh \frac{\theta - \phi}{2}, 0, 0 \right), \quad \tilde{u}'_2 = \left(\cosh \frac{\theta - \phi}{2}, \sinh \frac{\theta - \phi}{2}, 0, 0 \right), \quad (\text{C3})$$

while the symmetric arrangement of Fig. 7 still holds. The corresponding Euclidean four-vectors are defined in such a way that each one retains its direction, while the magnitude is rescaled to unity with respect to the metric signature $(+ + + +)$. The result is

$$\begin{aligned} u_1 &= \left(\frac{\cosh \frac{\theta-\phi}{2}}{\sqrt{\cosh(\theta-\phi)}}, \frac{\sinh \frac{\theta-\phi}{2}}{\sqrt{\cosh(\theta-\phi)}}, 0, 0 \right), \\ u_2 &= \left(\frac{\cosh \frac{\theta-\phi}{2}}{\sqrt{\cosh(\theta-\phi)}}, \frac{-\sinh \frac{\theta-\phi}{2}}{\sqrt{\cosh(\theta-\phi)}}, 0, 0 \right). \end{aligned} \quad (\text{C4})$$

Again, one verifies that $u_1^2 = u_2^2 = 1$.

We now proceed to examine inner products. In Minkowski space we have

$$w \equiv \tilde{u}_1^\mu \tilde{u}_{2\mu} = \cosh^2 \frac{\theta-\phi}{2} + \sinh^2 \frac{\theta-\phi}{2} = \cosh(\theta-\phi). \quad (\text{C5})$$

Analogously, we determine in Euclidean space

$$\theta \equiv u_{1\mu} u_{2\mu} = \frac{\cosh^2 \frac{\theta-\phi}{2} - \sinh^2 \frac{\theta-\phi}{2}}{\cosh(\theta-\phi)} = \frac{1}{\cosh(\theta-\phi)}. \quad (\text{C6})$$

Then it follows

$$\tilde{u}_1 \tilde{u}_2 = \frac{1}{u_1 \cdot u_2} \quad (\text{C7})$$

or

$$w = \frac{1}{\theta}. \quad (\text{C8})$$

Finally,

$$\frac{\theta}{\sqrt{1-\theta^2}} = \frac{1}{\sqrt{\frac{1}{\theta^2}-1}} = \frac{1}{\sqrt{w^2-1}}, \quad (\text{C9})$$

which completes the validity of (72).

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FIGURES

FIG. 1. (a) Example of a contour complying with the smoothness requirement involved in the calculation of the one-fermion Green function in the IR regime. (b) A path for which $x(\tau) \approx x(\tau')$ does not imply that $\tau \approx \tau'$; such contours are excluded in the calculation of the Green function.

FIG. 2. Feynman diagram representing a heavy meson decay in which a heavy quark of one flavor undergoes a transition to a heavy quark of another flavor by emitting a weak current A_μ .

FIG. 3. Illustration of the broken contour involved in the evaluation of the three-point function discussed in the text. The disjoint pieces of the contour correspond to different infraparticle states labelled by their respective four-velocities.

FIG. 4. Contours that enter the calculation of the infraparticle vertex function $\xi(w)$. The lhs shows those pieces of the contour which correspond to an independent propagation of two infraparticles. The rhs shows the nontrivial part of the contour which contains a minuscule self-intersecting loop at point z and serves to simulate the soft interactions.

FIG. 5. Illustration of the dynamical pinch at the vertex, simulated by a loop with a size C_{zz} shrinking to zero. The in- and out-going segments correspond to uniform propagation with different four-velocities u_1 and u_2 which, respectively, enter and leave the loop smoothly.

FIG. 6. Comparison of selected theoretical expressions for the Isgur-Wise function $\xi(w)$ in terms of the four-velocity change $w = u_1 \cdot u_2$. The curves correspond to our result (solid line), and to those obtained by Isgur [30] (dotted line), Radyushkin [31] (dashed-dotted line), and Neubert [7] (dashed line). Also shown is the data of different collaborations [32,33] as indicated in the figure. For the CKM matrix element the average value $|V_{cb}| = 0.039$ has been used.

FIG. 7. Illustration of the symmetric configuration of the two four-velocities u_1 and u_2 on the unit hyperboloid which serves to facilitate their transcription to Euclidean space.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7