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Bird's IP View of Limits of Conventional

e^+e^- Linear Collider Technology*

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Abstract

Scaling laws appropriate to future e^+e^- linear colliders in the high γ regime are examined assuming that the luminosity must increase as the square of the energy. Limits on achievable energy for these colliders are identified under the assumption that no exotica such as energy recovery, superdisruption, or four-beam charge compensation are employed, and all technology is foreseeable and has an apparent cost within the bounds of a large international collaboration. Following these guidelines, an upper energy limit appears around 15 TeV in the center of mass as the normalized emittance required to produce ever smaller vertical spot sizes become unattainable with conventional damping ring technology.

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Introduction

This paper is an attempt to look into the future and so one should view these results with a large amount of skepticism. On the other hand, as we sit on the verge of choosing a technology for the next linear collider, it seems important that each technology be assessed regarding its capabilities for expansion to higher energies. The current linear collider design projects have settled on an initial energy of 500 GeV in the center-of-mass (c.m.), with expansion to 1 TeV c.m., and in some cases expansion to 1.5 TeV c.m. is contemplated. This paper takes the point of view that a subsequent project would have at least five times this energy; namely, be able to achieve at least 5 TeV in the center-of-mass. A third-generation machine would be envisioned to have a center-of-mass energy of 25 TeV.

This investigation is based on five premises, that:

1. no known physics be violated,
2. luminosity must scale as energy squared and be equal to $10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$ at 1 TeV c.m.,
3. the technologies employed must be known and conceivably extrapolated to the limits assumed,
4. no exotica like energy recovery, charge compensation with four beams, or superdisruption be invoked, and
5. the "first glance" total cost not be obviously off-scale for a large international project.

The perspective of this study is heavily weighted on interaction point (IP) dynamics, and indeed a second motivation for this study is the investigation of the IP scaling laws at very large γ parameters. When designing a complete machine, trade-offs between optimal parameters derived from any single perspective are invariably balanced, and this usually drives parameters into a "corner" of the natural parameter space as viewed from any one perspective.

Thus the results of this study should not be expected to match one-to-one with the outcome of a more complete design process.

Figure 1 shows the schematic layout of this project, and Table 1 and Table 2 give the general and IP parameters, respectively.

Luminosity Scaling Law

We start with the well-known luminosity formula

$$L = \frac{1}{4\pi} \frac{N^2 f n_b}{\langle \sigma_x \rangle \langle \sigma_y \rangle} \quad (1)$$

where $\langle \sigma_x \rangle$ and $\langle \sigma_y \rangle$ indicate the average size of the spot during the collision including disruption effects. In fact, $\langle \sigma_x \rangle$ is usually very close to the linear σ_x for acceptable parameters, whereas $\langle \sigma_y \rangle$ is usually about two-thirds of σ_y .

If, in this formula, the beam power, P_b , and wall-plug power, P_w , and the energy in the center-of-mass, $E_{c.m.}$, are inserted through the relationships $E_{c.m.} = 2E_b$, $Nfn_b E_b = P_b$, and $2P_b = \eta_{w \rightarrow b} P_w$ where $\eta_{w \rightarrow b}$ is the wall-plug-to-beam-power efficiency, one obtains

$$L = \frac{1}{4\pi} \frac{P_w}{E_{c.m.}} \eta_{w \rightarrow b} \left(\frac{N}{\langle \sigma_x \rangle} \right) \left(\frac{1}{\langle \sigma_y \rangle} \right). \quad (2)$$

We will often refer to the parameters quoted for the next linear collider (NLC) project at SLAC.

We will show that the factor $\frac{N}{\langle \sigma_x \rangle}$ is restricted by a desire to have a clean interaction, where,

by clean we mean not too much energy spread in the final center-of-mass energy of colliding primary particles, low numbers of background hadrons per bunch collision and good polarization retention throughout the collision. One will, of course, seek to maximize the efficiency, but there are limits to what can be done with presently conceivable technology. The

Figure 1. Schematic layout of a linear collider.

Table 1.

Long Bunch Train Working Parameter List for a 250 and 500 GeV NLC
General Parameters

Parameter	500 GeV CM Energy	1 TeV CM Energy	Comments
E	250 GeV	500 GeV	Beam Energy
L ₀	6×10^{33}	1.2×10^{34}	Luminosity
L _H	9×10^{33}	2.0×10^{34}	w/Enhancement
2 L _{Linac}	15 km	15 km	Total X-Band Linac Length
E _z	50 MV/m	100 MV/m	Accelerating Gradient
f _{rf}	11.4 GHz	11.4 GHz	rf Frequency
N	0.65×10^{10}	1.3×10^{10}	Particles/Bunch
n _B	90	67	Number of Bunches
s _B	1.4 ns	1.4 ns	Bunch Spacing
f	180 Hz	120 Hz	Bunch Trains/second
P _{beam}	4 MW	8 MW	Average Beam Power at IP
P _{wall}	90 MW	140 MW	Modulator Wall-Plug Power for Both Linacs
σ _x	300 nm	425 nm	Variable
σ _y	3 nm	2 nm	$\sqrt{\gamma}$ scaling
σ _z	100 μ	100 μ	Bunch Length
δ _{rms}	2×10^{-3}	2×10^{-3}	Intra-bunch Energy Spread
δ̄ _{rms}	$\leq 2 \times 10^{-3}$	$\leq 2 \times 10^{-3}$	Bunch-to-bunch Energy Centroid Deviation

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only remaining free factor, $\frac{1}{\langle \sigma_y \rangle}$, will be limited by stability considerations and normalized emittance limits. If we require the luminosity to scale as the square of the energy and to equal $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ then L must equal

$$L = 10^{26} \gamma^2 \text{ m}^{-2} \text{ s}^{-1}. \quad (3)$$

Putting this together with the luminosity formula [Eq. (2)] one arrives at

$$\gamma^3 = 0.5 \times 10^{-14} P_w \eta_{w \rightarrow B} \left(\frac{N}{\langle \sigma_x \rangle} \right) \left(\frac{1}{\langle \sigma_y \rangle} \right) \quad (4)$$

The product of four factors must increase with the cube of the energy to achieve an acceptable design for a higher-energy machine. These factors are: (i) the wall-plug power; (ii) the

efficiency; (iii) $\left(\frac{N}{\langle \sigma_x \rangle} \right)$; and (iv) the inverse of the vertical spot size. For the NLC 1 TeV

parameter set for which $\gamma^3 = 10^{18}$ and $L = 1.7 \times 10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$, we have

$$P_w = 1.4 \times 10^8, \eta_{w \rightarrow B} = 0.12, \frac{N}{\langle \sigma_x \rangle} = 3 \times 10^{16}, \text{ and } \frac{1}{\langle \sigma_y \rangle} = 0.7 \times 10^9.$$

Table 2.

Long Bunch Train Working Parameter List for a 250 and 500 GeV NLC
IP Region

Parameter	500 GeV CM Energy	1 TeV CM Energy	Comments
σ _x	300 nm	425 nm	variable $\rightarrow \times \frac{1}{2}$
σ _y	3 nm	2 nm	$\sqrt{\gamma}$ scaling
ε _x	10^{-11}	$1/2 \times 10^{-11}$	$\gamma \epsilon_x = 5 \times 10^{-6} \text{ meter-rad}$
ε _y	10^{-13}	$1/2 \times 10^{-13}$	$\gamma \epsilon_y = 5 \times 10^{-8} \text{ meter-rad}$
β _x	10 mm	40 mm	For σ _x , $2 \times \sqrt{\gamma}$ scaling
β _y	100 μ	100 μ	
σ _{x',y'}	30,30 μrad	15,20 μrad	IP Divergent Angle
σ _z	100 μ	100 μ	Bunch Length
θ _d	3 mrad	4.2 mrad	Bunch Diagonal Angle
δ _{rms}	2×10^{-3}	2×10^{-3}	Energy Spread
D _{x,y}	.08, 8.3	.04, 8.8	Disruption Parameter
θ _D	25 mrad	.17 mrad	Maximum Disruption Angle at Beam Energy
Y (upsilon)	0.1	0.28	Beamstrahlung Parameter
δ _B	.03	.077	Mean Energy Loss to Beamstrahlung γs
n _γ	0.9	1.1	Number of Photons per Electron
N _{had}	.04	.34	Number of Hadronic Events per Bunch Crossing [†]
N _{jet5}	.001	.03	Number of Mini-jet Events per Bunch Crossing [†]

[†]Calculated following arguments of Chen, Barklow, and Peskin.

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Limits on Luminosity Factors

The $P_w \eta_{w \rightarrow B}$ ($= 2P_B$) Limit

The wall-plug-to-beam efficiency achievable with present conventional rf technology is about 6%. The 1-TeV technology efficiency is projected to be 12%. This gain is achieved by going to a gridded klystron, or improving modulator efficiency. We take as a “foreseeable” technology an efficiency of 19% which would be achieved by eliminating the SLED pulse compression

unit, using instead something like the cluster klystron now under development by SLAC and BNL.¹

The component efficiencies associated with present technology, the projection for 1 TeV, and the “foreseeable” technologies are indicated in Table 3.²

Table 3

	Power prep	rf source	Pulse comp.	Power transm.	rf-to-beam	Wall-to-beam
0.5 TeV	.75	.6	.76	.95	.27	.09
1.0 TeV	.95	.65	.82	.95	.25	.12
Foreseeable	.95	.70	1.0	.95	.3	.19

The total wall-plug power that might be available for a future linear collider is difficult to predict. It does not seem unreasonable to assume that one would locate such a facility near a large source of cheap power (hydroelectric?) or that one would build a power plant dedicated to providing energy to the collider as needed, and otherwise feeding a power grid. Under such assumptions a power in the range of 1 or 2 gigawatts would seem to be a maximum limit.

Assuming the availability of 2 gigawatts of power, one achieves a factor of 25 improvement in the $P_w \eta_{w \rightarrow B}$ product as compared to the NLC 1-TeV parameters. Since we will be contemplating two future collider generations with a factor of 5 increase in center-of-mass energy each, this assumption of available beam power corresponds roughly to a linear increase of beam power with center-of-mass energy. Thus for the purposes of this paper we will assume

$$P_w \eta_{w \rightarrow B} = 2 P_B = 80 \times 10^6 \left(\frac{\gamma}{\gamma_5} \right) \text{ W.}$$

Here we have introduced the notation $\gamma_5 = 5 \cdot 10^6$ for the value of γ at 5 TeV c.m. since we will scale our results from a 5 TeV c.m. platform lying within the high upsilon regime.

$\frac{N}{\langle \sigma_x \rangle}$ Limits

In the large upsilon limit³ there is a rather simple relationship between n_γ , the number of photons produced per electron during the beam-beam collision, and the quantity $\frac{N}{\sigma_x}$.⁴

$$\frac{N}{\sigma_x} = c_1 \left(\frac{\gamma}{\sigma_z} \right)^{1/2} n_\gamma^{3/2} \quad (5)$$

where $c_1 = 1.4 \times 10^{11} \text{ m}^{-1/2}$. For orientation, if one inserts the 1 TeV c.m. parameters, which are not yet in the large upsilon regime, into the rhs of Eq. (5), one obtains an $\frac{N}{\sigma_x}$ about half of

its value in the 1-TeV design. Additionally, in the large upsilon limit, one finds the simple relationship

$$\delta_B = \frac{2}{9} n_\gamma \quad (6)$$

where δ_B is the beamstrahlung energy spread. The polarization loss during the beam-beam collision, $\langle \Delta P \rangle$, has been shown to be⁵:

$$\langle \Delta P \rangle = .04 n_\gamma \quad (7)$$

and the distribution function for the crossing-averaged number of electron-positron collisions as a function of electron energy has a delta-function spike at the initial electron beam energy given by⁶

$$\Psi_{AV}(E) = \left[\frac{(1 - e^{-n_\gamma})}{n_\gamma} \right] \delta(E - E_0) + \dots \quad (8)$$

The coefficient of the delta function spike at $n_\gamma = 0.5$ is 0.79 while at $n_\gamma = 1.0$ it is 0.63. The expressions for δ_B , $\langle \Delta P \rangle$, and $\Psi_{AV}(E)$ indicate the value of n_γ must lie between 0.5 and 1.0.

If relation (5) is inserted into the luminosity formula of Eq. (4), one obtains:

$$\gamma^{5/2} = 0.7 \times 10^{-3} n_\gamma^{3/2} 2P_B \frac{1}{\sigma_z^{1/2}} \frac{1}{<\sigma_y>}. \quad (9)$$

We see here that, except for a mild $\sigma_z^{-1/2}$ dependence, one must rely on increased beam power and a small vertical spot size to achieve the $\gamma^{5/2}$ scaling.

The choice for n_γ also has an important impact on the number of hadronic background events per crossing. The number of hadronic events per crossing, N_{Had} , is well represented by⁷.

$$N_{Had} = L_X n_\gamma^2 \sigma_{\gamma\gamma \rightarrow Had} \quad (10)$$

where the luminosity per crossing, L_X , is given by

$$L_X = \frac{1}{4\pi} \frac{N^2}{\sigma_x <\sigma_y>} = \frac{1}{4\pi} \left(\frac{N}{\sigma_x} \right)^2 \frac{\sigma_x}{<\sigma_y>} \quad (11)$$

Figure 2 shows the predicted $\sigma_{\gamma\gamma \rightarrow Had}$ cross section as a function of energy. It grows very slowly as the function $\ln(s)^2$. Hence, if $L_X n_\gamma^2$ is held constant, hadronic backgrounds of future colliders will be no worse than the design value $N_{Had} = 0.3$ for the NLC 1-TeV collider.

Thus

$$c_2 \equiv \left[\frac{4\pi N_{Had}}{\sigma_{\gamma\gamma \rightarrow Had}} \right]^{1/2} = n_\gamma \frac{N}{\sigma_x} \left(\frac{\sigma_x}{<\sigma_y>} \right)^{1/2} \quad (12)$$

must remain approximately equal to its value at 1 TeV: $c_2 = 3 \times 10^{17} \text{ m}^{-1}$.

At this point, one can use relations (5) and (12) to solve for n_γ or $\frac{N}{\sigma_x}$ in terms of the aspect ratio and $\frac{\sigma_z}{\gamma}$ with the result:

$$n_\gamma = \left(\frac{c_2}{c_1} \right)^{2/5} \left(\frac{\sigma_z}{\gamma} \right)^{1/5} \left(\frac{<\sigma_y>}{\sigma_x} \right)^{1/5} \quad (13)$$

Figure 2. The predicted $\sigma_{\gamma\gamma \rightarrow Had}$ cross section.

or

$$\frac{N}{\sigma_x} = c_2^{3/5} c_1^{2/5} \left(\frac{\gamma}{\sigma_z} \right)^{1/5} \left(\frac{<\sigma_y>}{\sigma_x} \right)^{3/10} \quad (14)$$

If the NLC 1-TeV aspect ratio and σ_z are inserted into the right-hand side of Eq. (13), a value of $n_\gamma = 1.0$ is achieved at $\gamma = 4.6 \times 10^6$, nearly 5 TeV in the center of mass. For these parameters, Eqs. (5) or (14) evaluate, as they must, to their value at 1 TeV: $\frac{N}{\sigma_x} = 3 \times 10^{16}$.

Interestingly, for higher energies, the condition of Eq. (13) allows for aspect ratios closer to unity. In other words, σ_x can be allowed to decrease faster than σ_y . However, this is not easy to achieve. The beta ratio at the IP tends to be very close to the emittance ratio, for when this is true the horizontal and vertical divergent angles are equal. If one tries to further reduce β_x at the

IP then the horizontal divergent angle increases and the vertical spot size is degraded by additional synchrotron radiation emitted in the horizontal focusing element of the final doublet.

The other option, reducing the horizontal emittance, is also difficult if this is accomplished in a damping ring, because, as we shall see below, the intrabeam scattering conditions become quite severe. Assuming then that the aspect ratio is constant, Eq. (14) sets the condition for the growth of $\frac{N}{\sigma_x}$; namely, as $\gamma^{1/5}$, n_y will fall below unity as $\gamma^{-1/5}$ implying that the beam quality

will improve with additional energy. Instead of Eq. (9) we now have

$$\gamma^{14/5} = 0.9 \times 10^{-20} 2P_B \left(\frac{1}{\sigma_z} \right)^{1/5} \left(\frac{\langle \sigma_y \rangle}{\sigma_x} \right)^{3/10} \left(\frac{1}{\langle \sigma_y \rangle} \right) \quad (15)$$

If we assume, as suggested in the section on beam power, that beam power scales linearly with energy, and assume σ_z and $\frac{\langle \sigma_y \rangle}{\sigma_x}$ are almost constant, then Eq. (15) requires that the luminosity be achieved by a $\gamma^{9/5}$ reduction of $\langle \sigma_y \rangle$. Since by Eq. (14) $\frac{N}{\sigma_x}$ grows as $\gamma^{1/5}$, assuming again that $\frac{\langle \sigma_y \rangle}{\sigma_x} = 10^{-2}$ is almost constant, N must be decreasing as $\gamma^{-8/5}$ and will satisfy

$$N = 3 \times 10^8 \left(\frac{\gamma_s}{\gamma} \right)^{8/5} \quad (16)$$

This will be helpful for wakefield considerations and will allow smaller rf structures. It implies, however, that, to put more average power into the beam, the number of bunches per second must be increasing dramatically. One can expect longer bunch trains and higher repetition rates.

The σ_y Oide Limit

The vertical beam size is limited by considerations of machine stability and conditions set by the Oide limit. Following the scaling laws proposed in the previous paragraph, Eq. (15) leads to the vertical beam heights presented in Table 4 (based on $\frac{\langle \sigma_y \rangle}{\sigma_x} = 10^{-2}$ and $\sigma_z = 10^{-4}$ m):

Table 4

	P_B (MW)	$\langle \sigma_y \rangle$ (nm)
5 TeV c.m.	40	0.1
Beyond 5 TeV	40 (γ/γ_s)	0.1 (γ_s/γ) ^{9/5}

The Oide limit⁸, based on a calculation of the effects of synchrotron radiation emitted in the final doublet, requires that

$$\sigma_{y,\min} = 2.6 \times 10^{-4} F^{1/7} \epsilon_{Ny}^{5/7} \quad (17)$$

at

$$\beta_y = \frac{5 \sigma_{y,\min}^2}{7 \left(\frac{\epsilon_{Ny}}{\gamma} \right)} \quad (18)$$

where F is a slowly varying function of the final doublet design parameters. For typical doublets, including the effects of horizontal motion, $F \approx 100$ and $F^{1/7} \approx 1.9$.⁹ This limit can be improved by a factor of about 3 by observing that the distribution from the synchrotron radiation is not Gaussian, as assumed in Oide's first note.¹⁰ These considerations lead to a minimum

$$\sigma_{y,\min} = 1.7 \times 10^{-4} \epsilon_{Ny}^{5/7} \quad (19)$$

Assuming an enhancement $\sigma_y / \langle \sigma_y \rangle = 1.5$, the normalized emittance corresponding to the 5 TeV c.m. parameters is $\epsilon_{Ny} = 3.3 \cdot 10^{-9}$ m-rad, a factor of 16 smaller than the normalized vertical emittance for the 1-TeV design. For smaller energies, according to Table 4 and Eq. (19), the scaling law must be $(\gamma^{-9/5})^{7/5} = \gamma^{-63/25} \approx \gamma^{-5/2}$, which may be written

$$\epsilon_{Ny} = 3.3 \times 10^{-2} \left(\frac{\gamma_s}{\gamma} \right)^{5/2} \quad (20)$$

For $E_{cm} = 25$ TeV, $\epsilon_{Ny} = 6.0 \times 10^{-11}$ m-rad, almost three orders of magnitude smaller than its value for the 1-TeV NLC parameters.

The scaling for β_y which now may be deduced from Eqs. (18) and (20) is $\beta_y \sim \gamma^{-1/10}$, very close to being independent of energy. This result is welcome, as the chromaticity of the final doublet, and the tolerances of the final focus system, scale with β_y .

ϵ_{Ny} Limits

A quantum mechanical limit on emittance was noted by J. Seeman¹¹

$$\Delta y \Delta p_y = p \Delta y \Delta y' = \gamma mc \Delta y' > \hbar \quad (21)$$

implying

$$\gamma \epsilon_y = \epsilon_{Ny} > \hbar / mc = \lambda_e = 4 \times 10^{-13} \text{ m - rad} . \quad (22)$$

This quantum limit is comfortably small. Indeed it yields, through Eq. (19), a limit on the energy of $E_{cm} = 180$ TeV at $\sigma_y = 0.25$ pm.

The real barrier to smaller normalized emittances comes from consideration of damping ring beam dynamics. The intrabeam scattering growth rate¹² scales as

$$\frac{N}{\gamma_D \sigma_{zD} \epsilon_{Nx}^{5/2}} . \quad (23)$$

where D indicates the parameter value in the damping ring. Using the scaling law for N from the paragraph following Eq. (15), namely $N \sim \gamma^{-8/5}$, and the emittance scaling of Eq. (19), we get

$$\tau^{-1} \sim \gamma^{4.15} . \quad (24)$$

This is a very severe scaling law since this growth rate must be balanced by a rapidly increasing damping rate. A preliminary damping ring design for the 5-TeV parameters has been proposed by T. Raubenheimer¹³ that would achieve $\epsilon_{Ny} = 10^{-9}$ m-rad. It consists of a dogbone design with a 1-km hybrid wiggler. The wiggler period λ_w would be 5 cm and have a field strength of 1.5 Tesla. The energy of the ring would be 3 GeV. Raubenheimer estimates that the limit of this technology is perhaps a factor of 4 smaller yet than this at $\epsilon_{Ny} = 2.5 \times 10^{-10}$. If that is true, Eq. (19) can be used to estimate a maximum achievable energy.

$$E_{cm, max} \leq 14 \text{ TeV} . \quad (25)$$

Equation (24) implies that the maximum achievable energy varies only weakly (1/4 power) with the maximum growth rate that can be overcome with additional damping. In this sense, one would expect the result quoted in Eq. (25) to be a rugged upper bound.

The emittance $\epsilon_{Ny} = 2.5 \times 10^{-10}$ corresponds to a vertical beam size at the IP of $\sigma_y = 0.04$ nm.

The σ_y Stability Limit

Figure 3 shows a frequency scale with four significant frequencies present in the design and operation of a linear collider. From right to left, we have: (1) the repetition frequency, which is contemplated to be 120 Hz for the 1-TeV NLC; (2) the effective frequency of feedback, about 1/10 of the repetition frequency; (3) the frequency at which the aberrations contributing to spot size growth are tuned, perhaps once every 15 minutes; and (4) the frequency at which the overall machine alignment is checked with beam-based procedures, perhaps every few days.

Figure 3. Significant design frequencies for a linear collider.

The repetition frequency for higher energy can be estimated by rewriting $P_B = N f n_b E_B$ as

$$f n_b = \frac{P_B}{(E_B N)}. \quad (26)$$

Because of our assumption that P_B scales linearly with energy, $\frac{P_B}{E_B} = 10^{14}$ is a constant. Using

Eq. (16), which shows that N scales as the inverse of the energy to the 8/5 power, we have

$$f n_b = 3.3 \times 10^5 \left(\frac{\gamma}{\gamma_5} \right)^{8/5}. \quad (27)$$

Details of rf design would be required to determine f and n_b separately, but a first guess would be to take the square root for each, giving:

$$f = 600 \left(\frac{\gamma}{\gamma_5} \right)^{4/5} \text{ and } n_b = 550 \left(\frac{\gamma}{\gamma_5} \right)^{4/5} \quad (28)$$

Since feedback is unable to cope with motion at frequencies higher than the feedback frequency, estimated to be 1/10 the repetition frequency, jitter must be suppressed at and above this frequency by either passive elements that filter the jitter, or active elements that measure element jitter directly and remove it. Typically the jitter tolerance of the final doublet is

somewhat smaller than the spot size, and the jitter tolerance on all other quadrupoles is about 10 times the spot size.

A rather simple-to-execute idea which has been used in Taiwan¹⁴ to reduce quadrupole jitter is sketched in Fig. 4. A seismometer placed on the quadrupole measures the vibration. The signal is amplified and fed to a steering coil mounted within or next to the quadrupole which compensates for the steering of the quadrupole center offset. Effectively, one keeps the magnetic center of the quadrupole fixed although the quadrupole itself is moving. The Taiwan group¹⁴ claims they have been able to reduce quadrupole motion by a factor of 100 in this way.

Figure 4. System to correct quadrupole jitter.

The disadvantage of the above method is that it is a dead-reckoning method which ultimately depends on the accuracy of the seismometer and the gain and stability of the amplifier. This idea may be improved upon¹⁵ by inertially suspending a coil in the quadrupole (see Fig. 5) and measuring the quadrupole field motion directly. The correction coil is mounted on the same

quadrupole and is driven to balance out the motion of the quadrupole field. Thus the system is now a feedback device. The limitation is the difficulty of inertially suspending the measuring coil. Preliminary estimates by S. Smith¹⁶ on the sensitivity of this device, based on the assumption of a 150-turn coil with resistance of 2.6 k Ω , indicate an ability to detect 12-pm motion at 1 Hz and 1-pm motion at 10 Hz. This, of course, assumes that the coil is suspended inertially.

Figure 5. A system to correct quadrupole jitter employing feedback.

Figure 6 indicates the range of typical ground motion as measured at “noisy” and “quiet” sites. The collider would be optimally located at a quiet site, but the presence of humans and an accelerator would immediately make the site somewhat noisy. Let us assume an average site.

At 1 Hz, the ground motion is indicated to be 1.6 nm. This is somewhat quieter than SLAC, which is measured to have about 10 nm, depending on specific location. However, no site-wide precautions have been taken to preserve “quiet,” and weekends and nights are indeed quieter. Also to be reckoned is that the motion of quadrupoles which are mounted on stands above the ground is inevitably somewhat larger than the motion of quadrupoles on the ground.

Figure 6. Typical ground motion spectra.

Nevertheless, the factor of 100 achieved in Taiwan is enough to more than meet stability requirements for all quadrupoles other than the final doublet quadrupole. Let us assume these have motion in the 20-nm range.¹⁶ To obtain 40-pm stability requires a factor of 500. In Fig. 7, we plot suppression functions associated with various feedback strategies, taking into account the suspension of the inertial coil. Inertial quad plus deadbeat at 360 Hz achieves a suppression of 2000 at 1 Hz and 300 at 10 Hz. We conclude that the 40-pm stability can be achieved, considering the high repetition rate of these machines.

Conclusions

Interaction point considerations give clear indications of how future conventional linear collider parameters must scale with energy. The resulting scaling laws are achievable only if the vertical spot size decreases as the 9/5 power of the energy. The Oide limit, based on synchrotron radiation in the final doublet, requires that the normalized emittance decrease as the 5/2 power of the energy. If the normalized emittance is achieved conventionally within a damping ring, this requirement of normalized emittance translates into an increase of the intrabeam scattering growth rate as roughly the fourth power of the final energy. Inevitably, at some energy, methods introduced to create radiation damping can no longer overpower this intrabeam growth rate. We believe this limitation sets a maximum energy achievable in conventional colliders somewhere in the vicinity of 14 TeV c.m. This would suggest a search for better ways to achieve small normalized emittances. There is a quantum limit on normalized emittance, but the quantum limit would allow energies of up to $E_{cm} = 180$ TeV if one could achieve the required machine stability.

Figure 7. Transfer functions associated with feedback strategies for beam and field stabilization.

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