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Abstract

The growth rates of unstable modes in nuclear matter are investigated using a quantal dispersion relation. Calculations carried out with a realistic finite range potential show that unstable modes with wave number larger than the Fermi momentum are strongly suppressed, due to the quantal effect, in the mean field evolution. The observed consequence is the shift of the most important modes towards longer wave lengths.

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Over the past several years significant progress has been made on the description of nuclear dynamics within the framework of semi-classical transport theories. In particular, the Boltzmann-Uhling-Uhlenbeck (BUU) model has proved to be a very good basis for understanding a variety of features associated with nuclear collisions at intermediate energies [1]. In these transport models, the effect of residual interactions is taken into account only in an average manner, and therefore, the description is well suited for processes exhibiting only small density fluctuations. However, such a description is inadequate for processes involving large fluctuations, in which the nuclear system may develop into widely different configurations. In particular, under suitable conditions, nuclear collisions may lead to situations in which bulk or surface instabilities may be agitated [2, 3]. In such cases the stochastic Boltzmann-Langevin (BL) transport model has emerged as a promising microscopic approach for describing the dynamics of large density fluctuations [4, 5]. According to this approach, the density fluctuations generated by the stochastic collision term may be amplified in the spinodal zone by the unstable self-consistent mean field, leading towards catastrophic transformation of the system into an ensemble of nuclear clusters [6].

In order to understand the early development of the spinodal instabilities a lot of work has been carried out in the linear response framework of the semi-classical BL model [7, 8, 9]. The dynamics of the collective modes associated with density fluctuations is governed by a simple transport equation, in which the modes are agitated by a source term arising from the stochastic part of the collision integral and propagated by the mean-field. On the other hand, the work carried out earlier using the Landau's kinetic model is restricted to the development of already existing fluctuations at the initial state, since this model does not contain any stochastic source term for exciting the collective modes [10]. These studies have been mostly focused on a semi-classical description of spinodal

instabilities in nuclear matter. In infinite nuclear matter, the collective modes are plane waves characterized by a wave number $\mathbf{k}$, and the characteristic frequency of a collective mode corresponding to a given wave number is determined by a semi-classical dispersion relation. It turns out that the fastest growing collective modes, which are those that will become predominant, have wave numbers that are comparable to the Fermi momentum [7]. For example, for densities $n \approx 0.3 n_0$ and typical temperatures $T = 3-5 \text{ MeV}$, the wave numbers of the fastest growing modes are in the order of $k \approx 0.8 \text{ fm}^{-1}$, and the Fermi momentum for the same densities is $k_F \approx 1.0 \text{ fm}^{-1}$. This indicates that the quantal effects associated with the mean field evolution can have an important influence on the growth of spinodal instabilities. Therefore, the quantal BL equation for the single particle density matrix $\rho(t)$ provides a more suitable framework for studying the dynamics of density fluctuations in nuclear systems [11],

$$i \frac{\partial \rho(t)}{\partial t} - [h[\rho], \rho(t)] = K[\rho] + \delta K(t). \quad (1)$$

Here the left hand side describes the mean field evolution in terms of an effective one-body Hamiltonian $h[\rho] = -\frac{1}{2m} \nabla^2 + U[\rho]$ where $U[\rho]$ is the density-dependent mean-field potential; in the right hand side $K(\rho)$ is a binary collision term in the quantal representation, and $\delta K(t)$ denotes its stochastic part.

The small fluctuations $\delta\rho$ of the single particle density matrix around a finite temperature equilibrium characterized by ρ_0 are determined by the linearized quantal BL equation,

$$i \frac{\partial \delta\rho}{\partial t} - [h_0, \delta\rho] - [\delta U, \rho_0] = I_0 \cdot \delta\rho + \delta K_0 \quad (2)$$

where h_0 is the mean field Hamiltonian at the equilibrium state, δU denotes its fluctuating part and $I_0 \cdot \delta\rho$ is the linearized two-body collision term. This linear response treatment provides a quantal basis for describing the early evolution of

the spinodal instabilities in nuclear matter, including both the mean-field and the dissipation-fluctuation effects due to the residual interactions. In this investigation, we are mainly interested in studying the quantal effect on the growth of instabilities in infinite nuclear matter, therefore we neglect the r.h.s. of eq.(2). Furthermore, we assume that the mean-field Hamiltonian h_0 at the equilibrium state is uniform, and hence, the equilibrium single particle density matrix is diagonal in the plane wave representation, $\langle \mathbf{p} | \rho_0 | \mathbf{p}' \rangle = \delta(\mathbf{p} - \mathbf{p}') \rho_0(\mathbf{p})$ where $\rho_0(\mathbf{p})$ is a finite temperature Fermi-Dirac function. Carrying out a Fourier transform with respect to time, the mean-field part of the linearized BL equation in the plane wave representation becomes:

$$(\omega - \epsilon_{p_1} + \epsilon_{p_2}) \delta\rho(\mathbf{p}_1, \mathbf{p}_2; \omega) = \delta U(\mathbf{p}_1, \mathbf{p}_2; \omega) [\rho_0(\mathbf{p}_2) - \rho_0(\mathbf{p}_1)] \quad (3)$$

where $\delta\rho(\mathbf{p}_1, \mathbf{p}_2; \omega)$ denotes the Fourier transform of the fluctuating part of the single particle density matrix,

$$\delta\rho(\mathbf{p}_1, \mathbf{p}_2; \omega) = \int dt e^{i\omega t} \langle \mathbf{p}_1 | \delta\rho(t) | \mathbf{p}_2 \rangle \quad (4)$$

and similarly for $\delta U(\mathbf{p}_1, \mathbf{p}_2; \omega)$. When the effective mean-field is generated from the local density $n(\mathbf{r})$ via a finite range interaction $g(\mathbf{r})$, $U[n] = g \otimes \tilde{U}$ where $\tilde{U}[n(\mathbf{r})]$ is a function of the local density, then the fluctuating part of the mean-field can be expressed as:

$$\delta U(\mathbf{p} + \mathbf{k}/2, \mathbf{p} - \mathbf{k}/2; \omega) = \frac{\partial U(k)}{\partial n} \delta n(\mathbf{k}, \omega) \quad (5)$$

Here $\delta n(\mathbf{k}, \omega)$ denotes the Fourier transform of the local density fluctuations in space and time,

$$\delta n(\mathbf{k}, \omega) = \int d\mathbf{r} dt e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \delta n(\mathbf{r}, t) = \int \frac{d\mathbf{p}}{(2\pi)^3} \delta\rho(\mathbf{p} + \mathbf{k}/2, \mathbf{p} - \mathbf{k}/2; \omega) \quad (6)$$

and $U(k) = g(k) \tilde{U}$, where $g(k)$ is the Fourier transform of the convolution function $g(\mathbf{r})$. Hence, writing the momenta in eq.(3) as $\mathbf{p}_1 = \mathbf{p} + \mathbf{k}/2$, $\mathbf{p}_2 = \mathbf{p} - \mathbf{k}/2$

and integrating over $\mathbf{p}$, we obtain a quantal dispersion relation for the frequency of the collective mode corresponding to the wave number $\mathbf{k}$,

$$1 = \frac{\partial U(k)}{\partial n} \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{\rho_0(\mathbf{p} - \mathbf{k}/2) - \rho_0(\mathbf{p} + \mathbf{k}/2)}{\omega_k - \epsilon_{\mathbf{p}+\mathbf{k}/2} + \epsilon_{\mathbf{p}-\mathbf{k}/2}} \quad (7)$$

The characteristic frequencies occur in pairs $\pm\omega_k$, and they are real for the stable modes and imaginary for the unstable ones, $\omega_k = i/\tau_k$ where $\pm\tau_k$ is the characteristic growth or decay time of the mode. At zero temperature, the momentum integral in the dispersion relation can be evaluated exactly to give [12],

$$\frac{1}{F_0(k)} + \frac{1}{2} = \frac{1}{8s_0} \left\{ [(s + s_0)^2 - 1] \ln \frac{s + s_0 + 1}{s + s_0 - 1} - [(s - s_0)^2 - 1] \ln \frac{s - s_0 + 1}{s - s_0 - 1} \right\} \quad (8)$$

where $s = \omega_k/(k v_F)$, $s_0 = k/(2k_F)$ and the k -dependent Landau parameter is indicated by $F_0(k) = (\partial U(k)/\partial n) \cdot (3n / 2\epsilon_F)$. We notice that in the limit of small wave numbers $s_0 \ll 1$, eq.(8) reduces, as expected, to the semi-classical dispersion relation at zero temperature,

$$\frac{1}{F_0(k)} + 1 = \frac{s}{2} \ln \frac{s + 1}{s - 1}. \quad (9)$$

We determine the characteristic frequencies of the collective modes by numerically solving the quantal dispersion relation (7) at finite temperatures. In the calculations we employ two different finite range interactions, which are adjusted to reproduce the ground state properties of finite nuclei [13, 14]. As an example, figure 1 shows the growth rate of the unstable modes as a function of the wave number in the spinodal region, corresponding to $n = 0.05 \text{ fm}^{-3}$ and $T = 3 \text{ MeV}$, calculated with the effective interaction employed in ref.[13]. The solid and the dashed lines are, respectively, the quantal and the semi-classical results with the finite range interaction. The dotted lines show the quantal results obtained with a local mean-field. Figure 2 shows the same quantities calculated with the effective interaction employed in ref.[14]. As seen from these figures, the dispersion

relation does not depend very strongly on the employed effective interaction, but is substantially modified by the quantal effect. The unstable modes are confined to a narrower range centered around a wavelength $\lambda \approx 8 - 10 \text{ fm}$, as compared to a broader range concentrated around $\lambda \approx 6 \text{ fm}$ in the semi-classical case. Also the maximum of the dispersion relation is reduced by the quantum effects by about a factor $2/3$. Therefore fluctuations take more time to develop when the quantum effects are introduced. Figure 3 shows the boundary of the spinodal region in the (n, T) plane corresponding to a mode with a wave length $\lambda = 6 \text{ fm}$. The solid and dashed lines are the quantal and the semi-classical results obtained with the effective interaction of ref.[13], respectively. It is seen that the spinodal region shrinks to a smaller size in the quantal case, indicating that the mode is quite suppressed by quantal effects. Therefore finite size effects should be more important when quantum effects are introduced.

It is interesting to notice that the influence of the quantal effect on the dispersion relation is analogous to that of a finite range interaction in the semi-classical calculation. This can be qualitatively understood by recalling the Heisenberg uncertainty principle, which states that, in order to introduce fluctuations with a typical wave length λ , the associated momentum must exhibit a variation in the order of $\Delta p = k = 2\pi/\lambda$. This gives rise to an additional energy cost of about $k^2/(2m) = (2\pi)^2/(2m\lambda^2)$ for the zero-point motion of the density fluctuations [15], which plays the role of a k -dependent potential energy in a semi-classical description. Therefore, as far as the spinodal decomposition is concerned, the quantal effect associated with the mean-field propagation can partly be taken into account in a semi-classical treatment by a suitable renormalisation of the range of the effective interaction.

In conclusion, we have shown that the inclusion of the quantal effect in the mean-field propagation changes significantly the nuclear dispersion relation and

the boundary of the k -dependent spinodal region. The characteristic growth rates of the unstable modes, in particular with wave numbers larger than the Fermi momentum, are strongly suppressed and the size of the spinodal zone corresponding to these modes is reduced by the quantal effect. As a result, the most unstable collective modes, which essentially determine the predominant size of the primary clusters, are shifted to longer wave lengths than those obtained in a semi-classical description with the same effective interaction. Therefore, the quantal effect in the mean field evolution appears to be important for a quantitative description of the spinodal instabilities of an expanding nuclear system. This effect can be partly incorporated into the semi-classical description by increasing the range of the effective interaction or by including an additional Laplacian term in the one-body Hamiltonian.

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Figure 1:

The dispersion relation for the unstable nuclear matter at $\rho = 0.05 \text{ fm}^{-3}$ and $T = 3 \text{ MeV}$, obtained with the effective interaction of ref. [13]. The solid and dashed lines show the quantal and the semi-classical results, respectively. The dotted line shows the quantal results obtained with a local mean-field.

Figure 2:

The dispersion relation calculated using the effective interaction of ref.[14]. The same quantities of fig.1 are displayed.

Figure 3: Phase Diagram

The boundaries of the spinodal region in the (n, T) plane associated with the wave number $\lambda = 6 \text{ fm}$, obtained in the quantal (solid line) and in the semi-classical (dashed line) case, using the effective interaction of ref.[13].