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Testing the principle of equivalence with neutrinos *

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I discuss a novel theoretical possibility of using neutrinos to probe into nonuniversality in neutrino's gravitational couplings and thereby test the principle of equivalence *à la* Einstein. The optimal sensitivities one can expect with this method are $10^{-14} - 10^{-15}$ for long baseline accelerator neutrino experiments, and $10^{-15} - 10^{-16}$ for solar neutrino observation by high-statistics water Cherenkov detectors.

The principle of equivalence is one of the fundamental building block of Einstein's general theory of relativity. The importance of experimentally testing the principle, therefore, cannot be overemphasized. There already exist a series of very precise experiments which test this principle that was initiated long time ago by Eötvös[1] and substantially improved by Dicke et al [2]. The best bound obtained by this type of experiment is $\sim 10^{-12}$ [3] for the difference in the ratio of inertial and gravitational masses between differing materials. A recent fifth-force experiment[4] produces a similar bound which is comparable in accuracy to Dicke et al.'s.

In this talk I shall describe a new method for testing Einstein's equivalence principle using neutrinos, a quite suitable subject for this conference. The method aims to test the possible difference between gravitational couplings of different flavor neutrinos. The optimal sensitivity one can expect with this method is $10^{-14} - 10^{-15}$ for long-baseline accelerator neutrino experiments, and $10^{-15} - 10^{-16}$ for solar neutrino observation.

The method heavily relies on the observation by Gasperini[5]. He observed that if the electron and the muon neutrinos differ in their gravitational couplings they experience neutrino oscillation in an exactly analogous way as the usual vacuum flavor mixing case due to masses of neu-

trinos. This is true for massless as well as massive neutrinos. His proposal was followed by several analyses[6-10] which contains more realistic consideration of how it can be realized experimentally.

The physics behind the gravity-induced neutrino oscillation is exactly parallel to that of the vacuum flavor mixing case. Since they differ in their gravitational couplings electron and muon neutrinos develop different phases in their wave functions while traversing in weak gravitational fields of the sun, or of the earth. The difference in phases of the wave functions results in the neutrino flavor oscillation as in the usual vacuum mixing case.

With masses of neutrinos we become enriched with two different ingredients for neutrino oscillation. Among other things we have two independent mixing angles θ_M and θ_G , which relate flavor (or gauge) eigenstate to the mass and the gravity eigenstates, respectively.

The neutrino evolution equation, however, takes a relatively simple form. For solar neutrinos it can be written as [10]

$$i \frac{d}{dr} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} \quad (1)$$

where

$$\begin{aligned} H_{11} &= \sqrt{2} G_F N_e - \Delta_M \cos 2\theta_M - \Delta_G \cos 2\theta_G \\ H_{12} &= H_{21} = \frac{1}{2} (\Delta_M \sin 2\theta_M + \Delta_G \sin 2\theta_G) \\ H_{22} &= 0 \end{aligned} \quad (2)$$

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with

$$\begin{aligned}\Delta_M &\equiv \Delta m^2/2E, \\ \Delta_G &\equiv \Delta f\phi(r)E.\end{aligned}\quad (3)$$

Here $\phi(r)$ is the gravitational potential felt by neutrinos expressed with $\bar{G} = \frac{1}{2}(G_1 + G_2)$ and $\Delta f \equiv (G_2 - G_1)/\bar{G}$, where $G_i (i = 1, 2)$ are the gravitational couplings of two gravity eigenstates.

Can one distinguish the effects due to the mass- and the gravity-induced neutrino mixing? The answer is fortunately yes. To see this clearly let us examine the case of constant gravitational potential ϕ and $\theta_M = \theta_G \equiv \theta$. (Nothing deep in the latter equality but just for illustrative purpose.) Then the transformation probability of $\nu_\mu \rightarrow \nu_\tau$ after traversing a distance L is given by [7]

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{4E} + \frac{1}{2} \Delta f \phi E \right) L \quad (4)$$

Notice that the energy dependence comes in a different way; the gravity-induced effect is proportional to the neutrino energy E whereas the mass-induced effect is inversely proportional to E . This difference arises due to the dimensional reason, G has dimension length^2 . In other word, it reflects the tensor nature of gravitational interactions[11]. Thank to the difference in energy dependence one can distinguish, at least in principle, the effects due to masses and gravitational interaction of neutrinos. I will examine in more detail how it actually works.

What is the reason behind great sensitivity we are going to discover with this method? The transformation probability (4) is sizable if $\Delta f \phi E L \simeq 1$. Putting into the numbers for long-baseline neutrino experiments; $E \simeq 10 \text{ GeV}$, $L \simeq 10^3 \text{ Km}$, $\phi(\text{earth}) \simeq 10^{-9}$, we find $\Delta f \simeq 10^{-14}$ as an order-of-magnitude estimation of the sensitivity. So the great sensitivity is a combined effect of linear energy dependence, a long path length, and the moderately strong earth's gravitational field. The gravity-induced neutrino oscillation acts as an amplifier of the tiny difference between gravitational couplings of electron and muon neutrinos.

To realize the test of the equivalence principle we need "gravitationally dressed" neutrino beams. The natural candidates are high energy accelerator neutrino beam [7,8] and the low energy solar neutrinos.[6,8–10] Despite that it is a low energy beam it can be used for our purpose because

of the presence of the MSW-like mechanism. We do not know the way how to utilize neutrinos from supernova to place a stringent limit comparable with that of the Eötvös-Dicke type experiments.

There are good and bad news in using these two beam sources. For solar neutrinos the good news is that the new generation high-statistics experiments, Superkamiokande and SNO, are ready to go. The bad news is the well-known solar neutrino problem which acts as an uncertainty for our present purpose. On the other hand, the good news for accelerator neutrinos is, of course, the beam is controllable with no missing beam problem! The bad news is, however, the funding is not yet decided in spite of the many proposal including Fermilab $\rightarrow$ SOUDAN2, CERN $\rightarrow$ Gran Sasso, KEK $\rightarrow$ Superkamiokande, and Brookhaven experiments.

Let us start our analysis with solar neutrinos for which a detailed analysis is available[10]. We intend to make a pilot computation to have a feeling on what would be the ultimate sensitivity in detecting violation of the equivalence principle. To utilize solar neutrinos for our purpose we have to identify what is the cause of the solar neutrino problem. Since the problem is not yet settled I assume here the MSW solution of the solar neutrino problem. It makes our analysis model-dependent. Of course, the different options for the solar neutrino solution are possible.

In our pilot computation we pinpoint the MSW parameter as $\Delta m^2 = 6 \times 10^{-6} \text{ eV}^2$ and $\sin^2 \theta_M = 0.01$ for nonadiabatic solution, and $\Delta m^2 = 2 \times 10^{-5} \text{ eV}^2$ and $\sin^2 \theta_M = 0.7$ for large-mixing-angle solution. Considering the new-generation high-statistics solar neutrino experiments, it is not so unrealistic to assume the situation that the MSW mechanism is identified as the cause of the solar neutrino deficit and the vacuum mixing parameter is determined up to the uncertainty within which the neutrino spectrum essentially remains unchanged.

I present here a part of our results obtained in Ref.[10] to which I refer for further details of our analysis. In Fig. 1 presented is the sensitive region of parameters, Δf and $\sin^2 2\theta_G$, to possible nonuniversality in gravitational couplings of neutrinos at 90% confidence level. The case of MSW parameters for nonadiabatic solar neutrino solution is examined. Asterisks and open circles correspond, respectively, to 20000 and

Fig. 1 The regions of sensitivity at 90 % confidence level to the violation of the equivalence principle are drawn on the parameter plane of $\Delta f - \sin^2 \theta_G$. The result is for Super-Kamiokande and, 40000 (open circles) and 20000 (asterisks) solar neutrino events are assumed, roughly corresponding to its operation over 4 and 2 years, respectively. The flavor mixing parameter for nonadiabatic MSW solution are taken as (i) $\Delta m^2 = 6 \times 10^{-6} \text{ eV}^2$ and $\sin^2 2\theta_M = 0.01$.

Fig. 2 The 90 % confidence level regions of sensitivity are drawn for flavor mixing parameter (ii) $\Delta m^2 = 2 \times 10^{-5} \text{ eV}^2$ and $\sin^2 2\theta_M = 0.7$ of large-mixing angle MSW solution. The result is for Super-Kamiokande and assumes 40000 (open circles) and 20000 (asterisks) solar neutrino events.

Fig. 3 Plotted are 90% confidence-level sensitivity contours for the long-base line appearance experiment, $\nu_\mu \rightarrow \nu_\tau$, with massless neutrinos. The total event number is assumed to be 10000. The solid line is for the distance 800km and the dashed line for 6000km. The neutrino energy spectrum of the Fermilab main injector [14] is used

40000 events in a water Cherenkov detector which roughly amounts to 2 and 4 years operation of Superkamiokande. Notice that the result depends on the sign of Δf and the relative sign of θ_G to θ_M is meaningful. The analysis relies on the spectral shape analysis which imitates the one done by the Kamiokande group to exclude the adiabatic MSW solution[12].

One can see that the sensitivity goes down to $|\Delta f| = 10^{-15} - 10^{-16}$ depending upon $\sin 2\theta_G$ for both signs of Δf . A similar computation is repeated for the heavy-water Cherenkov detector, SNO. The detector is advantageous in using the charged current reaction to measure the neutrino energy spectrum. On the other hand its statistics is about 1/3 compared with Superkamiokande. It turns out that the merit and the demerit almost cancel and we obtain the result practically indistinguishable with that of Fig. 1. So we do not present it here, referring [10] for interested readers.

Fig. 4 90% confidence-level sensitivity contours for the short-base line experiments, $\nu_\mu \rightarrow \nu_\tau$, with massless neutrinos. The three lines, from left to right, indicate the expected sensitivities of CHORUS, Fermilab E531, and a long-base line disappearance experiment (for the purpose of comparison) corresponding to 10% deficit analyzed in Ref. [7]

We obtain a qualitatively different result with the large-mixing-angle MSW parameters. The region of sensitivity is at most $10^{-13} - 10^{-14}$ as can be seen in Fig. 2. Again we present only the result for Superkamiokande because the result for SNO looks very similar. The reason for such great insensitivity (compared with the non-adiabatic MSW case) is basically due to the difficulty in perturbing the flavor MSW mechanism working in its full adiabaticity by the tiny non-universal gravity effects.

Let us turn to the analysis with accelerator neutrinos. It appears that the most sensitive way of probing nonuniversal gravitational couplings is by doing appearance experiments, $\nu_\mu \rightarrow \nu_\tau$ or $\nu_\mu \rightarrow \nu_e$. While the long-baseline experiments are desirable and powerful in deriving a stringent constraint, $\Delta f = 10^{-14} - 10^{-15}$, the short-baseline experiments can compete in their accuracy with the Eötvös-Dicke type experiments by achieving a great amount of statistics. I shall indicate this by performing the analysis by using the parameters of the CHORUS experiment.

Since the space is limited I won't describe any detail of our analysis[13]. The analysis essentially follows that of Ref.[7] but we have used a realistic density profile of the earth. In Fig. 3 I present 90% confidence-level sensitivity contours for long-baseline experiments using the channel $\nu_\mu \rightarrow \nu_\tau$ in the case of massless neutrinos. The event number is taken as 10000, a typical number for long-baseline neutrino experiments. It anticipates the Fermilab main injector as neutrino beam source[14] and assume the detectors 800 km (solid line) and 6000 km (dashed line) away from Fermilab, imagining SOUDAN2 and DUMAND, respectively. The former set-up is similar to CERN $\rightarrow$ Gran Sasso in its distance apart from the fact that the CERN neutrino beam will be more energetic. One can see from Fig. 3 that the region of $\sin^2 2\theta_G$ one can explore is up to 10^{-3} in this type of experiment due to the limitation of statistics. Depending upon the path length one can expect the sensitivity $\Delta f = 10^{-14} - 10^{-15}$ as represented in Fig. 3.

Finally I analyze the CHORUS experiment which is ongoing and aims at ultimately taking the data of 5×10^5 events. Again I examine only the massless neutrino case and assume the channel $\nu_\mu \rightarrow \nu_\tau$. As a simplified treatment we assume the mono-energetic neutrino beam of 27 GeV and roughly estimate the ratio of charged current μ cross section to τ cross section by linear extrapolating the numbers given in Ref.[15]. The result of our analysis is given in Fig. 4. With enormous amount of statistics (this is a neutrino experiment!) one can explore the region of $\sin^2 2\theta_G$ well beyond 10^{-4} . The sensitivity on Δf is also comparable with that of the Eötvös-Dicke type experiments, $\Delta f = 10^{-11} - 10^{-13}$.

Thus the gravity-induced neutrino oscillation provides a sensitive probe into violation of the equivalence principle. I concluded my talk by saying that the neutrinos are useful probe into gravitational physics as well as to particle physics and astrophysics.

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