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A New Estimate of the Uncertainties in the Primordial Helium Abundance: New Bounds on Ω Baryons

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ABSTRACT

We re-examine the systematic errors in the determination of the primordial helium abundance, Y_P . We find that the systematics are significantly larger than the statistical errors. The uncertainty in (the determination of) Y_P , is thus, larger than is currently claimed. Furthermore, all these systematics lead to an underestimate of Y_P . We argue that the true upper bound on Y_P could be as large as 0.255, which allows cosmological models with no non-baryonic dark matter in which $\Omega_{total} = \Omega_{baryons} = \Omega_{BBN} = 0.2h_{50}^2$.

Subject headings: cosmology: observations—H II regions—radiative transfer

1. Introduction

The current limit (Kernan & Krauss 1994) on Ω_{BBN} , the baryonic density parameter that matches big bang nucleosynthesis, $\Omega_{BBN} < 0.06 h_{50}^{-2}$ (where h_{50} is the Hubble constant in units of $50 \text{ km s}^{-1} \text{ Mpc}^{-1}$) is in conflict with other observations of the density parameter Ω . A dynamical estimate to Ω from inferred motions of virialized systems on small scales

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gives $\Omega_{dyn} = 0.1 - 0.2$ (Dekel 1994). In order to satisfy the above BBN limit an unknown non-baryonic dark matter was invoked. Theoretical prejudice, based on the inflationary scenario, favor a flat universe with $\Omega_{total} = 1$. For $\Omega_{total} = 1$ the observed ratio of matter+gas to total mass in clusters (White *et al.* 1993) $\Omega_{baryons}/\Omega_{total} \geq 0.01 + 0.14h_{50}^{-1.5}$ is in conflict with the above constraint on Ω_{BBN} . This uncomfortable low limit has provoked many attempts to increase the bound on Ω_{BBN} , most of which were unsuccessful, while the few successful ones (Gnedin & Ostriker 1992; Gyuk & Turner 1994) are based on drastic modifications of the standard picture.

In this paper we reexamine the observational basis for the constraints imposed on Ω_{BBN} , with a particular emphasis on the primordial abundance of helium, Y_P . The uncertainties are dominated by systematic errors which generally escape the attention of those who use Y_P limits for theoretical constraints. During the last few years there has been substantial improvement in the atomic data for helium, as well as of the methods for solving detailed radiative transfer. In this paper we take advantage of both in order to obtain a quantitative estimate of the systematic errors in the derivation of primordial helium abundances.

We feel that the abundance of ^4He continues to be an important observational constraint on Ω_{BBN} , because of recent uncertainties in the determination of ^7Li and D . While ^6Li in old stars could be a constraint to ^7Li , progress there is still slow. The D abundance could play a major role in studying BBN, if constrained more strongly. Unfortunately, current values of D_P from interstellar medium determinations have a large scatter around $D/H \approx 1.5 \times 10^{-5}$ (Boesgaard & Steigman 1985; Linsky *et al.* 1993), while the extragalactic measurements in QSOs (Songaila *et al.* 1994; Carswell *et al.* 1994) are still inconclusive.

2. The ^4He Limit

The observational determination of Y_P relies exclusively on the use of the optical recombination spectra of metal-poor, extragalactic ionized nebulae – H II regions and H II galaxies (Peimbert & Torres-Peimbert 1974; Pagel *et al.* 1992 [PSTE]; Skillman *et al.* 1993). In the absence of complicating effects, the He/H abundance ratio could be deduced from relative intensities of hydrogen and helium lines using theoretical effective recombination coefficients. The current quoted values of Y_P are based on assumption of Case B radiative recombination theory (Baker & Menzel 1938) and the emissivities of Brocklehurst (1972). Case B is a simple approximation in which one assumes that the line photons (mostly resonance lines of abundant ions) are scattered so many times that their downward radiative transitions can effectively be omitted from consideration. The other extreme assumption, Case A, is that all emitted line photons escape and there is no radiative transfer problem. The

real situation, however, is intermediate (Osterbrock 1989, Ch.4), non-linear and depends on the ions involved. The important sources of errors inherent to the H II galaxy method have been already identified and discussed (see *e.g.*, Skillman & Kennicutt 1993 [SK]; Davidson & Kinman 1985, and PSTE). However, they were not always quantified and used to set limits. Most of these are of systematic nature and fall generally into the category of radiative transfer problems. We consider three of them: **(1)** The invalidity of Case B for H ; **(2)** The detailed radiative transfer in $He\ I$; and **(3)** The amount of neutral helium.

We examine the above systematic effects with the help of detailed radiative transfer calculations for H, He I & II. We solve simultaneously the coupled statistical equilibrium and transfer equations in a plane-parallel illuminated slab geometry using the approximate Λ -operator method (Scharmer & Carlsson 1985). This is a full multi-level non-LTE calculation for the atomic models of H, He I, and He II. We take special care of the helium resonance lines transfer, *e.g.*, He II $\text{L}\alpha$ 304 Å, which can be an important ionization source. In constructing the slabs we used photoionization equilibrium calculations with a mean escape probability formulation after Proga *et al.* (1994). The helium atomic model is similar to that of Almog & Netzer (1989), but contains some new atomic data from the Opacity Project database, the new term values from Drake (1993), and the collisional rates evaluated by P.J. Storey from the cross sections of Berrington & Kingston (1987) (see Sasselov & Lester (1994) for some details on the atomic data). We compare the results of our computation to the calculations using Case B analysis with the standard “corrections” (see PSTE) for the same physical problem. The comparison gives us a *realistic* estimate of the systematics in the determination of Y_p from the observed data.

We apply the results of our calculations of systematic effects to four recent values of Y and Y_p as derived by different observers: Y_p by Pagel *et al.* (PSTE) derived from He *vs.* O and He *vs.* N; Y_p by Olive & Steigman (1994) from a new set of low-metallicity H II regions observed and analyzed by Skillman *et al.* (1993) and combined with the set of Pagel *et al.* (for both O and N); and the abundance determination in the most low-metallicity system – I Zw 18 by SK.

We proceed by describing briefly the specific problems and their overall contribution to the individual systematic errors for each case. Additional tabular data from our calculations will be given in a separate paper.

[(1)] *The invalidity of Case B for H*: The low-metallicity H II regions have high enough electron temperatures, T_e , that some of the observed $H\alpha$ flux may be due to collisional excitation (Davidson & Kinman 1985). This is a deviation from Case B in the sense of its assumption of independence between the level populations for $n \geq 3$ and those of levels $n = 1$ and 2. Thus, upward collisional transitions may enhance $H\alpha$ above its Case B

intensity. The rate depends on the ground-state population, N_{H^0} , which in principle can be obtained from detailed radiative transfer calculations given a well constrained geometry by high-resolution observations. Unfortunately, such observations are beyond the capability of current telescopes.

To demonstrate the magnitude of this effect we consider the NW H II region in I Zw 18 for which high resolution data and an *HST* (Kunth *et al.* 1994) constraint on the Lyman continuum flux are available. We find an amount of H^0 : $N_{H^0}/N_{H^+} \approx 8 \times 10^{-4} f_2 f_1^{-1/2}$ (f_1 and f_2 are volume filling factors for the ionized gas (0.02) and ionizing photons (0.1) respectively). The value is larger than previous estimates (Davidson & Kinman 1985; SK) by more than a factor of 2. Unless the above filling factors (Dinerstein & Shields 1986; Shields 1987) are wrong by an even larger factor, this amount of H^0 leads to an increased $H\alpha$ flux with a resulting 2% effect on the derived helium abundance.

The collisional excitation of $n = 3$ of H^0 increases with T_e (see Table 3 of Hummer & Storey 1987), and thus is more of a problem for the sets of most metal-poor H II regions, like that of Olive & Steigman (1994), where temperatures higher than 15,000 K are common. Also, the collisional correction for $H\beta$ (the line used in PSTE) is smaller, than that for $H\alpha$ (used by SK and Olive & Steigman). In summary, though small, the error is systematic; always in the direction of increasing Y of up to 3%. The volume filling factors cannot be constrained better at present, hence our uncertainty remains as a bound. This error was discussed by SK, but neglected for lack of constraining information, while PSTE did not consider it at all.

[(2)]: *The detailed radiative transfer in He I*: In general, recombination theory is rarely simply valid for neutral helium, because of the distinct singlet and triplet species of He I, with the strongly metastable 2^3S state of the triplets. In the calculations of effective recombination coefficients the triplet series is often treated only as Case B, while the singlets are treated in both Case A and B (e.g. Brocklehurst 1972; Smits 1991). The intersystem transitions coupling the singlet and triplet series are not included, except for the latest models (Smits 1994) which include only those between the $n = 2$ levels. Through this coupling the accurate calculation of the singlets of He I becomes dependent on the detailed treatment of the triplets. The triplets are more sensitive to collisional and self-absorption effects in low-density plasmas, and hence to the varying optical depth. Therefore, except under very low- N_e and low- T_e conditions (Clegg & Harrington 1989), it is necessary to solve simultaneously for all species, as well as non-locally. The use of simple recombination theory excludes also any consideration of inter-species radiative transfer effects (see problem 3 below).

In order to illustrate the problem we have calculated detailed models, extending to lower densities the models of Almog & Netzer (1989). In Figure 1 our results are compared to

Case B calculations, and to two observed low-metallicity H II galaxies for which we found published data. This line-ratio diagram is a diagnostic diagram which illustrates best the dependence of line transfer on plasma conditions, and thus show the physics involved in the resulting differences. It juxtaposes the singlets (He I λ 6678) to the triplets (He I λ 7065). Through the fluorescent enhancement of λ 7065, this diagram can also indicate large optical depth, τ_{3889} , in the He I λ 3889 line. In the lowest density regime ($1\text{--}100\text{ cm}^{-3}$), both the singlet λ 6678 line and the He I λ 5876 line are sensitive to T_e and N_e in the same way, hence their ratio stays practically constant, instead of going up with the strengthening of the He I λ 6678 line. This is not true for the triplet λ 7065 transition which is fluorescent (coupled to λ 3889), its emissivity increases fast with τ_{3889} , as collisions and optical depth become more important.

Thus Figure 1 illustrates the degree to which the singlet line He I λ 6678 can be affected by increasing optical depth in the triplets: if one eliminates λ 5876 and uses λ 7065 as a τ_{3889} -indicator, one can see that for $N_e > 10\text{ cm}^{-3}$ the λ 6678 line starts to behave differently. Between 100 and 300 cm^{-3} at $T_e = 20,000\text{ K}$ (*e.g.* SK derive for I Zw 18: $T_e = 19,600\text{ K}$, $N_e \approx 100\text{ cm}^{-3}$) the rate of change of the λ 6678 line is the largest and away from the Case B predictions. Practically the same is true at $T_e = 15,000\text{ K}$, while at $10,000\text{ K}$ this happens in the $600\text{--}1000\text{ cm}^{-3}$ densities range. As Fig. 1 shows all Case B models underestimate the lines sensitivity to N_e (for both He I systems), as is inherent to Case B. Moreover, in our detailed models the density sensitivity increases with T_e , as expected, while no such trend is discernible in the Case B models.

The line ratios for the observations plotted in Fig. 1 have been corrected by the original authors (Skillman *et al.* (1994); PSTE) for absorption by assuming Case B for the Balmer lines and assuming some stellar absorption in each Balmer line. The fact that both galaxies are displaced towards the upper right corner (away from our models) could be due to unaccounted absorption along the line-of-sight (A_V). Despite the uncertainties of these observed line ratios, the two observations in Fig.1 appear to support the new features of the detailed models. If Case B models were correct (T_e -dependence, no N_e -dependence), UGC 4483 and II Zw 40 would have had to swap places in Fig. 1 (since II Zw 40 is cooler). Obviously, that could not be due to reddening, A_V , which acts orthogonally in this diagnostic diagram. It is due to the insensitivity of Case B models to density (II Zw 40 is more than 2 times denser). From Fig.1 one can see that, less the absorption uncertainty, our models describe qualitatively (and perhaps, quantitatively) the observations, while all other models fail to do so.

Although systematics in the radiative transfer of He I were known to exist, they were usually accounted for, partially, in terms of collisional corrections to the Case B values.

Fig. 1.— He I intensity ratios: tick marks for densities of 1, 10, 100, 10^3 , and 10^4 cm^{-3} ; solid line for $T_e = 20,000 \text{ K}$. Observed ratios for UGC 4483 ($T_e = 16,600 \text{ K}$, $N_e \approx 100 \text{ cm}^{-3}$) and II Zw 40 ($12,700 \text{ K}$, 225 cm^{-3}). The plotted model of Almog & Netzer (1989) was specially designed for comparison to Brocklehurst (1972). Absorption along the line of sight, A_v , displaces observed ratios towards the upper right corner.

Such treatment does not account for differential optical depth effects and is only valid under conditions in which decoupled behaviour (see above) is assured (below 50 cm^{-3} in our calculations). Unfortunately, although small, the discrepancy is systematic (underestimating Y). PSTE acknowledged it (to a few percent) without quoting a specific number; and SK noted that it could be larger than a few percent but still did not quote limits. In summary, the systematic effect due to problem (2) could increase Y by as much as 4%–6% (for He I λ 6678) and 5%–8% (for He I λ 4471) for T_e between 15,000 and 20,000 K, and N_e between 100 and 300 cm^{-3} . Such values apply to the most metal-poor H II regions, which determine Y_P .

[(3)] *Neutral helium*: This problem arises from the lack of observational indication of the amounts of neutral helium (similarly to hydrogen) in an H II region. Yet any N_{He^0} not accounted for underestimates the final abundance Y . Very detailed model calculations constrained by the best H II region observations, estimate N_{He^0} at 3.2% in the Orion Nebula (Baldwin *et al.* 1991). In the extragalactic H II regions the problem becomes extremely difficult to solve correctly and consistently, because of its strong dependence on geometry and composite H II regions (Dinerstein & Shields 1986). However, one effect is likely to be present often – the coupling between the hydrogen Lyman continuum (λ 912) and the He I (λ 504) continuum, which can become optically thick only in accord with the optical depth of hydrogen, τ_{912} , because the dominant opacity at 504 Å is due to the Lyman continuum. Once the Lyman continuum becomes optically thick, the ionization equilibrium of hydrogen shifts in favor of H I and affects N_e ; hence – the total recombination rate of He I. This is an example of inter-species effects, which we include in the calculation. With the conditions used in Problem 1, the systematic error due to the presence of neutral helium can vary from zero to 3%. PSTE considered this systematic error to be negligible for most of their sample; SK did not add a correction for neutral helium.

The corrections for each Y value due to each radiative transfer problem, relevant for the particular lines observed by each of the groups, are listed in Table 1. The final upper bounds result from the systematic effects added, observers’ collisional corrections subtracted, and the 2σ observational statistical bound included. Our approach is to add all uncertainties and derive upper bounds, because all effects we studied, plus some we do not examine (e.g., consideration of underlying stellar absorption), always lead to a systematic underestimate of Y_P (see SK). For each upper bound on Y_P we derive a corresponding bound on the baryon-to-photon ratio, η_{median} in terms of $\eta_{10} = \eta \times 10^{10}$, from the most recent theoretical relation of Kernan & Krauss (1994) and their 95% confidence limits, $\eta_{95\%c.l.}$.

TABLE 1
THE UPPER BOUNDS ON Y_P , $\eta_{10} = \eta \times 10^{10}$, AND Ω_{BBN} .

	Pagel et al. He <i>vs.</i> O	Pagel et al. He <i>vs.</i> N	Olive & Steigman	I Zw 18 NW
Y_P	0.228	0.233	0.232	0.231
1σ	± 0.005	± 0.004	± 0.003	± 0.006
systematics:				
Problem 1	+0.005	+0.005	+0.006	+0.005
Problem 2	+0.013	+0.013	+0.010	+0.008
Problem 3	+0.002	+0.002	+0.002	+0.002
U.B. on:				
Y_P	0.257	0.260	0.255	0.258
$\eta_{10, median}$	17.72	25.06	14.00	19.85
$\eta_{10, 95\% c.l.}$	19.43	28.22	15.52	22.06
$\Omega_{BBN}(h = 0.5)$	0.284	0.412	0.226	0.322
$\Omega_{BBN}(h = 0.7)$	0.145	0.21	0.116	0.164

3. The Implications for Ω

From the above analysis we put most weight on two of our derived upper bounds: the one on Y_P by Olive & Steigman, and the one on Y of the NW H II region of I Zw 18 by SK. Their essence is different. While Olive & Steigman give a value of Y_P based on careful extrapolation of the relation through the set of most recent H II region abundances, there is only a Y value for I Zw 18 which is by no means truly primordial Y_P . However, Y in I Zw 18 is very carefully derived by SK from their modern high-quality data, hence we are more confident about the size of the systematic errors (0.015) applied to it. The gas in I Zw 18 is not primordial, but is the closest to primordial we can get in H II regions (Kunth *et al.* 1994), and it is good to keep its Y value in mind. As a result of all this, we suggest that current ^4He abundance derivation methods do not exclude a $Y_P \leq 0.255$, and perhaps even a value as high as $Y_P \leq 0.258$. We would like to point out that $Y_P \leq 0.255$ is allowed by the values and bounds for ^7Li , which are calculated from stellar models that include diffusion and rotational mixing (Chaboyer & Demarque 1994), as well as by the current uncertain limits on D and $\text{D}+^3\text{He}$ (see section I).

We calculate an upper bound on Ω_{BBN} using the connection between η_{10} and Ω_{baryon} : $\Omega_{baryon} h_{50}^2 = 0.00365 \eta_{10} 4T_{2.726}^{-3}$ (where $T_{2.726}$ is the temperature of the CMBR in units of 2.726 K). From the above limits on Y_P we get that Ω_{BBN} can be as high as $0.22 h_{50}^{-2}$, and it might be even as large as $0.32 h_{50}^{-2}$. Our results using $\eta_{10,95\%c.l.}$ are summarized in Table 1.

The new upper limits on Ω_{BBN} would enable us to reconsider many cosmological models which were in conflict with the old limits. A universe with $\Omega_{total} = \Omega_{BBN} = \Omega_{dyn}$ is in agreement with the new bound and there is no need to invoke non-baryonic dark matter. One such model is the primeval baryon isocurvature (PBI) model of Peebles (1987a,b) with $\Omega \approx 0.1$. Moreover, in the model that was suggested by Gnedin and Ostriker (1992), in which $\Omega_{BBN} = \Omega_{dyn}$, unconventional scenarios were needed in order to justify the required high values of Y_P . These Y_P values are well inside our new limit and therefore this model can rely on conventional cosmology. Finally, a value of Y_P near 0.251 is all that is needed in order that the fraction of baryons in clusters (White *et al.* 1993) will agree with the value of Ω_{BBN} in a flat universe dominated by cold dark matter.

4. Conclusions

In this paper we do not determine a new value for the primordial helium abundance, Y_P ; we demonstrate that some important systematic errors may have been underestimated. When accounted for, they provide an upper bound on Y_P which is less confining than currently assumed. The new bound on Y_P would relax substantially the previous tight constraints on the number of effective light neutrino species (Walker *et al.* 1991; Kernan & Krauss 1994). Alternatively, the new bound on Y_P can provide an upper limit to Ω_{BBN} which is much less restrictive for cosmological models.

The new relatively high upper bound on Ω_{BBN} would remove the need to turn to exotic models in order to reconcile between various observations of the density parameter. Moreover, the simplest and most esthetic model in which $\Omega_{total} = \Omega_{BBN} = \Omega_{dyn} \approx 0.1$, a model that does not rely on unobserved entities (dark matter) or unconventional physics and is consistent with many observations (Peebles 1993, and references there), is also consistent with BBN limits.

Because of the logarithmic dependence of Y_P on η , a very accurate determination of Y_P is needed in order to determine Ω_{BBN} precisely. Unfortunately, because of the observational limitations of the H II galaxy method, all we were able to calculate in this paper are upper bounds on Y_P and, therefore, we are unable to constrain further the cosmological models. Two of the important sources of systematic errors in the H II galaxy method discussed here are related to the geometry and stellar content of the H II regions. Such constraints cannot

be achieved with the resolving power of any current telescope. Even with the best-observed nearest H II region (the Orion Nebula) the uncertainties in the helium abundance are still difficult to reduce below 7% (Baldwin *et al.* 1991). Therefore, modern sophisticated radiative transfer techniques cannot be applied with equal success to the extragalactic H II regions. As an alternative, the oldest stars in our galaxy, metal-poor halo dwarfs, which are much closer (by 2 orders of magnitude) to primordial abundances in their unprocessed atmospheres, may provide a way to derive Y_{P} with precision interesting for cosmology. A project to derive helium abundances in extreme halo dwarfs is underway (Sasselov 1992; Sasselov & Lester 1994). Once we can determine accurately the value of Y_{P} we will be able to reject/verify the validity of the various cosmological models.

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