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A thermodynamical bag model for the nucleon spin

V. Devanathan

*Institute for Nuclear and Particle Physics
University of Virginia, Charlottesville, Virginia 22901, U.S.A.
and*

*Department of Nuclear Physics, University of Madras
Guindy Campus, Madras 600 025, India*

J .S. McCarthy

*Institute for Nuclear and Particle Physics
University of Virginia, Charlottesville, Virginia 22901, U.S.A.*

ABSTRACT

A thermodynamical bag model has been developed to study the nucleon spin. Equations of state for the bag are deduced and solved self-consistently. The recent experimental data of the SMC and the E143 collaboration and the Bjorken Sum Rule are completely explained within the framework of the model.

PACS numbers: 13.60Hb, 12.38Qk, 12.39Ba

Ever since the publication of the EMC results [1] in 1988 on the deep inelastic scattering of polarized leptons on polarized proton, there has been considerable excitement as to what constitutes the nucleon spin since the data implied that only a small fraction of the proton spin is carried by the quarks. This is in contradiction with the naive expectation from the low-energy constituent quark models which were so successful in explaining the entire hadronic spectrum with all its static properties such as mass, charge, spin and magnetic moment that the startling result of the EMC experiment has caused what is known as 'the spin crisis'. This has generated considerable activity among the theorists and experimentalists. A second generation of experiments [2–5] was planned and executed. The results have just started arriving. Besides the proton spin structure function $g_1^p(x)$, the neutron spin structure function $g_1^n(x)$ has been extracted [3] from a measurement on 2D and 3He . Their integrals are related by the Bjorken Sum Rule (*BSR*) [6].

$$BSR = \Gamma_1^p - \Gamma_1^n = \frac{1}{6}(g_A/g_V)(1 - \frac{\alpha(Q^2)}{\pi}), \quad (1)$$

which follows rigorously from QCD.

In the parton model, the spin structure function of the nucleon can be expressed in terms of the parton spin distribution functions.

$$g_1(x) = \frac{1}{2} \sum_i e_i^2 [q_i^+(x) + \bar{q}_i^+(x) - q_i^-(x) - \bar{q}_i^-(x)], \quad (2)$$

where e_i is the charge of the quark (antiquark) of flavor i and $q_i^+(x)$ ($\bar{q}_i^+(x)$) is the quark (antiquark) distribution function of momentum fraction x , having the helicity parallel to that of the nucleon and $q_i^-(x)$ ($\bar{q}_i^-(x)$) is that with helicity antiparallel to that of the nucleon. Restricting our consideration to u and d quarks only, the proton and neutron spin structure functions can be written explicitly.

$$g_1^p(x) = \frac{1}{2} \left[\frac{4}{9} U(x) + \frac{1}{9} D(x) \right], \quad (3)$$

$$g_1^n(x) = \frac{1}{2} \left[\frac{1}{9} U(x) + \frac{4}{9} D(x) \right], \quad (4)$$

where

$$U(x) = [u^+(x) + \bar{u}^+(x)] - [u^-(x) + \bar{u}^-(x)], \quad (5)$$

$$D(x) = [d^+(x) + \bar{d}^+(x)] - [d^-(x) + \bar{d}^-(x)], \quad (6)$$

with the usual convention

$$d^n(x) = u^p(x) = u(x); \quad u^n(x) = d^p(x) = d(x). \quad (7)$$

The integrals Γ_1^p and Γ_1^n of the proton and neutron spin functions have special significance since their difference is known as the Bjorken sum rule defined in Eq.(1).

$$\Gamma_1^p = \int_0^1 g_1^p(x)dx; \quad \Gamma_1^n = \int_0^1 g_1^n(x)dx. \quad (8)$$

Although the Bjorken sum rule is obtained from general considerations, there is no rigorous theory by which the parton distribution functions can be obtained. Brodsky, Burkardt and Schmidt [7] have recently developed simple analytic representations for polarized quark and gluon distributions in the nucleon incorporating certain constraints demanded by general properties. They are of the nature of parametrized forms given by Martin, Roberts and Stirling [8] given for unpolarized parton distributions. Several phenomenological models [9-14] have been proposed to obtain the parton distribution functions, both unpolarized and polarized and they have been only partially successful in explaining the distribution functions in the valence quark region. At small x region, these models fail miserably and the sea quarks and gluons have to be put in by hand to explain the small- x features.

Recently, a thermodynamical bag model has been developed by Devanathan et al [15,16] and it yields the parton distribution functions with correct asymptotic behavior, viz., vanishing at $x = 1$ and increasing as $x \rightarrow 0$. The purpose of this letter is to include the spin degree of freedom and develop the model to study the spin structure of the nucleon.

The nucleon is considered to be a MIT bag consisting of quarks and gluons [15-18]. For simplicity, let us treat the quarks as Fermi gas and the gluons as Bose gas and use the appropriate statistical distribution functions to study their behavior. For quarks, we need to use the Fermi distribution function which includes the spin degree of freedom [19,20]. Let us write down explicitly the number densities [20] of u and $\bar{u}$ quarks with spin up and spin down.

$$n_u^{\uparrow} = \frac{1}{8\pi^3} \int \frac{d^3p}{\exp[\beta(p - \mu_u - \frac{1}{2}\gamma_u)] + 1}, \quad (9)$$

$$n_u^{\downarrow} = \frac{1}{8\pi^3} \int \frac{d^3p}{\exp[\beta(p - \mu_u + \frac{1}{2}\gamma_u)] + 1}, \quad (10)$$

$$n_{\bar{u}}^{\uparrow} = \frac{1}{8\pi^3} \int \frac{d^3p}{\exp[\beta(p + \mu_u - \frac{1}{2}\gamma_u)] + 1}, \quad (11)$$

$$n_{\bar{u}}^{\downarrow} = \frac{1}{8\pi^3} \int \frac{d^3p}{\exp[\beta(p + \mu_u + \frac{1}{2}\gamma_u)] + 1}. \quad (12)$$

A similar set of equations can be written for d quarks for which the chemical potential is μ_d and the spin parameter is γ_d . The energy densities ϵ for $u, \bar{u}, d$ and $\bar{d}$ quarks with positive and negative helicities can also be written in a similar way assuming the quarks to be of zero rest mass.

The Lagrangian multipliers $\beta(=1/T)$, μ_u , μ_d , γ_u and γ_d are determined from the constraints on energy, particle number and the spin of the system. For the proton, we have the following equations of state to determine the Lagrangian multipliers.

$$\epsilon(T)V + BV = W. \quad (13)$$

$$gV[(n_u^\uparrow + n_u^\downarrow) - (n_u^\uparrow + n_u^\downarrow)] = 2. \quad (14)$$

$$gV[(n_d^\uparrow + n_d^\downarrow) - (n_d^\uparrow + n_d^\downarrow)] = 1. \quad (15)$$

$$gV[(n_u^\uparrow + n_u^\downarrow) - (n_u^\uparrow + n_u^\downarrow)] = a. \quad (16)$$

$$gV[(n_d^\uparrow + n_d^\downarrow) - (n_d^\uparrow + n_d^\downarrow)] = b. \quad (17)$$

$$P = \frac{1}{3}\epsilon(T) - B = 0. \quad (18)$$

The multiplicative factor g denotes the color degeneracy which is 3 and V is the volume of the bag. The bag constant is denoted by B and the mass or energy of the system by W . The energy density $\epsilon(T)$ of the quark-gluon gas can be written explicitly in terms of the energy densities of the constituents.

$$\epsilon(T) = 3(\epsilon_u^\uparrow + \epsilon_u^\downarrow + \epsilon_u^\uparrow + \epsilon_u^\downarrow) + 3(\epsilon_d^\uparrow + \epsilon_d^\downarrow + \epsilon_d^\uparrow + \epsilon_d^\downarrow) + 16\epsilon_g. \quad (19)$$

Eqs.(14) and (15) have the following physical interpretation. For the proton, the number of u -valence quarks is 2 and the number of d -valence quark is 1. The pressure balance condition or the energy minimization condition is given by Eq.(18). Eqs.(16) and (17) have a deep physical meaning. They denote the separate spin contributions from u and d quarks to the spin of the proton. This is the crux of the problem. In the constituent quark model of the proton, the entire proton spin is attributed to the spin of the constituent quarks. Then $a + b = 1$. Close [21] argues that the u -quarks, being antisymmetric in color should be symmetric in spin (being in s -state) and consequently the spin contribution a from the u -quark should be $4/3$ and the contribution b from the d -quark $-1/3$ such that $a + b = 1$. In the parton model that we use, such a simple interpretation does not hold true. The quarks have considerable amount of transverse momentum and hence there is nothing in principle that will preclude the proton spin receiving contributions from the orbital angular momentum of its constituents. This has been suggested by Leader and Anselmino [22] and, in fact, we find that only 40 % of the proton spin is due to the spins of u and d quarks. Choosing $a = 1.1$ and $b = -0.7$ such that $a + b = 0.4$, the model explains completely the spin structure of both the proton and the neutron and yields the Bjorken Sum Rule accurately. This is supported by the recent E143 collaboration experimental data [2] which indicate that $0.34 \pm 0.04 \pm 0.09$ of the nucleon spin is carried by quarks.

Eqs.(13) to (18) are the equations of state for the bag and analytical expressions for these equations can be obtained in terms of the bag variables $V, T, B, \mu_u, \mu_d, \gamma_u$ and γ_d using the Fermi distribution functions (9-12) for the quarks and the Bose

distribution function for the gluons. The energy density that occurs in Eq.(13) has the analytical form

$$\epsilon(T) = \frac{37}{30}\pi^2 T^4 + \frac{3}{2}T^2(\mu_u^2 + \mu_d^2 + \frac{1}{4}\gamma_u^2 + \frac{1}{4}\gamma_d^2) + \frac{3}{4\pi^2}(\mu_u^4 + \mu_d^4 + \frac{3}{2}\mu_u^2\gamma_u^2 + \frac{3}{2}\mu_d^2\gamma_d^2 + \frac{1}{16}\gamma_u^4 + \frac{1}{16}\gamma_d^4). \quad (20)$$

Analytical expressions can be obtained similarly for Eqs.(14-17).

$$\frac{V}{2\pi^2}[2\pi^2\mu_u T^2 + 2\mu_u^3 + \frac{3}{2}\mu_u\gamma_u^2] = 2. \quad (21)$$

$$\frac{V}{2\pi^2}[2\pi^2\mu_d T^2 + 2\mu_d^3 + \frac{3}{2}\mu_d\gamma_d^2] = 1. \quad (22)$$

$$\frac{V}{2\pi^2}[\pi^2\gamma_u T^2 + 3\gamma_u\mu_u^2 + \frac{1}{4}\gamma_u^3] = a. \quad (23)$$

$$\frac{V}{2\pi^2}[\pi^2\gamma_d T^2 + 3\gamma_d\mu_d^2 + \frac{1}{4}\gamma_d^3] = b. \quad (24)$$

Using these analytical expressions, the equations of state of the bag have to be solved to yield the bag variables. For the ground state of the nucleon, $T = 0$ and W reduces to the mass of the nucleon $M = 938.7$ MeV. All other quantities $B, V, \mu_u, \mu_d, \gamma_u$ and γ_d are obtained by solving the set of six equations of state of the bag self-consistently. We need to stress here that there is no free parameter.

The invariant mass W of the final hadronic state in deep inelastic scattering is a function of x and Q^2 . By identifying W with the mass of the excited nucleon, the equations of state of the bag are solved self-consistently to yield $T, V, \mu_u, \mu_d, \gamma_u$ and γ_d . Here we have established an intimate relation between the Bjorken variable x and the temperature T of the thermodynamical bag. The smaller the value of x , the greater is the temperature; thereby explaining the copious production of sea quarks and gluons at smaller values of x . The volume of the bag is not a constant but increases with temperature. The chemical potentials μ_u and μ_d and the spin parameters γ_u and γ_d decrease with temperature. In the limiting case of $x = 1$ (elastic scattering), $W = M$, $T = 0$ and the bag radius $R = 0.985$ fm.

The quark spin distribution functions are obtained by transforming the Fermi distribution functions (9-12) to the infinite momentum frame (IMF) and integrating over the transverse momentum following the works of Mac and Ugaz [11], Cleymans and Thews [12] and Devanathan et al [15,20].

$$u^{\uparrow}(x) = A \ln[1 + \exp\{\frac{1}{T}(\mu_u + \frac{1}{2}\gamma_u - \frac{1}{2}MX)\}], \quad (25)$$

$$u^{\downarrow}(x) = A \ln[1 + \exp\{\frac{1}{T}(\mu_u - \frac{1}{2}\gamma_u - \frac{1}{2}MX)\}], \quad (26)$$

$$\bar{u}^{\uparrow}(x) = A \ln[1 + \exp\{\frac{1}{T}(-\mu_u + \frac{1}{2}\gamma_u - \frac{1}{2}MX)\}], \quad (27)$$

$$\bar{u}^{\downarrow}(x) = A \ln[1 + \exp\{\frac{1}{T}(-\mu_u - \frac{1}{2}\gamma_u - \frac{1}{2}MX)\}], \quad (28)$$

with

$$A = \frac{3M^2VT_x}{4\pi^2} \quad (29)$$

A similar set of equations are obtained for d quarks, replacing μ_u by μ_d and γ_u by γ_d . These equations evolve as a function of the Bjorken variable x . The Bjorken variable x has been initially introduced to define a specific kinematic configuration of the reaction but it acquires nice physical interpretations. It can be considered as the inelasticity parameter with values ranging from 0 to 1. The upper limit $x = 1$ corresponds to elastic scattering and the lower limit $x = 0$ corresponds to the extreme inelastic event when the entire energy of the incident lepton is transferred to the target nucleon. Alternatively, in IMF, the Bjorken variable x corresponds to the fraction of the momentum carried by the quark. The interplay between these two definitions of x leads to a new interpretation of the nucleon structure functions and their evolution as a function of x .

The hadronic tensor $W_{\mu\nu}$ that occurs in the study of inclusive reaction $e+p \rightarrow e'+h$ is the current - current interaction.

$$W_{\mu\nu} \sim \sum_h \langle p | J_\mu^\dagger | h \rangle \langle h | J_\nu | p \rangle, \quad (30)$$

where the summation is over all the possible final hadronic state h of invariant mass W . If one attempts to deduce the quark distribution function from the Fermi distribution function, in some way the excitation of the target nucleon to the final hadronic state and its subsequent deexcitation should be incorporated in the phenomenological model by identifying the invariant mass W of the final hadronic state with the mass of the excited nucleon and normalising the distribution function so obtained to yield the experimentally observed fraction of the momentum carried by the quarks by using a single parameter $\eta(Q^2)$. In Eqs. (25-29), the temperature T is replaced by $\eta(Q^2)T$ to obtain the quark spin distribution functions. This is the only parameter used in the theory and the numerical calculations are performed at $Q^2 = 3 \text{ GeV}^2$, choosing the value 0.6 for the parameter $\eta(Q^2)$. The results for unpolarized structure functions are given in Ref.[16] and the results for the polarized structure functions are presented here.

Figs. 1 and 2 depict our results for $xg_1^p(x)$ and $g_1^p(x)/F_1^p(x)$ along with the experimental data of E143 collaboration. The results for the neutron spin structure function $xg_1^n(x)$ are presented in Fig.3. In Table I, we compare our theoretical calculations with the available experimental data. The agreement is indeed remarkable. The theory gives the proton and neutron spin distribution functions as functions of x in agreement with the experimental data and also their integrated values yielding the Bjorken sum rule accurately.

Recently, Bourrely et al. [23] have used the Fermi distribution function for the study of polarized nucleon structure functions but the study involves a large number

of parameters. The present theory, although unconventional, is simple and able to explain the whole gamut of experimental observation with just one parameter $\eta(Q^2)$. The theory can be improved by taking the discrete energy spectrum for the quarks in a finite volume and also by including the massive strange quarks but the simplicity will be lost since analytical expressions for the bag equations will no longer be possible.

It is a pleasure to thank H.J. Weber, P.K. Kabir, M.K. Banerjee, F. Tabakin, L.S. Kisslinger, T.S. Santhanam, M. Rajasekaran and C.M. Ko for useful discussions. One of us (VD) thanks the Council of Scientific and Industrial Research (India), the University Grants Commission (India) and the Institute for Nuclear and Particle Physics, University of Virginia for partial support.

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TABLE I. Comparison of the present calculation ($Q^2 = 3\text{GeV}^2$) with the experimental data [2,3] on spin observables.

	Present Calculation	Experimental data
Γ_1^p	0.134	$0.129 \pm 0.004 \pm 0.010$
Γ_1^n	-0.033	$-0.033 \pm 0.008 \pm 0.013$
BSR	0.168	0.162 ± 0.024
Δu	0.685	0.821 ± 0.034
Δd	-0.320	-0.437 ± 0.035
Δs	—	-0.098 ± 0.037
Δq	0.365	0.287 ± 0.104

FIGURE CAPTIONS

FIG. 1. The graph $xg_1^p(x)$ Vs. x at $Q^2 = 3\text{GeV}^2$. The solid curve denotes the present calculation and the experimental data are from the E143 collaboration [2].

FIG. 2. The graph $g_1^p(x)/F_1^p(x)$ Vs. x at $Q^2 = 3\text{GeV}^2$. The solid curve denotes the present calculation and the experimental data are taken from the E143 Collaboration [2].

FIG. 3. The graph $xg_1^n(x)$ Vs. x at $Q^2 = 3\text{GeV}^2$ for the neutron spin structure function. The experimental points are from E142 data [3] and the solid curve denotes the present calculation.

Fig. 1

Fig. 2