**Cluster formation in disordered systems
and nuclear fragmentation**

B. Elattari¹, J. Richert², P. Wagner

Centre de Recherches Nucléaires, IN2P3-CNRS
and Université Louis-Pasteur
BP28, 67037 Strasbourg Cedex 2, France

and

Y.M. Zheng

China Institute of Atomic Energy, P.O. Box 275(18),
Beijing 102413, People's Republic of China

50095-17

We investigate an approach to the description of nuclear fragmentation relying on the concept of disorder which may characterise a fragmenting system of particles interacting by means of two-body short and long range potentials. We introduce different criteria in order to define clusters of bound nucleons. We work out the fragment content of events generated by means of numerical simulations. We analyse the moments of mass and charge distributions and related observables, as well as the energy content of the system. We discuss the outcome of the calculations in terms of the concept of universality which has been introduced in the field of nuclear fragmentation in the framework of percolation models.

PACS numbers: 25.70Pq, 05.40+j, 02.50Ng.

¹ Also: Université des Sciences Bouchaïb Doukkali,
Eljadida, Morocco

² Also: Laboratoire de Physique Théorique, Institut de Physique,
3-5, rue de l'Université, F-67084 Strasbourg Cedex

1. Introduction

The dynamics of nuclear fragmentation may be complex and the successes of many different phase space and dynamical models indicate that it shows many facets¹⁻⁴⁾. Important issues concerning the outcome of the process have been analysed and discussed in the recent past.

In the present work we start from the idea that complex systems like fragmenting nuclei are essentially completely disordered systems of interacting particles, some of which are bound to form finite clusters of different sizes. We assume that this property reflects in the final mass and charge distributions which are experimentally measured. It is however a non trivial but established fact that disorder may be connected with universality⁵⁾, a concept which reflects the fact that disordered systems are not sensitive to the details of their dynamical characteristics but show features which are common to many of them, such as scaling properties with respect to their size or other physical characteristics. In fact, the concept of universality properties of disordered systems is nicely illustrated by simple-minded percolation models⁶⁾ which are impressively able to reproduce experimental results related to charge distributions⁷⁻⁸⁾. These universality properties are related to the existence of a phase transition in the infinite size limit. But percolation models are purely geometrical descriptions of disordered systems. It is the aim of the present work to develop an approach which contains, at different levels, the microscopic phase space properties of a finite disordered system interpreted as a fragmenting nuclear system and to compare its properties to those predicted by percolation models and existing experimental data.

The content of the paper is the following. In section 2 we introduce the phase space description of the system, a specific bond criterion between neighbouring particles based on energy arguments and different criteria which identify bound clusters in the many-body system. Physical observables related to fragment size distributions and a discussion about the way to characterise them is the subject of section 3. In section 4 we present and discuss calculations of physical observables performed with two different cluster identification algorithms. The results of these calculations are confronted with percolation results and experimental available data. Section 5 is devoted to a detailed analysis of disordered systems by means of a third algorithm which relies on a deterministic characterisation of the bond property of particles. We present a quantitative comparison between the different algorithms and percolation and confront the fragment definition criteria already introduced

with further ones. This constitutes the central part of the work. In section 6 we discuss the influence of the geometry of the system, collective effects, energy-multiplicity correlations and fluctuations. Section 7 contains a summary of the results, further comments, discussions and further questions and perspectives.

2. Finite disordered systems and cluster identification

A fragmenting nucleus is considered as a disordered system of interacting particles which can be bound to form clusters (fragments). We introduce and describe the phase space characteristics of such a system, the criteria by means of which we identify clusters and the physical observables which characterise the fragment size distributions.

2a. Phase space configurations and bond criterion for pairs of neighbouring particles

We consider a finite, compact volume $\mathcal{V}$ in 3-dimensional space, such as for instance a cube. It is occupied by A particles located in cubic cells of linear dimension d such that $\mathcal{V} = Ad^3$. Each particle inside its cell is characterised by its position $\vec{r}_i$ ($i = 1, \dots, A$) and linear momentum $\vec{p}_i$. The particles are considered as classical. The uncertainty principle constraint $|\vec{r}_i - \vec{r}_j| |\vec{p}_i - \vec{p}_j| \geq \hbar$ between particles in neighbouring space cells characterised by $(\vec{r}_i, \vec{p}_i, ; \vec{r}_j, \vec{p}_j)$ is the only quantum mechanical feature which we introduce into the description. Neighbouring particles interact through a short range two-body potential $V(|\vec{r}_i - \vec{r}_j|)$ whose explicit form we shall present below. The total energy associated with a 2-particle system (i, j) is decomposed into a center of mass contribution and relative energy $\mathcal{E}$

$$E = \vec{P}^2/2M + \vec{p}'^2/2\mu + V(r) = \vec{P}^2/2M + \mathcal{E} \quad (1)$$

where $\vec{P} = \vec{p}_i + \vec{p}_j$, $\vec{p}' = \vec{p}_i - \vec{p}_j$, $r = |\vec{r}_i - \vec{r}_j|$, $M = 2m$, $\mu = m/2$, m being the nucleon mass.

In practical calculations the coordinates $(\vec{r}_i, \vec{r}_j)$ and $(\vec{p}_i, \vec{p}_j)$ are drawn from specific probability distributions. The coordinates of a particle located at $\vec{r}_k(x_k, y_k, z_k)$ are fixed as $x_k = x_{k0} + \varepsilon d/2$ and similarly for y_k and z_k , where $\vec{r}_{k0}(x_{k0}, y_{k0}, z_{k0})$ is the coordinate of the centre of the cell with respect to some reference frame and ε is a random number drawn from a uniform (or other) distribution in the interval $[0,1]$. The momenta coordinates $\vec{p}_k$ are randomly drawn in a sphere of radius $p_{\max}$. In the sequel we introduce also the quantities $E_{\min}$ which corresponds to the min-

imum of potential energy and $E_{\max}$ which is the maximum kinetic energy of two neighbouring particles, $E_{\max} = p_{\max}^2/m$.

Once the phase space coordinates of two neighbouring particles are fixed one calculates E and $\mathcal{E}$. If $\mathcal{E} < 0$ we consider the particles to be bound relatively to each other and unbound if $\mathcal{E} > 0$. By repetition of this procedure over all possible positions and momenta of the particles (typically 10^7 draws) we can construct $p(E)$, the probability for the two-body system to be bound if its total energy is E . In practice $p(E) = n(E)/\mathcal{N}(E)$ where $\mathcal{N}(E)$ is the total number of draws in a given energy interval $[E, E + \delta E]$ and $n(E)$ the total number of events out of $\mathcal{N}(E)$ for which the system is bound at energy E . If the uncertainty principle constraint is taken into account we reject all events which do not verify it. The typical behaviour of $p(E)$ is shown in Fig. 1 along with $\mathcal{N}(E)$.

2b. Cluster identification criteria and algorithms

There exists no unique way to define bound clusters in a compact system of interacting particles. We propose three different criteria and algorithms in order to identify these clusters.

The first procedure which is called A2 in the sequel is the following. In order to generate a given event we choose a random number η in the interval $[0,1]$. This quantity defines an energy interval $[E_{\min}, E_0]$ through $\eta = p(E_0)$ in which a pair of particles can be bound. The range over which this interval extends may be related to the conditions under which the system has been generated. The energy E of each pair of particles is drawn from $[E_{\min}, E_{\max}]$ by means of the distribution $\mathcal{N}(E)$ defined above (see Fig. 1). Knowing E , one obtains $p(E)$ which is compared to η . If $p(E) \geq \eta$ the pair is bound, if $p(E) < \eta$ it is unbound. This procedure is repeated for all couples of neighbouring particles, and clusters (fragments) are identified as sets of connected nucleons. The algorithm is repeated over all possible random values of η . The typical number of generated events is 10^6 . As it can be realised, this procedure which relies on bond probabilities $p(E)$ is closely related, in spirit and in practice to a bond percolation algorithm. The probabilities are taken in the interval $[0,1]$. The difference between A2 and the bond percolation algorithm is twofold. First, the comparative test which decides for the existence of a bond ($p(E) \geq \eta$) is reversed. Second, $p(E)$ which lies in the interval $[0,1]$ is not drawn from a uniform distribution. We shall come back to this point in section 3 where we analyse the outcome of the realisations of this algorithm.

Algorithm A2 neglects correlations since the bond test for any pair of neigh-

bouring particles, say $(i, i + 1)$ is performed independently of the bond test for the pair $(i + 1, i + 2)$, although both pairs share in principle the phase space coordinates of particle $i + 1$. In order to take these correlations into account we introduce the following algorithm. As before, we draw a random number η . In contrast with A2, we do not fix the energy E by means of $\mathcal{N}(E)$ but generate first the phase space coordinates $(\vec{r}_i, \vec{p}_i)$ of each particle in its configuration cell, which fixes E for each pair. The knowledge of E fixes $p(E)$ and the bond criterion $p(E) \gtrless \eta$ is applied as in the case of A2. This bond fixing procedure and cluster determination is called algorithm A1 in the sequel.

The cluster identification and algorithms described above ground on a probabilistic determination of bonds and algorithm A1 shows some hybrid character since it relies on the knowledge of the microscopic phase space which allows in practice to decide about the bond property of any pair but introduces and uses the probability $p(E)$ which would in fact not be needed. A1 is somewhat academic. It has been introduced as a way to study correlation effects which are absent from A2.

In fact, correlations are consistently and fully taken into account by means of the introduction of a third algorithm. In this framework one simply generates events for which the microscopic phase space $(\{\vec{r}_i, \vec{p}_i ; i = 1, \dots, A\})$ is known and with which one decides about bound pairs by investigating whether the relative energy $\mathcal{E}$ (see Eq. 1) is negative or positive. This algorithm which corresponds to a deterministic description of the disordered system is called A0 in the sequel. It will play a central role in the present work.

3. Physical observables related to mass and charge distributions and their characterisation

3a. Physical observables

The knowledge of the particle and fragment content of a given event k allows the calculation of mass distributions and related moments of order p , $m_p^{(k)}$ for fixed multiplicity m , i.e.

$$m_p^{(k)}(m) \equiv \sum_l l^p P_l^{(k)}(m)$$

$$S_p^{(k)}(m) \equiv m_p^{(k)}(m)/m_1^{(k)}(m) \quad (2)$$

where $P_l^{(k)}$ is the number of clusters of size l and the prime index indicates that the heaviest fragment is omitted. The multiplicity m is an interesting parameter since

it is directly accessible from the experiment. From these moments several other observables can be constructed, such as the combination⁷⁻⁸⁾

$$\begin{aligned}\gamma_2^{(k)} &\equiv m_0^{(k)}(m) m_2^{(k)}(m) / m_1^{(k)^2}(m) \\ &= 1 + \sigma^{(k)^2} m_0^{(k)^2}(m) / m_1^{(k)^2}(m)\end{aligned}\quad (3a)$$

where

$$\sigma^{(k)^2} \equiv (m_0^{(k)}(m) m_2^{(k)}(m) - m_1^{(k)^2}(m)) / m_1^{(k)^2}(m) \quad (3b)$$

is a width. Use of these observables on the outcome of different models³⁾ shows that they can constitute sensitive tests of theoretical approaches. In the sequel we shall also work out the relation between $A_{max}^{(k)}$ and $m_2^{(k)}$ ⁸⁾.

In practice a significant amount of information is already contained in the average of the observables over N_{ev} ($k = 1, \dots, N_{ev}$). We denote the average moments by $\overline{m}_p(m)$. For γ_2 one can define two types of averages

$$\overline{\gamma}_2(m) = \overline{m}_0(m) \overline{m}_2(m) / \overline{m}_1^2(m) \quad (3c)$$

or

$$\langle \gamma_2(m) \rangle = N_{ev}^{-1} \sum_{k=1}^{N_{ev}} (m_0^{(k)}(m) m_2^{(k)}(m)) / m_1^{(k)^2}(m) \quad (3d)$$

In most cases we shall use expression (3d). It has been shown that in the framework of percolation models $\delta\gamma_2 \equiv \langle \gamma_2(m) \rangle - \overline{\gamma}_2(m)$ measures fluctuations which may arise from finite size effects⁹⁾.

3b. Characterisation of the observables

Percolation models^{7,10,11)} have been extensively used as a reference for other model approaches. There are two reasons for this. First, simple-minded bond percolation calculations reproduce quite successfully the up to now known available experimental data¹²⁻¹⁴⁾. Second, percolation algorithms describe infinite systems which show a phase transition between a regime with essentially a large infinite cluster and a multi-cluster regime⁹⁾. The existence of this phase transition induces universality properties and scaling laws corrected for finite size effects which could also characterise high-energy fragmentation processes.

At the bond probability p_c corresponding to the phase transition the moments $m_i(p)$ for $i \geq 2$ and $\gamma_2(p)$ show power law singularities with specific exponents. These singularities are of course smoothed out and appear as maxima in finite

systems. Hence it is tempting to try to extract the location of these maxima and the value of the exponents in order to compare the outcome of different models and confront theoretical calculations to experimental results. An attractive method which fulfills this program has been proposed and worked out recently for finite systems in the framework of bond percolation where the order parameter is the bond probability p ¹⁵⁾. It was applied in order to obtain, among others, the exponent γ which characterises $m_2(p) \sim |p - p_c|^{-\gamma}$ in the vicinity of p_c , in the infinite size limit. The method was further used in order to work out the behaviour of $m_2(m) \sim |m - m_c|^{-\tilde{\gamma}}$ which allows a direct comparison of percolation results with those obtained from the experiment¹⁶⁾.

A close analysis of this procedure raises however several problems and questions. The method which allows the determination of γ works reasonably consistently for large systems, where one approaches infinity. The outcome of the parameter p_c (resp. m_c) and exponent γ (resp. $\tilde{\gamma}$) is stable with respect to the numerical procedure which is used, i.e. the choice of intervals in the variable p (resp. m) in the neighbourhood of the critical point. However, for small systems which are typically of the size of nuclei this is no longer the case, the extracted γ ($\tilde{\gamma}$) is not stable with respect to the choice of the $p(m)$ intervals. This is qualitatively understandable since the maximum of m_2 gets more and more shallow when the size of the system decreases, for small systems it gets more and more difficult to identify the characteristic features of the infinite system. Finally, the analysis in terms of the bond probability p does not lead to the same critical exponent γ as the exponent $\tilde{\gamma}$ which characterises the parametrisation of m_2 in terms of the multiplicity m . This is due to the fact that there is no one-to-one mapping between p and m (see Fig. 2) and a non linear behaviour of the average of m as a function of p . These facts distort the power law behaviour of m_2 , when one goes from one variable to the other $\gamma \neq \tilde{\gamma}$, which shows that exponents extracted from experimental results obtained in terms of m should not be compared to those obtained from percolation results which are related to p , but to those obtained by means of m .

In view of these difficulties we propose a different approach in order to obtain quantitative comparisons of physical observables extracted from calculations and to compare with experimental results. In practice we work out the location of the maximum $m_{\max}$ of m_2 and τ_2 , the strength of the observables at this point and the lowest moments which characterise the average distribution of the observables as a function of m . In this way, this simple-minded procedure leads to detailed com-

parisons of the outcome of any model calculation to any other and to experimental results when they are available. Such comparisons will be shown below.

4. Application of algorithm A2 and correlation effects induced by means of A1

4a. Physical observables. Comparison with the bond percolation approach

Numerical simulations have been performed with algorithm A2, the two-body potential taken from Ref.¹⁷⁾ and the uncertainty principle constraint acting on two neighbouring particles is taken into account. The average mass distribution shows a perfect power law behaviour over a large range of intermediate size clusters. It is, like the observables $\overline{m}_2$, $\overline{\gamma}_2$ and $A_{\max} = f(\overline{m}_2)$ in perfect agreement with bond percolation results for systems of the same space dimensionality and size, see Fig. 3. This quantitative agreement remains true for an extended set of values of cell sizes d which fix the average density, for different choices of $E_{\max}$ and strengths of the interaction. Trivial deviations appear only if the potential strength is changed and gets too weak or too strong. For too weak potentials low multiplicity events are missing, and $\overline{m}_2$ and $\overline{\gamma}_2$ are cut off on the left side. For too strong potentials, the attraction generates large fragments, hence high multiplicity events, $\overline{m}_2$ and $\overline{\gamma}_2$ are cut off on the right side. This effect is exemplified in Fig. 4.

The agreement between A2 and bond percolation for $\overline{m}_2$ and $\overline{\gamma}_2$ expressed as a function of the multiplicity m is amazing. How can it be explained? Algorithm A2 proceeds in a similar but not the same framework as bond percolation, see section 2b. The fact is that the fixed bond probability $\eta = p(E_0)$ used in A2 is not chosen in a uniform distribution like p in bond percolation since η is characterised by a probability distribution $\rho(\eta)$ which is not uniform but varies strongly with the choice of the parameters which define the physical system, i.e. $E_{\max}$, d and the strength of the two-body interaction. The actual variable $\tilde{\eta}$ which corresponds to p in bond percolation is identified with the quantity

$$\tilde{\eta} = 1 - \int_0^\eta \rho(\eta') d\eta'$$

In terms of this quantity the correspondence between m and p in percolation calculations can be mapped onto the correspondence between m and $\tilde{\eta}$ in A2, i.e. m and $\tilde{\eta}$ are related to each other in the same way as are m and p in bond percolation. This correspondence is insensitive to the choice of the physical parameters, except

of course for possible cut-off effects discussed elsewhere. One obtains a unique value $\tilde{\eta}_c$ corresponding to p_c and $\tilde{\eta}_c \simeq p_c$ where p_c is the percolation threshold probability. We checked these points. The close relationship between percolation and A2 in terms of (p, m) and $(\tilde{\eta}, m)$ explains the close agreement between the observables $\overline{m_2}(m)$ and $\overline{\gamma_2}(m)$ obtained by means of both algorithms.

The fact that the frequency with which $p(E)$ is selected is not uniform in $[0,1]$ because $\mathcal{N}(E)$ is not, reflects in the yield of events with multiplicity m , $P(m)$. Fig. 5 shows a comparison between the outcome of percolation calculations and algorithm A2. One observes indeed differences, particularly in the high multiplicity sector of the distribution. It would be interesting to compare the yields $P(m)$ with those obtained from the experiment. Whether this is possible, f.i. with sophisticated detectors like INDRA, could be investigated in the near future.

We have also worked out cases in which the potential differentiates between protons and neutrons¹⁸⁾. Here $p-p$ and $n-n$ interactions are repulsive, i.e. bond probabilities $p_{pp}(E) = p_{nn}(E) = 0$ and the $p-n$ interaction shows an attractive contribution, hence $p_{pn}(E) \neq 0$ behaves like $p(E)$ above. For each pair which is tested the particles are taken to be protons with probability $(Z/A)^2$, neutrons with probability $(N/A)^2$ and non-identical particles with probability $2NZ/A$ where $Z(N)$ is the number of protons (neutrons), $A = N + Z$. The results shown in Fig. 6 are again very close to percolation results, if not identical to them. In fact protons also feel a long range Coulomb interaction which has to be added to the strong short range nucleon-nucleon interaction. We do this in an approximate way. Considering the two-particle system, we check whether one of the particle is a proton. If this is the case we add the Coulomb interaction between this particle and all other charged particles in the system when we calculate the pair energy E , i.e. under the assumption that these charges are uniformly distributed in a sphere of volume $\mathcal{V}$ corresponding to the size of the total system. We separate E into a *c.m.* contribution and the relative energy $\mathcal{E}$ and construct $p_{pn}(E)$ as before. This quantity depends in fact on the localisation of the pair in the system, i.e. close to the center or close to the surface. It comes out of the calculations that the behaviour of the moments is unaffected by the Coulomb effects for all possible cases between the two extremes. This is again understandable, the Coulomb interaction is much weaker than the short range potential and hence acts as a weak renormalization factor on the strong interaction. But, as was shown above, moments and related quantities are robust with respect to the strength of the interaction.

4b. Confrontation with experimental information

Multifragmentation data available from experiments made by the ALADIN collaboration^{12,13,19,20)} can be used in order to confront it with our approach. In practical calculations we use algorithm A2 with the potential from Ref.¹⁷⁾ and go over from mass number A to charge number Z by means of the empirical formula²¹⁾

$$Z = A(1.98 + 0.015 A^{2/3})^{-1} \quad (4)$$

For $A = 3$ fragments the isobaric ratio ${}^3\text{H}/{}^3\text{He}$ is fixed to the value 2.2 following Ref.²²⁾. The experiments we consider correspond to collisions of Au on C, Al and Cu targets at 600 MeV/nucleon²³⁾. The mass of the spectator part of the fragmenting system depends on the impact parameter and the relation between this quantity and the involved mass is obtained by means of dynamical calculations^{24,25)}. Recent investigations show that this should be a sensible procedure. The mass range is divided into 10 sets $A_i = 67, 73, 84, 98, 113, 133, 150, 166, 181$ and 189 corresponding to 10 intervals of impact parameters. Each system with A_i nucleons is put in a cube of 7^3 cells in an as compact and symmetric configuration as possible. Each calculated observable O is averaged over the whole impact parameter range $\{b_i\}$.

$$\overline{O} \equiv \sum_{i=1}^{10} b_i \Delta b_i O(b_i) / \sum_{i=1}^{10} b_i \Delta b_i \quad (5)$$

The analysis of the experiment is done with a set of 3.10^4 events²³⁾. In Fig. 7 we show the behaviour of the average of different quantities as a function of Z_{bound} , the number of charges in fragments with charge $2 \leq Z \leq 30$ and the similar quantity Z_{b_6} for which $6 \leq Z \leq 30$. As it can be seen, the agreement of our calculations with the experiment are good and, as expected, quite indistinguishable from percolation results. Notice that the experimental multiplicity of He species, $\langle M_{\text{He}} \rangle$ shown in Fig. 7, may be larger than shown here for technical reasons related to detection efficiency²⁶⁾. The sensitivity of the calculations with respect to the strength of the potential can clearly be seen in Fig. 7c. A decrease of this strength by a factor 1/2 does practically not affect the results. If however it is increased by a factor 2 the behaviour of the observables changes drastically. This is due to the same mechanism as described in section 4a. In this case the distribution $\mathcal{N}(E)$ (see Fig. 1) moves its centroid to lower E values. Hence most particles are bound in large clusters whose charge exceeds $Z = 30$. This decreases drastically the amount of intermediate mass fragments.

We mentioned above that the outcome of algorithm A2 and percolation results differ in the yields of events $P(m)$. These yields depend on the strength of the interaction, i.e. on the details of the dynamics. One observes sizable differences for different strengths of the potential. Percolation results and those obtained with the potential of Ref.¹⁷⁾ are quite close and in reasonably good agreement with the experimental result. A detailed comparison of event yields as a function of the observable $A_{23} = (Z_2 - Z_3)/(Z_2 + Z_3)$ where $Z_2(Z_3)$ is the second (third) heaviest charged fragment is shown in Fig. 8. Again the results of our calculations are in fair, even if not in perfect agreement with the experiment.

4c. Introduction of correlations: algorithm A1

Algorithm A1 which we described in section 2 introduces correlations into a probabilistic determination of bonds. Its application leads to typical results shown in Fig. 9 when one uses the potential of Ref.¹⁷⁾. The qualitative behaviour of the observables is similar to the one obtained by means of A2. These are quite robust with respect to the choice of the physical ingredient d , $E_{\max}$ and the strength and type of the potential. But the location of the maxima of $\overline{m}_2$ and $\overline{\gamma}_2$ definitely show that these quantities do not reproduce the properties predicted by percolation calculations. For the potential used in Fig. 9 this maximum is located in the vicinity of $n = m/A = 0.45$ which is quite far from the expected value ~ 0.30 . Correlations which induce deviations from percolation can be tested by means of correlation functions between different points of the system²⁷⁾. These criteria are measures of short range correlation effects. They indicate whether and how far these correlations drive the system which is analysed in terms of percolation concepts away from the standard percolation universality class.

5. Deterministic fragment identification: algorithm A0

5a. Physical observables related to mass distributions

The results obtained with algorithm A2 raise a problem. On the one hand A2 which is close to percolation should also be closest to the experiment. On the other hand it evidently neglects important correlations which should be present in the system. If the experiment confirms percolation results it indicates that the effect of correlations may be washed out in some way. In order to test this point we present and discuss now the analysis of events generated by means of algorithm A0 which takes these correlations into account.

Indeed, the generation of the explicit phase space coordinates $\{\vec{r}_i, \vec{p}_i ; i =$

$1, \dots, A\}$ and the use of the bond criterion based on the sign of the relative energy $\mathcal{E}$ allows for a deterministic determination of bonds by means of A0 (see section 2b) which enables to avoid the probabilistic arguments for the existence of bonds used in A2 and A1. Furthermore, this algorithm allows for an easy treatment of the long range Coulomb interaction whose effect can be explicitly taken into account in the bond criterion of pairs of neighbouring particles. This is done by summing up the contribution of all charged particles of the system interacting with the possibly charged particles of the considered pair.

Calculations have been performed for a system in a cubic geometry, with fixed $E_{\max}$ and the two types of potentials introduced above^{17,18)}. For different, fixed values of the density one obtains moments of mass distributions which differ sensibly from those obtained by means of A2 and percolation. If, however, one adds up events corresponding to values of d which are uniformly distributed over a given interval of average densities ρ (say $[5.10^{-3}, 0.17] \text{ fm}^{-3}$) and averages over them one obtains mass distributions and related observables which are remarkably close to those obtained with algorithm A2, see Fig. 10. The quantitative agreement between results obtained by means of different algorithms will be presented below. We also find out that the observables are robust with respect to modifications of the strength of the short range potential, like in the case of A2. If the strength gets however too weak, one observes cuts in the multiplicity similar to those obtained and discussed above in the case of A2.

Finally we have investigated the correlation between the energy content of the system and the fragment distribution. Fixing the energy $E_{\max}$ of a pair leads to a total energy distribution of the system which is strongly peaked around a fixed value, hence mixes events with different energies even though most events are close in total energy, see Fig. 11. We have analysed the behaviour of the system in two different cases. First, for a given potential, we fixed the density of the system and mixed up the contribution of events with different $E_{\max}$, hence different total energies. The results show that the observables which characterise the fragment size distribution are very close to those observed before. Second, we also used algorithm A0 for a given potential and a fixed total energy E_{tot} . We tested the behaviour of the system in two windows, $E_{tot}/A \in [-0.5, +0.5] \text{ MeV}$ and at higher energy, $E_{tot}/A \in [-4.5, +5.5] \text{ MeV}$ with the potential of Ref.¹⁷⁾ and in the presence of the Coulomb interaction. The observables are little affected, except for cut-off effects of the high multiplicity tail for the lower energy interval, see Fig. 11. The yields of

events for fixed multiplicity are of course sensibly different in both cases.

Up to now we have no solid explanation for the remarkable similarity of results obtained with algorithm A2 and A0. Algorithm A0 would be equivalent to A2 for the same set of physical parameters $E_{\max}$, d and interaction strengths if we would determine the existence of bonds between neighbouring particles for each couple by redefining the phase space coordinates of each couple independently before performing the bond test. In reality A0 takes by definition full account of the correlations between particles. Since the calculations with A0 which mix up events corresponding to different average densities lead to very similar results as these obtained by means of A2, one may conclude that it is the averaging over densities which acts as if it would wash out the effect of the correlations. The same conclusion may also hold if one mixes events of fixed density and different values of $E_{\max}$.

5b. Quantitative analysis of moments

The agreement of the experimental fragment distribution analysis with percolation results on one hand and the prediction and possible experimental observation of a liquid-gas phase transition^{16,28,29,30)} on the other hand raises two questions. First, one may ask whether nuclear fragmentation effectively shows a phase transition corrected for finite size effects which would explain the universality properties of the process. And if so, one must raise the question whether this transition is compatible with percolation theory or whether it is a typical liquid-gas transition or even of another type. For the moment we have no definite answer to these questions. One way to investigate it is to try to extract the location of a critical point and the value of critical exponents. As already stated above this program has been implemented in the close past¹⁶⁾. Since this method and its application, although attractive and nice, is plagued with technical difficulties, we define $\overline{m}_2(m)$ and $\overline{\gamma}_2(m)$ by the characteristic quantities introduced at the end of section 3b. In Table 1 we compare these quantities which were obtained by using the different algorithms. The agreement between percolation A0 and A2 is rather impressive. As expected, A1 shows sizable deviations which were discussed above.

5c. Confrontation with further fragment definition criteria

There exists no unique definition of fragments in a dense system. Algorithms A2 and A0 generate fragments by means of a nearest neighbour bond criterion. We confront the criterion used in A0 with two others.

The first one has been proposed and used by Dorso and Randrup³¹⁾. There, a particle is supposed to belong to a cluster if the sum of its kinetic and potential

energies due to its interaction with all other particles of the defined cluster is negative. This criterion is very constraining and leads to smaller clusters than in the case A0. We tested the energy of each particle in the clusters obtained by means of A0 and considered that it is a free particle if the Dorso-Randrup criterion is not fulfilled. From a set of events which was analysed in this way we calculated $\overline{m}_2$ and $\overline{\gamma}_2$. As it can be seen in Table 1, the location of the maxima is shifted to larger values and the strengths at maximum are reduced with respect to those obtained with A0. The difference, even if sensible, is however not dramatic.

The second criterion grounds on the use of the internal energy of a fragment which is the sum of the kinetic energy of the particles relative to the centre of mass of the fragment and the two-body potential which acts between the particles. If this energy is negative the fragment is considered to be bound. If not, the event is not taken into account. A comparison of the size distribution obtained with this criterion and the distribution from A0 shows that approximately 3% of the clusters with mass less than 10 and about 10% of those with mass larger than 10 which are qualified by A0 as fragments are unbound with respect to the criterion introduced above. The distortion produced by the discrepancy between the two criteria can again be appreciated in Table 1. As one can see, the observables $\overline{m}_2$ and $\overline{\gamma}_2$ stay rather close to those obtained with A0. It should be mentioned here that a rigorous comparison of the criteria quoted above should proceed by means of a proper identification of clusters based on a sampling of particles in clusters in such a way that the energy of the system is minimum. In practice such an identification could be realised by means of a Metropolis-Monte Carlo treatment of each event, as proposed for instance by Dorso and Randrup³¹⁾. Such a procedure represents a huge amount of computer time which we did not try to perform up to now.

Different cluster identification criteria lead to distortions in the physical observables related to fragment size distributions. But even for the most constraining criterion the consequences are never drastic, the observables are not very sensitive to the choice of a specific criterion.

6. Further tests of the description of the disordered system by means of A0

6a. Surface geometry effects

Up to now we analysed the behaviour of disordered systems contained in a somewhat restricted and naive cubic volume. In order to analyse the influence

of the topology of the surface on the fragment formation and identification we investigated other forms going from quasi-spherical shapes to all kinds of more or less exotic shapes such as parallelepipeds, systems with streaked, crenellated and crumpled surfaces. These last surfaces which have a fractal character have been generated by means of the following procedure. We start from an occupied cell and decide randomly which of the six neighbouring cells would be occupied. The space occupation is further tested by choosing the next occupied cell in the neighbourhood of the two already occupied ones by means of the same procedure as defined above. If this operation is pursued up to the moment where all A particles have been located in space one obtains systems with random fractal surfaces and possibly empty cells in the bulk. The effect of the regular surfaces as well as the crumpled surface generated by the random procedure defined above can be appreciated in Table 2 where we show $\bar{m}_2$ and $\bar{\gamma}_2$ for all the different cases. Clearly one observes distortions with respect to the reference cubic system. These distortions are rather weak for all cases except maybe the last one. One observes a cut-off at $m/A \leq 0.12$ which corresponds to the fact that low multiplicity events are scarce, which is certainly due to the presence of empty cells in the volume. But even in this extreme and more or less realistic case, these distortions are never dramatic and do not lead to qualitative changes in any observable related to the fragment size distribution which we introduced above.

6b. Collective effects

In the preceding sections we have shown and discussed at length the possible connections between a completely disordered and excited system of particles which breaks into fragments and the average power law behaviour of its mass distribution. This is the typical behaviour which is experimentally observed in peripheral collisions, whereas central collisions show charge distributions which are close to an exponential shape, at least over a large range of fragment sizes²²⁾. Furthermore, it has been observed that central collisions are characterised by a sizable collective radial flow²³⁾.

This observation prompted us to investigate whether our approach is able to reproduce the correlation between collective flow and exponential decay of fragment size distributions. Collective flow is clearly a dynamical effect and it is out of scope to reproduce it faithfully in the framework of our approach. It is nevertheless possible to mimic it in some way. In practice we generate events with algorithm A0 in a range of cell sizes $1.8 \leq d \leq 6$ fm for a system with $A = 216$ particles. The

strengths of the momenta of individual particles is chosen randomly as before, but their directions are fixed along a line which joins the centre of the system to their actual position, in the direction pointing out of the system. The typical average fragment size distribution is represented in Fig. 12, in a semi-log representation. The largest fragment has not been taken into account. The shape of this distribution is very close to an exponential for fragment sizes reaching from mass 20 to about mass 90. Deviations are observed for lighter masses. Other but similar scenarii have been tested. They lead to similar results.

As stated above, the present investigations rely on a rather schematic description of possible flow effects. Our crude description shows nevertheless a strong discrepancy with the ordinary power law behaviour and a strong similarity with the expected exponential behaviour. This comforts, to our feeling, the coherence of the picture which associates in energetic reactions total disorder with peripherality on one hand, collective effects with centrality on the other hand.

6c. Energy - multiplicity correlations

Energy governs the fragment content of the disordered system in our approach which grounds on algorithm A0. Hence it is of some interest to look at the correlations between these two observables. One must however keep in mind that these correlations correspond to systems which are still far from their asymptotic freeze-out configurations, from which on these quantities can in principle be compared to experimental results.

We define an energy content E_{cont} of the system as the sum of the kinetic energy of the fragments and the particles, and the internal energy of the fragments, hence the total energy except for the total interaction energy between all the fragments and the particles

$$E_{cont} = \sum_{\{part\}} E_{kin}(part) + \sum_{\{frag\}} E_{kin}(frag) + E_{int}(frag)$$

with

$$E_{int}(frag) = \sum_{\{frag\}} \left(\sum_i \frac{p_i'^2}{2m} + \sum_{i < j} V_{ij} \right)$$

in an obvious notation. The p_i' are the momenta of the particles in a given fragment, in the c.m. system of the fragment. Fig. 13 shows the correlation between E_{cont}/A and the fragment and particle multiplicity obtained by means of calculations done

in the framework of algorithm A0 for a fixed total energy E_{tot} of the system. The energy raises with the multiplicity as expected and the raise is monotonous. It is however neither linear nor quadratic and it is difficult to know whether the system is thermodynamically equilibrated or not at this stage.

We have also calculated the correlation discussed above by using the criterion which defines a fragment as a set of particles with negative internal energy, see section 5c. The qualitative behaviour of the evolution of E_{cont} with respect to multiplicity is the same as before, the absolute values however are quite different. Fragments are more bound than they are when one uses the A0 algorithm. Hence the energy is quite sensitive to the definition of clusters even if observables related to mass distributions are not so much. As already stated in section 5c, our procedure $E_{frag} < 0$ rejects the events where at least one fragment does not obey this criterion. In our application of the Dorso-Randrup criterion, we do not explore all the possible combinations of bound fragment formation leading to minimum total energy. In fact, for each given event, one should implement a full-fledged Monte Carlo (or better, Metropolis - Monte Carlo) procedure in order to fix the fragment distributions as rigorously as possible. This is a very time-consuming procedure which we have not yet implemented.

6d. Fluctuations

Fluctuations are generally a source of information about complex systems. There exists a good deal of work which concerns the nature and the physical significance of fluctuations in fragment size distribution³⁴⁾. In particular, the correlation between an intermittent behaviour of fluctuations in a finite system and a power law behaviour of the average mass distribution has been established³⁵⁾. Recent work³⁶⁾ indicates that there is no well established connection between the observation of intermittent fluctuations in mass distributions of finite nuclei and the possible existence of a phase transition in an infinite system. It remains nevertheless interesting, in our opinion, to analyse the fluctuation content of mass distributions in terms of factorial moments which constitute a further observable by means of which one can compare different theoretical approaches.

Hence we used horizontal factorial moments³⁷⁾ in order to analyse the fluctuation content of mass distributions generated by means of A2. Using the variable $\tilde{h}$ defined in section 4a, we observe intermittency signals when the events which are used in order to construct the average horizontal factorial moments are taken from $\tilde{q}$ intervals which are large enough and contain or are close to $\tilde{\eta}_c$, where $\tilde{\eta}_c$ is the

equivalent of p_c in the bond percolation procedure. Under these conditions, the multiplicity distribution of events gets large and centered around a sizable value of the multiplicity m , see Fig 14. The signals disappear when these conditions are not fulfilled. This confirms what we said above.

The same conditions are observed when one uses algorithm A0. No signal is seen when corresponding multiplicity distributions are narrow. One observes a more or less clean intermittent signal for specific selections of events which correspond to broad multiplicity distributions centered around a large value like in the case of A2, see Fig. 15. This again confirms the conditions under which the effect can be observed.

7. Summary, comments, further developments and prospective

In the present work we have tried to establish a connection between nuclear fragmentation, disorder and universality. Admittedly, our investigations ground on numerical simulations, hence they cannot be used as rigorous and decisive answers. They show nevertheless that many signs argue for the existence of these connections.

We proceeded to describe the properties of fragmentation by means of different algorithms which lead in principle to different definitions of clusters of bound particles, starting from the closest to bond percolation. We discussed the physical content of these algorithms and confronted them by means of analytical and numerical arguments. We were able to show that the most realistic definition of fragments in terms of nearest neighbour bonds in the framework of a fully deterministic definition of these bonds leads to results which are close to those predicted by percolation, at least as far as reference observables related to mass distributions are concerned. Other quantities like the event yield for fixed multiplicity of particles and fragments can be quite sensitive to the cluster definition. Such an observable could perhaps work as a test of one or the other approach of the fragmentation process, at least if the experimental conditions are good enough to get a precise quantitative information about it. Our fragment identification method has also been confronted with other available cluster definitions. It comes out that observables related to cluster size distributions are only moderately affected which indicates that the system is relatively robust with respect to the definition of bound species in the fragmenting system.

The systems which we generate can be highly excited, not necessarily thermodynamically equilibrated. They contain clusters which are far from being nuclei

with the smooth surface of leptodermous objects, the surface may show fractal properties. The observation that the generated fragment mass distributions could be so close to the final asymptotic distributions which are experimentally observed is a remarkable fact. Our description which covers a range of average densities covers many different fragmentation configurations. In a time-dependent expansion picture it indicates that the remnants of early stages show a high probability of survival which may indicate that the process is fast and (or) that the fragments survive even when they are far from their ground state, being only affected by light particle emission which does not sensibly influence the final distributions.

Our description raises also the question concerning the importance of the Coulomb interaction which we took into account in our deterministic determination of fragments. Multifragmentation models based on the concept of thermodynamical equilibrium in the framework of the microcanonical ensemble¹⁾ rely strongly on the importance of the long range Coulomb interaction. In our case, this interaction does not affect observables related to the fragment size. That this is so can be guessed from the fact that this long range force is much weaker than the strong short range force. We have no good explanation for the seemingly different role played by the Coulomb interaction at these different stages. Indeed, our approach describes a very different phase of the fragmentation process. Multifragmentation models are concerned with freeze-out configurations, when clusters are already fully formed nuclei with smooth surfaces which correspond to systems in which the surface energy is balanced by the internal Coulomb energy, preventing both collapse and disintegration. The stages described by our approach may be far from this situation, fragments are far from showing smooth surfaces. The transition from one regime to the other is not clear, in principle it can only be established in the framework of a full time-dependent description.

In section 6, the approach related to algorithm A0 has been further tested and its implications have been analysed. We have shown first that observables related to mass distributions are rather robust with respect to the topology of the surface of the system, which comforts us in the idea that our results do not really depend on the naive cubic geometry we used in our numerical simulations. Further we have shown that it is possible to establish a correlation between shapes of fragment size distributions and the degree of disorder which prevails in the system. This fact is in nice agreement with recent experimental results which correlate the formation of complete disorder with peripheral collisions and the observation of collective effects

with central collisions. We have also studied the correlation between the energy content of the fragmenting system and multiplicities and seen that different fragment identification criteria lead to sizable changes, although the quantitative features of the correlation remains preserved for all criteria we introduced. Finally we have analysed the properties of mass distribution fluctuations. Under specific conditions these fluctuations behave like intermittency signals, as it is the case in percolation calculations concerned with finite systems. As it has been said elsewhere, the physical significance of these signals is not completely clear.

An important open question is related to the fact that the fragmenting system may be generated in two different regimes which are related by a phase transition corrected for finite size effects. The fact that our results and also experimental results are close to percolation predictions is *a priori* an indication for the existence of a phase transition of a percolating type. On the other hand, however, there are recent experimental indications that one could observe a liquid-gas phase transition. If this is so, we are confronted with some contradiction, since a percolation transition does not belong to the same universality class as a liquid-gas transition. In order to progress on this point, it would be of major importance to obtain precise values of the critical exponents. As we discussed in section 5b, this is a difficult task for as small a system as a nucleus. Two directions may be pursued at this point. First, one may try to find out the exponents obtained with A_0 on a large system, and compare them with the liquid-gas exponents. Second, one may try to work out the thermodynamical properties of our disordered system and look for the signs of a first order phase transition. If our approach is a good description of the fragmenting system, i.e. if it describes the correct features of the process at least when this process is induced by peripheral collisions we hope to learn about the nature of the transition, whether percolation or liquid-gas or even another type. In order to progress on this path we aim to construct the thermodynamic functions and to study the phase diagrams in the framework of the microcanonical ensemble.

Finally, nuclear fragmentation is an irreversible mechanism which develops in time. It would certainly be interesting to confront our description with the outcome of dynamical models like molecular dynamics at different steps corresponding to different average densities in order to compare the phase space content of the two approaches at different stages of the fragmentation process. We aim to consider this point also in the future.

Acknowledgments

The authors would like to thank W. Trautmann for providing them with experimental data concerning Au fragmentation experiments which were performed by the ALADIN Group, GSI, Darmstadt. They would also like to express their gratitude to W. Trautmann, W.F.J. Müller, R. Peschanski, M. Ploszajczak, X. Campi and H. Krivine for their interest in this work.

References

- [1] D.H.E. GROSS — *Rep. Prog. Phys.* **53** (1990) 605
- [2] SA BEN HAO, ZHENG YU-MING AND ZHANG XIAO-ZE — *Int. J. Mod. Phys.* **A5** (1990) 843
- [3] J. RICHERT — *Int. J. Mod. Phys.* **E2** (1993) 679
- [4] L.G. MORETTO AND G. WOZNIAK — *Ann. Rev. Nucl. Part. Sci.* **43** (1993) 379
- [5] E.P. WIGNER — *Proc. Cambridge Phil. Soc.* **47** (1951) 790, reprinted in C.E. Porter, Statistical Theories of Spectra: Fluctuations, *Academic Press*, New York, 1965
- [6] D. STAUFFER AND A. AHARONY — Introduction to Percolation Theory, Taylor and Francis, London (1994) 2nded., and refs. therein
- [7] X. CAMPI — *J. Phys. A : Math. Gen.* **19** (1986) L917
- [8] X. CAMPI — *Phys. Lett.* **B208** (1988) 351
- [9] X. CAMPI AND H. KRIVINE — Contribution to the International Workshop on Dynamical Features of Nuclei and Finite Fermi Systems, Sitges, Spain (1993) edited by *World Scientific Editions*
- [10] W. BAUER, D.R. DEAN, U. MOSEL AND V. POST — *Phys. Lett.* **B150** (1985) 53
- [11] T. BIRO, J. KNOLL AND J. RICHERT — *Nucl. Phys.* **A459** (1986) 692
- [12] U. LYNEN ET AL. — *Nucl. Phys.* **A545** (1992) 329c

- [13] P. KREUTZ ET AL. — *Nucl. Phys.* **A556** (1993) 672
- [14] P. DÉSESQUELLES ET AL. — *Phys. Rev.* **C48** (1993) 1828
- [15] J.B. ELLIOTT ET AL. — *Phys. Rev.* **C49** (1994) 3185
- [16] M.L. GILKES ET AL. — *Phys. Rev. Lett.* **73** (1994) 1590
- [17] L. WILETS, E.M. HENLEY, M. KRAFT AND A.D. MC KELLAR — *Nucl. Phys.* **A282** (1977) 341
- [18] R.J. LENK, T.J. SCHLAGEL AND V.R. PANDHARIPANDE — *Phys. Rev.* **C43** (1990) 372
- [19] W. TRAUTMANN — Proceedings of the International Workshop XXII on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Austria, January 1994, H. Feldmeier and W. Nörenberg eds., GSI, Darmstadt, 1994
- [20] W. TRAUTMANN — Proceedings of the XXXIII International Winter Meeting on Nuclear Physics, Bormio, Italy, January 95, I. Iori ed., Ricerca Scientifica ed Educazione Permanente
- [21] P. MARMIER AND E. SHELDON — Physics of Nuclei and Particles, *Academic Press*, INC, London, 1969
- [22] K.G.R. DOSS ET AL. — *Phys. Rev.* **C37** (1988) 163
- [23] Data file for the reaction of ^{197}Au on Cu at $E = 600 \text{ MeV/nucleon}$ from the ALADIN Collaboration (communicated by W. Trautmann)
- [24] J. HUBELE ET AL. — *Phys. Rev.* **C46** (1992) R1577
- [25] X.Z. ZHANG, D.H.E. GROSS, S.Y. XU AND Y.M. ZHENG — *Nucl. Phys.* **A461** (1987) 668;
B.A. LI, A.R. DE ANGELIS AND D.H.E. GROSS — *Phys. Lett.* **B303** (1993) 225
- [26] W. TRAUTMANN — private communication
- [27] M.B. ISICHENKO — *Rev. Mod. Phys.* **64** (1992) 981
- [28] J.P. BONDORF, R. DONANGELO, I.N. MISHUSTIN AND H. SCHULZ — *Nucl. Phys.* **A444** (1986) 460

- [29] D.H.E. GROSS AND ZHANG XIAO-ZE — *Nucl. Phys.* **A495** (1989) 231c
- [30] W.F.J. MÜLLER — Proceedings of the XXXIII International Winter Meeting on Nuclear Physics. Bormio. Italy, January 95, I. Iori ed., Ricerca Scientifica ed Educazione Permanente
- [31] C. DORSO AND J. RANDRUP — *Phys. Lett.* **B301** (1993) 328
- [32] J.G. KUNDE ET AL. — *Phys. Rev. Lett.* **74** (1995) 38
- [33] W.C. HSI ET AL. — *Phys. Rev. Lett.* **73** (1994) 3367
- [34] See f.i.: R. BOTET AND M. PLOSZAJCZAK. — Intermittency Patterns of the fluctuations in disaggregating systems. in "Complex Fluids" p.303. L. Garrido ed., Springer Verlag, 1993
- [35] B. ELATTARI, J. RICHERT AND P. WAGNER — *Nucl. Phys.* **A560** (1993) 603
- [36] X. CAMPI AND H. KRIVINE — preprint IPNO/TH 95-13, IPN Orsay, Paris
- [37] A. BIALAS AND R. PESCHANSKI — *Nucl. Phys.* **B273** (1986) 703; ib. *Nucl. Phys.* **B308** (1988) 857

Table caption

Table 1: Comparison of the characteristics of $\overline{m}_2(n)$ and $\overline{\gamma}_2(n)$ (n = multiplicity normalized to 1) corresponding to calculations in the framework of bond percolation and using algorithms A0, A1, A2 for a system of size 6^3 .

The next to the last column corresponds to a cluster identification for which the total energy of the cluster has to be negative, the last column corresponds to the criterion of ref.³¹⁾ (see text, section 5c). The 6 upper lines show the 6 lowest moments of the observables, the variance σ and skewness χ are shown in lines 7-8, the position of the maxima $m_{\max}$ and the corresponding strengths $m_{2\max}$ and $\gamma_{2\max}$ in lines 9-10.

Table 2: Characteristics of $\overline{m}_2(n)$ and $\overline{\gamma}_2(n)$ calculated with algorithm A0. The lines correspond to the same quantities as in Table 1. The columns correspond to the following shapes: (a) cube (6^3); (b) approximate sphere; (c) parallelepiped ($3 \times 8 \times 9$); (d) streaked surface; (e) crenelated surface; (f) random surface. See comments in the text.

Table 1

$$\overline{m}_2(n)$$

Moment	Percolation	A2	A1	A0	A0 $Ef < 0$	A0 ref. ³¹⁾
1	0.454	0.443	0.547	0.462	0.467	0.504
2	0.252	0.238	0.334	0.257	0.260	0.292
3	0.164	0.148	0.222	0.165	0.165	0.190
4	0.118	0.101	0.158	0.117	0.115	0.135
5	0.092	0.075	0.118	0.089	0.086	0.102
6	0.075	0.058	0.092	0.071	0.068	0.081
σ	0.216	0.205	0.187	0.209	0.203	0.196
χ	0.673	0.620	0.192	0.626	0.576	0.561
$n_{\max}$	0.294	0.296	0.468	0.314	0.332	0.359
$m_2 \max$	5.707	5.710	3.909	5.568	5.300	4.950

$$\overline{\gamma}_2(n)$$

Moment	Percolation	A2	A1	A0	A0 $Ef < 0$	A0 ref. ³¹⁾
1	0.435	0.435	0.488	0.438	0.444	0.463
2	0.254	0.254	0.294	0.256	0.260	0.276
3	0.174	0.174	0.200	0.175	0.178	0.190
4	0.132	0.131	0.147	0.132	0.134	0.143
5	0.106	0.105	0.115	0.106	0.108	0.115
6	0.088	0.088	0.094	0.089	0.090	0.096
σ	0.254	0.254	0.237	0.253	0.251	0.248
χ	0.465	0.463	0.122	0.453	0.431	0.348
$n_{\max}$	0.262	0.263	0.436	0.283	0.291	0.335
$m_2 \max$	3.368	3.332	3.407	3.379	3.277	3.265

Table 2

 $\overline{m_2}(n)$

Moment	(a)	(b)	(c)	(d)	(e)	(f)
1	0.462	0.480	0.428	0.519	0.569	0.499
2	0.257	0.271	0.227	0.309	0.358	0.292
3	0.165	0.174	0.142	0.205	0.245	0.192
4	0.117	0.123	0.098	0.147	0.179	0.139
5	0.089	0.093	0.074	0.112	0.138	0.107
6	0.071	0.073	0.058	0.090	0.111	0.086
σ	0.209	0.202	0.210	0.199	0.184	0.207
χ	0.626	0.565	0.705	0.405	0.391	0.485
$n_{\max}$	0.314	0.346	0.280	0.407	0.463	0.385
$m_{2\max}$	5.568	5.184	6.582	4.095	3.712	4.271

 $\overline{\gamma_2}(n)$

Moment	(a)	(b)	(c)	(d)	(e)	(f)
1	0.438	0.449	0.421	0.471	0.493	0.459
2	0.256	0.264	0.241	0.284	0.303	0.272
3	0.175	0.180	0.164	0.197	0.210	0.187
4	0.132	0.136	0.123	0.149	0.158	0.141
5	0.106	0.109	0.099	0.119	0.126	0.113
6	0.089	0.091	0.083	0.099	0.105	0.094
σ	0.253	0.249	0.252	0.249	0.244	0.248
χ	0.453	0.410	0.556	0.267	0.123	0.359
$n_{\max}$	0.283	0.315	0.249	0.373	0.428	0.340
$m_{2\max}$	3.379	3.324	3.892	2.809	2.883	2.948

Figure caption

Fig. 1: Distribution of events $\mathcal{N}(E)$ (normalised to 1) and bond probability $p(E)$ between two neighbouring particles which interact through a 2-body potential¹⁷. Here $E_{\max} = 20 \text{ MeV}$ and $d = 1.8 \text{ fm}$. The size of the cubic lattice is 6^3 .

Fig. 2: Upper part: mapping of the percolation probability p on the average fragment size multiplicity $\bar{m}$. Lower part: width of the multiplicity distribution for fixed values of p . Notice that the maximum lies at a value p which corresponds to the critical point p_c . See discussion in the text.

Fig. 3: Physical observables related to mass distributions. $n = m/A$ is the reduced multiplicity. P : bond percolation; A2: algorithm A2. $A_{\max}$ is the size of the largest cluster.

Fig. 4: Cut-off effects. Here the strength of the 2-body potential is multiplied by a factor 1.5.

Fig. 5: Yield of events with fixed (reduced) multiplicity $n = m/A$ obtained in the framework of bond percolation and A2.

Fig. 6: Physical observables calculated with algorithm A2 and the 2-body interaction of Ref.¹⁸.

Fig. 7: Comparison of different observables obtained from experiment²³) (open circles) with results obtained with A2. Full line: potential of Ref.¹⁷); dotted line: potential strength reduced by a factor 2; dashed-dotted line: potential strength enhanced by a factor 2. a: average value of the charge of the heaviest fragment; b: average number of fragments with charge $Z \geq 3$; c: average multiplicity of He species; d: average number of fragments with charge $Z \geq 6$. Calculations are performed with algorithm A2.

Fig. 8: Number of events with fixed Q_{tot} (see text) for different Z_{bond} intervals (in %). Dots: experiment; continuous line: calculations with A2.

Fig. 9: Physical observables calculated with algorithm A1. See comments in the text.

Fig. 10: Average mass yield, $\langle \bar{m} \rangle$, $\langle \bar{m}^2 \rangle$ and $\langle \bar{m}^3 \rangle$ (see section 3a) calculated with algorithm A0 for a cubic system of size 6^3 , the 2-body interaction of Ref.¹⁷) and Coulomb interaction. $E_{\max} = 20 \text{ MeV}$. d is uniformly chosen between 1.8 and 6 fm. See text.

Fig. 11: Distribution of the total energy per particle of events generated by means of A0 for a system 6^3 with the 2-body potential of Ref.¹⁷⁾ and Coulomb interaction. The observables $\overline{m}_2$ and $\overline{\gamma}_2$ are calculated for total energies per particle lying in the intervals $E0[-0.5, +0.5]$ MeV and $E5[+4.5, +5.5]$ MeV.

Fig. 12: Mass distributions for a 6^3 system. The upper part of the figure corresponds to calculations with A0 in a log-log representation, the lower part to systems with radially oriented momenta in a semi-log representation. The straight lines correspond to a power law (upper part) and an exponential distribution (lower part). See text.

Fig. 13: Average behaviour of $\overline{E}_{cont}/A$ as a function of the reduced multiplicity $n = m/A$. See comment in the text. The total energy per particle E_{tot}/A is restricted to the interval $[-0.5, +0.5]$ MeV.

Fig. 14: Distribution of event yields as a function of multiplicity and horizontal factorial moments represented as a function of the mass interval length δs . Calculations are done with algorithm A2 for a system 6^3 . The events correspond to a range of values of $\tilde{\eta} \in [0.18, 0.25]$.

Fig. 15: Same quantities as in Fig. 14 but calculated by means of algorithm A0 with a set of cell sizes $d \in [2.8, 3.4]$ fm.

Fig. 1

Fig. 1

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15