Electromagnetic Form Factors of the SU(3) Octet Baryons in the semibosonized SU(3) Nambu-Jona-Lasinio Model

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Abstract

The electromagnetic form factors of the SU(3) octet baryons are investigated in the semibosonized SU(3) Nambu–Jona-Lasinio model (chiral quark-soliton model). The rotational $1/N_c$ and strange quark mass corrections in linear order are taken into account. We find that the Wess-Zumino terms which are absent from the SU(2) model are numerically of importance. The electromagnetic charge radii of the nucleon and magnetic moments are also evaluated. It turns out that the model is in a remarkable good agreement with the experimental data.

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I. INTRODUCTION

In spite of the belief that Quantum Chromodynamics (QCD) is the fundamental underlying theory of the strong interaction, low energy phenomena such as static properties of hadrons defy solutions based on QCD. The pertinacity of QCD in the low energy region have led to efforts to construct an effective theory for the strong interaction. In pursuit of this aim, the chiral quark soliton model (CQSM)—also known as the semibosonized Nambu-Jona-Lasinio (NJL) model—emerged as a successful effective theory to describe the low energy phenomena without loss of important properties of QCD such as chiral symmetry and its spontaneous breaking. Originally, the model was introduced by Kahana, Ripka and Soni [1] and Banerjee and Birse [2]. The bound states of the valence quarks were well explored in the model while it suffered from the vacuum instability [3]. This problem of the vacuum instability was solved by Diakonov and Petrov [4]. Having investigated the instanton picture of the QCD vacuum in the low-momenta limit in Ref. [4], they have shown that the low-momenta theory is equivalent to the quark-soliton model free from the vacuum instability. The model was further elaborated in Ref. [5] so that it could roughly predict the static properties of the nucleon in the gradient approximation.

The baryon in this model is regarded as $N_c$ valence quarks coupled to the polarized Dirac sea bound by a non-trivial chiral field configuration in the Hartree approximation [6] [7] [8]. The identification of the baryon quantum numbers is acquired by the semiclassical quantization [5] [9] (in nuclear physics called the cranking method [10]) which is performed by integrating over the zero-mode fluctuations of the pion field around the saddle point. It makes the baryon carry the proper quantum numbers like spins and isospins. In SU(2), the model enables us to describe quantitatively a great deal of static properties of the nucleon such as $N-\Delta$ splitting [11], axial constants [13] [14], electromagnetic form factors [12], and to some extent also magnetic moments [8] [12].

Although the SU(2) version of the model was quite successful to explain many static properties of the nucleon, it is necessary to extend the model from SU(2) to SU(3) so that it can be possible to examine the same properties of hyperons and moreover to investigate the effects of hidden strangeness on the nucleon which are in particular manifested in the $\pi N$ sigma term [15] [16], the iso-splitting of the baryonic masses [18] and strange form factors [17]. B lotz et al. [21] [22] and Weigel et al. [23] have carried out the extension of the model from SU(2) to SU(3). Starting from the semibosonized NJL-type lagrangian, they have shown that the model describes hyperon spectra succesfully. The extended SU(3) model is distinguished from the SU(2) CQSM in two ways: Firstly, the quantization of the SU(3) chiral soliton provides us with the mixed terms of the pure SU(2) part and the strange vacuum part induced by the trivial imbedding of the SU(2) soliton into SU(3). The Wess-Zumino anomaly contribution appears due to this mixing in the model. Similarly, we can see the feature in the SU(3) Skyrme model where the Wess-Zumino term survives while it vanishes in SU(2) [25] [26]. Secondly, since the mass of the strange...
quark is not negligible, one has to take into account the mass term in the effective action explicitly. It plays an essential role in the mass splitting of the hyperons. These two differences determine the characteristic of the SU(3) NJL model.

Refs. [21]-[23] indicate that the SU(3) CQSM provides a more refined structure of the collective Hamiltonian than the pseudoscalar Skyrme model. A comparable structure can be obtained in the Skyrme model only by introducing explicit vector mesons and arranging the anomalous and symmetry breaking part of the effective action [24]. However, it is inevitable to import large numbers of parameters into the Skyrme model with vector meson, while the parameters in the NJL model can be fixed completely by adjusting mesonic masses and decay constants ($f_\pi$, $f_K$). The only free parameter we have is the constituent quark mass of the up-down quark.

It is of great importance that $1/N_c$ rotational corrections are taken into account. Starting from the path integral formalism, when we integrate over zero modes fluctuations around the saddle point, a time ordered product of collective operators appears. The $1/N_c$ contribution survives due to the noncommutivity of the collective operators. It was examined in ref. [12] by calculating the axial vector constants $g_A$ and isovector magnetic moments in SU(2). In the same spirit, the SU(3) model was applied to obtain the axial constants $g_A^{(3)}$, $g_A^{(8)}$, and $g_A^{(0)}$ [19] [20]. It predicted the experimental data within about 10%.

In recent papers, we have proceeded to evaluate the magnetic moments [29]. The magnetic moments predicted by the SU(3) model was remarkably enhanced by the contribution arising from $1/N_c$ corrections with the Wess-Zumino terms, so that we were able to predict the nucleon magnetic moments much better than the SU(2) model.

Now, we are in a position to study the electromagnetic form factors and other form factors such as strange form factors. It is important to investigate the form factors in our model, since it allows us to take a step forward in studying the dynamics. Hence, as a first phase, we will consider the electromagnetic form factors. It is of great significance to know them in the SU(3) CQSM in that they provide us with the electromagnetic informations but also they allow us to proceed to explore the techniques for the other form factors of the neutral($Z^0$) currents and charged weak($W^\pm$) currents.

The outline of the paper is as follows. In the next section, we develop the general formalism for the electromagnetic form factors in the SU(3) CQSM (semibosonized NJL model). In section 3, we discuss the electric form factors with related quantities such as electric charge radii. In section 4, we continue to study the magnetic form factors of the SU(3) octet baryons. In section 5, we summarize the work and draw conclusions.

II. THE GENERAL FORMALISM

In this section, we present the general formalism of the electromagnetic form factors of the SU(3) octet baryons in the CQSM, i.e. the semibosonized NJL model.
The SU(3) CQSM is characterized by a partition function in Euclidean space given by the functional integral over pseudoscalar meson and quark fields:

$$Z = \int \mathcal{D}\Psi \mathcal{D}^\dagger \pi \exp \left( - \int d^4x \Psi^\dagger iD\Psi \right),$$

where $D$ denotes the Dirac differential operator

$$iD = \beta(-i\partial + \hat{m} + MU),$$

with the pseudoscalar chiral field

$$U = \exp i\pi^a \lambda^a \gamma_5.$$

$\hat{m}$ is the matrix of the current quark mass given by

$$\hat{m} = \text{diag}(m_u, m_d, m_s) = m_01 + m_8 \lambda_8.$$

$\lambda^a$ represent the usual Gell-Mann matrices normalized as $tr (\lambda^a \lambda^b) = 2\delta^{ab}$. Here, we assume isospin symmetry, i.e. $m_u = m_d$. $M$ shows the dynamical quark mass arising from the spontaneous chiral symmetry breaking, which is in general momentum-dependent [31]. For the sake of convenience we shall look upon $M$ as a constant and introduce the ultra-violet cut-off via the proper time regularization which preserves gauge and chiral invariance [30]. The $m_0$ and $m_8$ in Eq. (4) are respectively defined by

$$m_0 = \frac{m_u + m_d + m_s}{3}, \quad m_8 = \frac{m_u + m_d - 2m_s}{2\sqrt{3}}.$$

The operator $iD$ is expressed in Euclidean space in terms of the Euclidean time derivative $\partial_t$ and the Dirac one-particle hamiltonian $H(U)$

$$iD = \partial_t + H(U) + \beta \hat{m},$$

with

$$H(U) = \frac{\bar{\alpha} \cdot \nabla}{i} + \beta MU.$$

$\beta$ and $\bar{\alpha}$ are the well-known Dirac hermitian matrices [31].

The electromagnetic form factors of the baryons $F_i(q^2)$ are defined by the expectation values of the electromagnetic current $V_\mu$ of the quark fields:

$$\langle B', p'| V_\mu(0)| B, p \rangle = \bar{u}_{B'}(p') \left[ \gamma_\mu F_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{2M_N} F_2(q^2) \right] u_B(p)$$

with

$$V_\mu(z) = \bar{\Psi}(z) \gamma_\mu \hat{Q} \Psi(z).$$
$M_N$ denotes the nucleon mass. $\hat{Q}$ designates the charge operator of the quark field $\Psi(z)$

$$
\hat{Q} = \begin{pmatrix}
\frac{2}{3} & 0 & 0 \\
0 & -\frac{1}{3} & 0 \\
0 & 0 & -\frac{1}{3}
\end{pmatrix}
$$

$$
= T_3 + \frac{Y}{2}.
$$

(10)

$T_3$ and $Y$ are respectively the third component of the isospin and hypercharge given by the Gell-Mann-Nishijima formula. The $q^2$ is just the four momentum transfer $q^2 = -Q^2$ with $Q^2 > 0$. Hence, the electromagnetic current $V_\mu$ can be decomposed into the third and eighth $SU(3)$ octet currents

$$
V_\mu = V^{(3)}_\mu + \frac{1}{\sqrt{3}}V^{(8)}_\mu
$$

(11)

with

$$
V^{(3)}_\mu = \frac{1}{2}\hat{Q}\gamma_\mu \lambda^3 \Psi
$$

$$
V^{(8)}_\mu = \frac{1}{2}\hat{Q}\gamma_\mu \lambda^8 \Psi.
$$

(12)

The electromagnetic form factors $F_i(Q^2)$ can be expressed in terms of the Sachs form factors, $G^B_E(Q^2)$ and $G^B_M(Q^2)$:

$$
G^B_E(Q^2) = F^B_1(Q^2) - \frac{Q^2}{4M_N^2}F^B_2(Q^2),
$$

$$
G^B_M(Q^2) = F^B_1(Q^2) + F^B_2(Q^2).
$$

(13)

In the non-relativistic limit $Q^2 << M_N^2$, the Sachs form factors $G^B_E(Q^2)$ and $G^B_M(Q^2)$ are related to the time and space components of the electromagnetic current, respectively:

$$
\langle B', p' | V_0(0) | B, p \rangle = G^B_E(Q^2)
$$

$$
\langle B', p' | V_i(0) | B, p \rangle = \frac{1}{2M_N}G^B_M(Q^2)i\epsilon_{ijk}q^j\langle \lambda' | \sigma_k | \lambda \rangle,
$$

(14)

where $\sigma_k$ stand for Pauli spin matrices. $|\lambda\rangle$ is the corresponding spin state of the baryon.

The matrix elements of the electromagnetic current can be represented by the Euclidean functional integral in our model associated with Eq. (1)

$$
\langle B', p' | V_\mu(0) | B, p \rangle = \frac{1}{Z_T}\lim_{T\rightarrow\infty} \exp \left( i\frac{p_\mu}{2} - i\frac{p'_\mu}{2} \right)
$$

$$
\times \int d^3x d^3y \exp \left( -i\vec{q} \cdot \vec{y} + i\vec{p} \cdot \vec{z} \right) \int D\Psi \int D\Psi' \int D\Psi
$$

$$
\times J_B(\vec{y}, T/2)\Psi^\dagger(0)\beta\gamma_\mu \hat{Q}\Psi(0)j^1_B(\vec{z}, -T/2)
$$

$$
\times \exp \left[ -\int d^4z \Psi^\dagger iD\Psi \right].
$$

(15)
The baryonic states $|B, p\rangle$ and $\langle B', p'|$ are respectively defined by

\[
|B, p\rangle = \lim_{x_4 \to -\infty} \exp(i p_4 x_4) \frac{1}{\sqrt{Z}} \int d^3 x \exp \left( i \vec{p} \cdot \vec{x} \right) J_B^\dagger |0\rangle
\]

\[
\langle B', p'| = \lim_{y_4 \to +\infty} \exp(-i p'_4 y_4) \frac{1}{\sqrt{Z}} \int d^3 y \exp \left( -i \vec{p}' \cdot \vec{y} \right) \langle 0| J_{B'}
\]

(16)

The baryon current $J_B$ can be constructed from quark fields with the number of colors $N_c$.

\[
J_B(x) = \frac{1}{N_c} \epsilon_{\alpha_1 \cdots \alpha_{N_c}} \Gamma_{J b T T_3 Y}^{\alpha_1 \cdots \alpha_{N_c}} \psi_{\alpha_1 i_1}(x) \cdots \psi_{\alpha_{N_c} i_{N_c}}(x).
\]

(17)

\(\alpha_1 \cdots \alpha_{N_c}\) denote spin–flavor indices, while \(i_1 \cdots i_{N_c}\) designate color indices. The matrices $\Gamma_{J b T T_3 Y}^{\alpha_1 \cdots \alpha_{N_c}}$ are taken to endow the corresponding current with the quantum numbers $J, b, T, T_3, Y$. The $J_B$ plays the role of creating the baryon state. With the quark fields being integrated out, Eq. (15) can be divided into two separated contributions:

\[
\langle B', p'| V_\mu(0)|B, p\rangle = \langle B', p'| V_\mu(0)|B, p\rangle_{\text{val}} + \langle B', p'| V_\mu(0)|B, p\rangle_{\text{sca}},
\]

(18)

where

\[
\langle B', p'| V_\mu(0)|B, p\rangle_{\text{val}} = \frac{1}{Z} \Gamma_{J b T T_3 Y}^{\beta_1 \cdots \beta_{N_c}} \Gamma_{J b T T_3 Y}^{\alpha_1 \cdots \alpha_{N_c}} \lim_{T \to -\infty} \exp \left( i p_4 \frac{T}{2} - i p'_4 \frac{T}{2} \right) \]

\[
\times \int d^3 x d^3 y \exp \left( -i \vec{p}' \cdot \vec{y} + i \vec{p} \cdot \vec{x} \right)
\]

\[
\times \int \mathcal{D}U \exp \left( -S_{\text{eff}} \right) \sum_{i=1}^{N_c} \beta_i \langle \vec{y} \cdot T / 2 | \frac{1}{i D} | 0 \rangle_{\gamma} \beta_j \langle \vec{y} \cdot T / 2 | \frac{1}{i D} | x, \vec{x} \rangle, \quad \gamma \gamma',
\]

\[
\times \langle 0 | \frac{1}{i D} | x, \vec{x}, -\vec{T} / 2 \rangle_{\alpha_i} \prod_{j \neq i}^{N_c} \beta_j \langle \vec{y} \cdot T / 2 | \frac{1}{i D} | x, \vec{x}, -\vec{T} / 2 \rangle_{\alpha_j},
\]

(19)

and

\[
\langle B', p'| V_\mu(0)|B, p\rangle_{\text{sca}} = \frac{1}{Z} \Gamma_{J b T T_3 Y}^{\beta_1 \cdots \beta_{N_c}} \Gamma_{J b T T_3 Y}^{\alpha_1 \cdots \alpha_{N_c}} \lim_{T \to -\infty} \exp \left( i p_4 \frac{T}{2} - i p'_4 \frac{T}{2} \right) \]

\[
\times \int d^3 x d^3 y \exp \left( -i \vec{p}' \cdot \vec{y} + i \vec{p} \cdot \vec{x} \right)
\]

\[
\times \int \mathcal{D}U \exp \left( -S_{\text{eff}} \right) Tr_{\gamma_{\lambda c}}(0, t) \frac{1}{i D} \langle \beta_{\mu} Q \rangle | 0, t \rangle
\]

\[
\times \prod_{i=1}^{N_c} \beta_i \langle \vec{y} \cdot T / 2 | \frac{1}{i D} | x, \vec{x}, -\vec{T} / 2 \rangle_{\alpha_i},
\]

(20)

\(S_{\text{eff}}\) is the effective chiral action expressed by

\[
S_{\text{eff}} = -Sp \log \left[ \partial_\tau + H(U) + \beta \dot{m} \right].
\]

(21)
Sp stands for the functional trace of the time-independent function.

The integral over bosonic fields can be carried out by the saddle point method in the large \( N \) limit, choosing the following Ansatz:

\[
U = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix},
\]

(22)

where \( U_0 \) is the SU(2) chiral background field

\[
U_0 = \exp \left[ \vec{n} \cdot \tau P(r) \right]
\]

(23)

with hedgehog symmetry. \( P(r) \) denotes the profile function satisfying the boundary condition \( P(0) = \pi \) and \( P(\infty) = 0 \). In order to find the quantum \( 1/N_c \) corrections, we have to integrate Eqs. (19, 20) over small oscillations of the pseudo-Goldstone field around the saddle point Eq. (22). This will not be done except for the zero modes. The corresponding fluctuations of the pion fields are not small and hence cannot be neglected. The zero modes are pertinent to continuous symmetries in our problem. Actually, there are three translational and three rotational zero modes. We have to take into account the translational zero modes properly in order to evaluate form factors, since the soliton is not invariant under translation. The rotational zero modes determine the quantum numbers of baryons [9]. Explicitly, the zero modes are taken into account by considering a slowly rotating and translating hedgehog:

\[
\tilde{U}(\vec{x}, t) = A(t)U(\vec{x} - \vec{Z}(t))A^\dagger(t).
\]

(24)

\( A(t) \) belongs to a SU(3) unitary matrix. The Dirac operator \( i\tilde{D} \) in Eq. (6) can be written as

\[
i\tilde{D} = \left( \partial_\tau + H(U) + A^\dagger(t)\dot{A}(t) - i\beta \vec{Z} \cdot \nabla + \beta A^\dagger(t)\dot{m}A(t) \right).
\]

(25)

The corresponding collective action is expressed by

\[
\tilde{S}_{eff} = -N_c Sp \log \left[ \partial_\tau + H(U) + A^\dagger(t)\dot{A}(t) - i\beta \vec{Z} \cdot \nabla \\
+ \beta A^\dagger(t)\dot{m}A(t) - \beta A^\dagger(t)V_\gamma^\mu \partial_\mu A(t) \right]
\]

(26)

with the angular velocity

\[
A^\dagger(t)\dot{A}(t) = i\Omega_E = \frac{1}{2}i\Omega^a E^a
\]

(27)

and the velocity of the translational motion

\[
\dot{\vec{Z}} = \frac{d}{dt}\vec{Z}
\]

(28)

The canonical quantization of the SU(3) soliton can be found in Ref. [32]. Expanding Eq. (26) in powers of angular and translational velocities, we end up with the action for collective coordinates:
\[ S_{\text{coll}} \approx -N_c \text{Tr} \log iD + S_{\text{rot}}[A] + S_{\text{trans}}[\tilde{Z}] \]

where

\[ S_{\text{rot}}[A] = \frac{1}{2} I_{ab} \int dt \Omega_a \Omega_b \]
\[ S_{\text{trans}}[\tilde{Z}] = \frac{1}{2} M_i \int dt \hat{\dot{Z}} \cdot \hat{\dot{Z}}, \]

with the moments of inertia \( I^{ab} \) defined in Ref. [22]. \( M_i \) is a classical mass of the soliton. Corresponding collective Hamiltonians have a form:

\[ H_{\text{rot}} = (I^{-1})_{ab} J_a J_b, \]
\[ H_{\text{trans}} = \frac{\vec{p} \cdot \vec{p}}{2M_i}, \]

where \( J_a \) are operators of angular momentum and \( \vec{p} \) are momentum operators.

Hence, Eq. (19) and Eq. (20) can be written in terms of the rotated Dirac operator \( i\hat{D} \) and chiral effective action \( S_{eff} \). The functional integral over the pseudoscalar field \( U \) is replaced by the path integral which can be calculated in terms of the eigenstates of the hamiltonian corresponding to the collective action given in Eq. (29). Then it becomes the ordinary integral due to the zero modes. Therefore, Eqs. (19, 20) can be rewritten as

\[ \langle B', p' | V_\mu(0) | B, p \rangle_{\text{val}} = \frac{1}{Z} \Gamma_{\mu_1 \cdots \mu_N_v \alpha_1 \cdots \alpha_N_v} \Gamma_{\mu_1 \cdots \alpha_N_v} \exp \left[ -(N_v E_{\text{val}} + E_{\text{sea}}) T \right] \]
\[ \times \lim_{T \to -\infty} \int d^3 x d^3 y \exp \left( -i \vec{p} \cdot \vec{y} + i \vec{p} \cdot \vec{x} \right) \]
\[ \times \int dA_f dAdA_i d\tilde{Z}_f d\tilde{Z} d\tilde{Z}_i \langle \tilde{Z}_f | \exp \left( -S_{\text{trans}} [\tilde{Z}] / 2 \right) | \tilde{Z} \rangle \]
\[ \times \langle \tilde{Z} | \exp \left( -S_{\text{rot}} [\tilde{Z}] / 2 \right) | A_f \rangle \exp \left( -S_{\text{rot}} [\tilde{Z}] / 2 \right) | A_i \rangle \]
\[ \times \sum_{k=1}^{N_v} \mathcal{T} \left[ \beta_f \langle \tilde{y} - \tilde{Z}_f, T/2 | A_f \frac{1}{iD} | -\tilde{Z} \rangle \gamma \left[ A^\dagger \beta_\gamma \gamma A_\gamma \right] \gamma' \right] \]
\[ \times \gamma' \langle -\tilde{Z} | \frac{1}{iD} A_i^\dagger | \tilde{x} - \tilde{Z}_i, -T/2 \rangle_{\alpha_i} \]
\[ \times \prod_{j \neq k} \beta_j \langle \tilde{y} - \tilde{Z}_j, T/2 | A_f \frac{1}{iD} A_i^\dagger | \tilde{x} - \tilde{Z}_i, -T/2 \rangle_{\alpha_j} \]

\[ \times \langle B', p' | V_\mu(0) | B, p \rangle_{\text{sea}} = \frac{1}{Z} \Gamma_{\mu_1 \cdots \mu_N_v \alpha_1 \cdots \alpha_N_v} \Gamma_{\mu_1 \cdots \alpha_N_v} \exp \left[ -(N_v E_{\text{val}} + E_{\text{sea}}) T \right] \]
\[ \times \lim_{T \to -\infty} \int d^3 x d^3 y \exp \left( -i \vec{p} \cdot \vec{y} + i \vec{p} \cdot \vec{x} \right) \]
\[ \times \int dA_f dA_0 d\tilde{Z}_f d\tilde{Z}_0 \langle \tilde{Z}_f | \exp \left( -S_{\text{trans}}^T /T /2 \right) | \tilde{Z}_0 \rangle \]
\[ \times \langle \tilde{Z} | \exp \left( -S_{\text{trans}}^T /T /2 \right) | Z_i \rangle \langle Z_i | \exp \left( -S_{\text{rot}}^T /T /2 \right) | A \rangle \]
\[ \times \langle A | \exp \left( -S_{\text{rot}}^T /T /2 \right) | A_i \rangle \]
\[ \times \mathcal{T} \left[ \text{Tr}_{\gamma\lambda\epsilon} \left( -\tilde{Z} \frac{1}{iD} A^\dagger \beta^\mu \hat{Q} A \right) \right] \]
\[ \times \prod_{k=1}^{N_{\epsilon}} \beta_k \langle \bar{y} - \tilde{Z}_f, T/2 | A_f \frac{1}{iD} A^\dagger | \bar{y} - \tilde{Z}_i, -T/2 \rangle_{\alpha_i}. \]  

(33)

\[ \mathcal{T} \[ \cdots \] \] denotes the time-ordered product of collective operators. This is due to the fact that the functional integral corresponds to the matrix elements of the time-ordered products of the collective operators. In particular, the time-ordering is very significant when we consider the magnetic form factors (as in case of the axial constants see [12] [19]), since the spin operator \( J^a \) does not commute with the SU(3) rotational unitary matrix \( A(t) \). As we integrate over zero modes in the final and initial states, we obtain the translational and rotational corrections of the classical energies of the soliton from the effective actions \( S_{\text{trans}} \) and \( S_{\text{rot}} \). Therefore, introducing the spectral representations of the quark propagator [5] expressed by the eigenfunctions of the Dirac Hamiltonian \( H(U) \) and making use of relations

\[ \int d\tilde{Z}_i \langle \tilde{Z} | \exp \left( -S_{\text{trans}}^T \right) | \tilde{Z}_i \rangle f(\tilde{x} - \tilde{Z}_i) \Longrightarrow \langle \tilde{Z} | \exp \left( -S_{\text{trans}}^T \right) | \bar{y} \rangle \int d^3 x' f(\bar{y}'), \]

(34)

\[ \Gamma_{\alpha^1 \cdots \alpha_{N_{\epsilon}}}^{I_{\gamma_{1}} \cdots I_{\gamma_{T}} T T_{a}} \int d^3 x' \prod_{k=1}^{N_{\epsilon}} [A_f \phi(\bar{y})]_{\beta_k} = \psi^{(8) \ast}_{\gamma_{1} \cdots \gamma_{T}} (A_f), \]

(35)

\[ \Gamma_{\gamma_{1} \cdots \gamma_{T}}^{I_{\alpha_{1}} \cdots I_{\alpha_{N_{\epsilon}}}} \int d^3 x' \prod_{k=1}^{N_{\epsilon}} [\phi(\bar{y}) A^\dagger_{\alpha_k}]_{\beta_k} = \psi^{(8) \ast}_{\gamma_{1} \cdots \gamma_{T}} (A_i), \]

(36)

\[ \langle A | \exp \left( -S_{\text{rot}} \right) | A_i \rangle = \sum_{(Y T T_3)_{\gamma_{1} \cdots \gamma_{T}}} \psi^{(n)}_{(Y T T_3)_{\gamma_{1} \cdots \gamma_{T}}} (A) \psi^{(n) \ast}_{(Y T T_3)_{\gamma_{1} \cdots \gamma_{T}}} (A_i) \exp \left( -\frac{J(J+1)}{2I} \right), \]

(37)

we obtain relatively simple expressions:

\[ \langle B', \bar{p}' | V_{\mu}(0) | B, p \rangle_{\text{val}} = N_f \int d^3 Z \exp \left( i\bar{p} \cdot \bar{Z} \right) \int_{\text{SU(3)}} dA \psi^{(n) \ast}_{\mu \nu} (A) \psi^{(n)}_{\mu \nu} (A) \]
\[ \times \mathcal{T} \left[ \mathcal{F}^{(8)}_{1}(A) + \mathcal{F}^{(8)}_{2}(A) + \mathcal{F}^{(8)}_{3}(A) \right] \]

(38)

\[ \langle B', \bar{p}' | V_{\mu}(0) | B, p \rangle_{\text{sea}} = N_f \int d^3 Z \exp \left( i\bar{p} \cdot \bar{Z} \right) \int_{\text{SU(3)}} dA \psi^{(n) \ast}_{\mu \nu} (A) \psi^{(n)}_{\mu \nu} (A) \]
\[ \times \mathcal{T} \left[ Tr \langle \tilde{Z} | \frac{1}{iD} A^\dagger \beta^\mu \hat{Q} A \right] \]

(39)
Here, we have considered contributions up to the first order of $\Omega_E$, \textit{i.e.} $1/N_c$ correction and linear correction of the strange quark mass $m_s$. The mixed term $O(m_s/N_c)$ is so tiny that it can be safely neglected [29]. It is performed by the expansion of the propagator $1/i\hat{D}$ in terms of $\Omega_E$ and $m_s$:

$$
\frac{1}{i\hat{D}} \approx \frac{1}{\partial_\tau + H} + \frac{1}{\partial_\tau + H}(-i\Omega_E) \frac{1}{\partial_\tau + H} + \frac{1}{\partial_\tau + H}(-\beta A^1 \hat{m} A) \frac{1}{\partial_\tau + H}. \quad (40)
$$

The collective SU(3) octet wave functions $\psi_{\mu'^{\nu'}}(A)$ are identified with the SU(3) Wigner functions

$$
\psi_{(Y'T'J_3)}(A) = \sqrt{dim(n)}(-1)^{Y'+2J_\lambda} \left[ \psi(Y, T, T_3 | D_{(n)}(A) | -Y', J, -J_3) \right]^* \quad (41)
$$
as eigenstates of the collective rotational Hamiltonian. The functions $\mathcal{F}_i(A)$ are defined as

$$
\mathcal{F}_1^{(q')} (A) = \langle val | \beta \gamma_\mu \lambda^a | val \rangle D_{Q_4}^{(s)} (A)
$$

$$
\mathcal{F}_2^{(\Omega')} (A) = \sum_n \left[ \langle val | \lambda^a | n \rangle \langle n | \beta \gamma_\mu \lambda^b | val \rangle i\Omega_E (A) D_{Q_4}^{(s)} (A) \right] \frac{1}{E_{val} - E_n}
$$

$$
\mathcal{F}_3^{(m')} (A) = -m_0 \sum_n \left[ \langle val | \beta | n \rangle \langle n | \beta \gamma_\mu \lambda^a | val \rangle D_{Q_4}^{(s)} (A) \right] \frac{1}{E_{val} - E_n}
$$

$$
- m_s \sum_n \left[ \langle val | \beta \lambda^a | n \rangle \langle n | \beta \gamma_\mu \lambda^b | val \rangle D_{Q_4}^{(s)} (A) D_{Q_{sa}}^{(s)} (A) \right] \frac{1}{E_{val} - E_n}
$$

$$
D_{Q_4}^{(s)} \text{ is defined as } \frac{1}{2} (D_{3a}^{(s)} + \frac{1}{\sqrt{3}} D_{8a}^{(s)}). \text{ The collective SU(3) octet wave function in Eq. (41) satisfies the orthonormality [33]}
$$

$$
\int dA \psi_{\mu'^{\nu'}}^{*(n)}(A) \psi_{\mu^{\nu}}^{(n)}(A) = \delta_{n'n} \delta_{\mu'\mu} \delta_{\nu'\nu}. \quad (43)
$$

The subscripts $\mu \nu$ of $\psi_{\mu^{\nu}}^{(n)}$ represent $(Y'T'J_3)(Y'J'J_3)$. (n) stands for the irreducible representation of SU(3). $Y'$ is the negative of the right hypercharge constrained by $Y_R = \frac{N_c B}{3} = 1$. Since Eq. (39), in particular, its real part diverges, we have to regularize it. We employ the well-known proper time regularization

$$
\text{Re} S_{eff} = \frac{1}{2} \text{Tr} \int_0^{\infty} du \frac{du}{u} e^{-u D_1 D} \phi (u; \Lambda_i) \quad (44)
$$

with
\[
\phi(u; \Lambda_i) = \sum_i c_i \theta \left( u - \frac{1}{\Lambda_i^2} \right). 
\] (45)

The cut-off parameter \(\phi(u; \Lambda_i)\) is fixed via reproducing the physical pion decay constant \(f_\pi = 93\text{MeV}\) and other mesonic properties [22]. As was done in case of the valence part, we take into account the \(1/N_c\) and linear \(m_s\) corrections (see Appendix A for detail).

Making use of the expansion Eq. (40) and the SU(3) octet wave functions and employing the proper-time regularization, we arrive at

\[
\langle B', p' | V_\mu(0) | B, p \rangle_{\text{sea}} = -\frac{N_c}{2} \sum_m \text{sign} E_m \langle D^{[8]}_{Qa} | B \rangle \left\{ \frac{\mathcal{R}(E_m) \delta_{\mu \nu}}{\mathcal{R}(E_m) \delta_{\nu \nu}} \right\} \mathcal{P}_{\mu \nu}^a(\tilde{q}) \\
+ \frac{N_c}{4} \sum_{n,m} \left\{ \frac{\mathcal{R}_Q(E_m, E_n) \langle D^{[8]}_{Qa} | i \Omega^{[8]}_E \rangle_B \delta_{\mu \nu}}{\mathcal{R}_I(E_n, E_m) \langle D^{[8]}_{Qa} | i \Omega^{[8]}_E \rangle_B \delta_{\nu \nu}} \right\} Q_{\mu \nu}^{ab}(\tilde{q}) \\
+ \frac{N_c}{2} \sum_{n,m} \left\{ \frac{\mathcal{M}_\mu^{ab}(\tilde{q})}{\mathcal{M}_\mu^{ab}(\tilde{q})} \right\} M_{\mu \nu}^{ab}(\tilde{q}) \\
+ \frac{N_c}{2} m_0 \sum_{n,m} \left\{ \frac{\mathcal{K}_{\mu \nu}^{ab}(\tilde{q})}{\mathcal{K}_{\mu \nu}^{ab}(\tilde{q})} \right\} K_{\mu \nu}^{ab}(\tilde{q}), 
\] (46)

where the quark matrix elements are written as

\[
\mathcal{P}_{\mu \nu}^a(\tilde{q}) = \int d^4x e^{i q \cdot x} \bar{\psi}_{n}^a(x) \gamma_\mu \gamma_5 \gamma_\nu \psi_{n}^a(x), \\
Q_{\mu \nu}^{ab}(\tilde{q}) = \int d^4x e^{i \tilde{q} \cdot x} \int d^4 y \bar{\psi}_{n}^b(x) \gamma_\mu \gamma_5 \gamma_\nu \psi_{m}^b(x) \gamma_5 \gamma_\nu \psi_{n}^b(y), \\
M_{\mu \nu}^{ab}(\tilde{q}) = \int d^4x e^{i q \cdot x} \int d^4 y \bar{\psi}_{n}^b(x) \gamma_\mu \gamma_5 \gamma_\nu \psi_{m}^b(x) \gamma_5 \gamma_\nu \psi_{n}^b(y), \\
K_{\mu \nu}^{ab}(\tilde{q}) = \int d^4x e^{i q \cdot x} \int d^4 y \bar{\psi}_{n}^b(x) \gamma_\mu \gamma_5 \gamma_\nu \psi_{m}^b(x) \gamma_5 \gamma_\nu \psi_{n}^b(y). 
\] (48)
The regularization functions are given by

\[
\mathcal{R}(E_n) = \int \frac{du}{\sqrt{\pi u}} \phi(u; \Lambda_i) |E_n| e^{-uE_n^2},
\]

\[
\mathcal{R}_Q(E_n, E_m) = \frac{1}{2\pi} c_i \int_0^1 \frac{d\alpha}{\sqrt{\alpha(1-\alpha)}} \frac{\alpha(E_n + E_m) - E_m \exp\left(-\alpha E_n^2 + (1-\alpha) E_m^2\right)}{\alpha E_n^2 + (1-\alpha) E_m^2},
\]

\[
\mathcal{R}_\beta(E_n, E_m) = -\frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{du}{\sqrt{u}} \phi(u; \Lambda_i) \left[ \frac{E_n e^{-uE_n^2} + E_m e^{-uE_m^2}}{E_n + E_m} + \frac{e^{-uE_n^2} - e^{-uE_m^2}}{u(E_n^2 - E_m^2)} \right],
\]

\[
\mathcal{R}_\lambda(E_n, E_m) = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{du}{\sqrt{u}} \phi(u; \Lambda_i) \left[ \frac{E_n e^{-uE_n^2} - E_m e^{-uE_m^2}}{E_n - E_m} \right].
\]

(49)

\(I_i\) are moments of inertia defined in Ref. [21]. \(\langle \cdot \rangle_B\) denotes the expectation value of the Wigner \(D\) functions in collective space spanned by \(A\). The expectation values of the \(D\) functions can be evaluated by SU(3) Clebsch–Gordan coefficients listed in Refs. [33] [34]. The index \(\mu\) is the Lorentz index and \(a\) and \(b\) denote the flavors, whereas \(i\) designates the space component of the electromagnetic current. We can here notice that in Eq. (47) \(1/N_c\) term includes two different commuting relations \(i.e.,\) the commutator and anti-commutator between the SU(3) Wigner function \(D^{(8)}\) and the angular velocity \(\Omega_E\) of the soliton. This is due to the time-ordering of the operators and the symmetric properties of the quark matrix elements under indices \(n\) and \(m\) or under \(G^\ast\)-parity [35]. If the quark matrix elements are antisymmetric, then the commutator survives, while if they are symmetric, then the anti-commutator does. The quark matrix elements for the electric form factors \((\mu = 4)\) are symmetric whereas some of the matrix elements for the magnetic form factors are anti-symmetric. However, note that on the whole the matrix element of the current is symmetric, since the regularization functions are symmetric under exchange of \(n\) and \(m\) except for \(\mathcal{R}_Q\).

The regularization functions in Eq. (49) are determined in the proper time regularization manifestly except for \(\mathcal{R}_\lambda\) which corresponds to the Wess–Zumino terms from the imaginary part of the action. In fact, \(\mathcal{R}_\lambda\) is not a regularization function. It is independent of the cut-off parameter \(\Lambda\).

With SU(3) symmetry explicitly broken by \(m_s\), the collective hamiltonian is no longer SU(3)-symmetric. Therefore, the eigenstates of the hamiltonian are not in a pure octet or decuplet but mixed states. Treating \(m_s\) as a perturbation, we can obtain the mixed SU(3) baryonic states:

\[
|8, B\rangle = |8, B\rangle + c_{10}^B |10, B\rangle + c_{27}^B |27, B\rangle
\]

(50)

with.
\begin{equation}
\epsilon^B_\text{fin} = \frac{\sqrt{5}}{15}(\sigma - r_1) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad I_2m_s, \quad \epsilon^B_\text{fin} = \frac{1}{15}(3\sigma + r_1 - 4r_2) \begin{bmatrix} \sqrt{6} \\ 3 \\ \sqrt{6} \end{bmatrix} I_2m_s. \quad (51)
\end{equation}

Here, $B$ denotes the SU(3) octet baryons with the spin 1/2. The constant $\sigma$ is related to the $\pi N$ sigma term $\Sigma = 3/2(m_u + m_d)\sigma$ and $r_i$ designates $K_i/I_i$, where $K_i$ stands for the anomalous moments of inertia defined in Ref. [22].

\section{III. The Electric Properties of the SU(3) Octet Baryons}

The electric form factors are straightforwardly obtained by the matrix elements of the time component of the electromagnetic current, as was defined in Eq. (14). Eq. (47) furnishes the final expression of the electric form factor. Since the SU(3) hedgehog solutions are obtained by means of the imbedding of the SU(2) hedgehog field $U_0$ as shown in Eq. (22), it is convenient to define the projection operators $P_T$ and $P_S$ [22]

\begin{equation}
P_T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (52)
\end{equation}

Having defined these projection operators, we can separate the pure SU(2) part from the SU(3) which are represented by the collective operators. Utilizing the projection operators and introducing SU(2)$_T \times$ U(1)$_Y$ invariant tensors

\begin{equation}
P_T \lambda^a = \begin{cases} \tau^a & \text{if } a = 1, 2, 3 \\ 0 & \text{if } a = 4, 5, 6, 7 \\ 1 & \text{if } a = 8 \end{cases}
\end{equation}

\begin{equation}
P_T \lambda^a P_S \lambda^b = \left[ i(f^{acb} - \epsilon^{abc}) - \frac{1}{\sqrt{3}}(\delta^{ac}\delta^{bs} + \delta^{as}\delta^{bc}) + \delta^{abc} \right] \lambda^c, \quad (53)
\end{equation}

we can find that the quark matrix elements include only the pure SU(2) components with transition matrix elements between the vacuum states with SU(2) flavors and the eigenstates of the one-body Hamiltonian Eq. (7). The SU(3) elements only appear in the collective parts. Hence, we can write the expression of the electric form factors

\begin{equation}
G^B_E(\vec{Q}^2) = \frac{N_c}{\sqrt{3}} \left[ \langle D_{Q_s}^{(8)} \rangle_B B(\vec{Q}^2) - \langle D_{Q_s}^{(8)} J_a \rangle_B \frac{2I_1(\vec{Q}^2)}{I_1} - \langle D_{Q_s}^{(8)} J_p \rangle_B \frac{2I_2(\vec{Q}^2)}{I_2} \right.
\end{equation}

\begin{equation}
+ \langle D_{Q_s}^{(8)} D_{Q_s}^{(8)} \rangle_B \frac{8m_s}{I_1\sqrt{3}} \left( I_1 K_1(\vec{Q}^2) - I_1(\vec{Q}^2)K_1 \right)
\end{equation}

\begin{equation}
+ \langle D_{Q_s}^{(8)} D_{Q_s}^{(8)} \rangle_B \frac{8m_s}{I_2\sqrt{3}} \left( I_2 K_2(\vec{Q}^2) - I_2(\vec{Q}^2)K_2 \right), \quad (54)
\end{equation}

13
where

\[
B(Q^2) = \int d^3 x \ j_0(Qr) \left[ \Psi_{val}^\dagger(x) \Psi_{val}(x) - \frac{1}{2} \sum_n \text{sign}(E_n) \Psi_n^\dagger(x) \Psi_n(x) \right],
\]

\[
I_1(Q^2) = \frac{N_c}{6} \sum_{n,m} \int d^3 x \ j_0(Qr) \int d^3 y \left[ \frac{\Psi_n^\dagger(x) \bar{r} \Psi_{val}(x) \cdot \Psi_m^\dagger(y) \bar{r} \Psi_n(y)}{E_n - E_{val}} + \frac{1}{2} \Psi_n^\dagger(x) r \Psi_m(x) \cdot \Psi_m^\dagger(y) \bar{r} \Psi_n(y) \mathcal{R}_{I}(E_n, E_m) \right],
\]

\[
I_2(Q^2) = \frac{N_c}{6} \sum_{n,m} \int d^3 x \ j_0(Qr) \int d^3 y \left[ \frac{\Psi_{n0}^\dagger(x) \Psi_{val}(x) \Psi_{m0}^\dagger(y) \Psi_{m0}(y) \mathcal{R}_{I}(E_n, E_m^0)}{E_{n0} - E_{val}} + \frac{1}{2} \Psi_n^\dagger(x) r \Psi_m(x) \cdot \Psi_m^\dagger(y) \beta \bar{r} \Psi_n(y) \mathcal{R}_{I}(E_n, E_m) \right],
\]

\[
K_1(Q^2) = \frac{N_c}{6} \sum_{n,m} \int d^3 x \ j_0(Qr) \int d^3 y \left[ \frac{\Psi_n^\dagger(x) \bar{r} \Psi_{val}(x) \Psi_m^\dagger(y) \beta \bar{r} \Psi_n(y)}{E_n - E_{val}} + \frac{1}{2} \Psi_n^\dagger(x) r \Psi_m(x) \cdot \Psi_m^\dagger(y) \bar{r} \Psi_n(y) \mathcal{R}_{I}(E_n, E_m) \right],
\]

\[
K_2(Q^2) = \frac{N_c}{6} \sum_{n,m} \int d^3 x \ j_0(Qr) \int d^3 y \left[ \frac{\Psi_{n0}^\dagger(x) \Psi_{val}(x) \Psi_{m0}^\dagger(y) \beta \Psi_{m0}(y)}{E_{n0} - E_{val}} + \frac{1}{2} \Psi_n^\dagger(x) r \Psi_m(x) \cdot \Psi_m^\dagger(y) \beta \Psi_n(y) \mathcal{R}_{I}(E_n, E_m) \right]
\]

(55)

with the regularization functions \( \mathcal{R}_I \) and \( \mathcal{R}_M \) defined in Eq. (49). The subscripts \( a \) and \( p \) denote the flavor indices \( a = 1, 2, 3 \) and \( p = 4, \cdots, 7 \), respectively, and \( m^0 \) denotes the vacuum state with the SU(2) flavor. \( j_0(Qr) \) is the spherical Bessel function of integral order 0. We can see that when \( Q^2 = 0 \) \( \mathcal{B} \) becomes the baryon number \( B = 1 \), while \( I_1 \) and \( K_1 \) become the usual and the anomalous moments of inertia, respectively. In that case, Eq. (54) is reduced to the Gell-Mann–Nishijima formula \( Q = T_3 + \frac{1}{2} Y \), using the relation

\[
\sum_{a=1}^{8} D_{3a}^{(s)} R^a = I_3 = T_3, \quad \sum_{a=1}^{8} D_{8a}^{(s)} R^a = I_8 = \frac{1}{2} \sqrt{3} Y.
\]

(56)

At \( Q^2 = 0 \), the mass corrections do not contribute to the electric form factors, since the fourth and fifth terms in Eq. (54) vanish at the zero momentum transfer.

In order to calculate the form factors and other observables numerically, we follow the well-known Ripka and Kahana method [36]. Since the isovector electric charge radii have a pole in the chiral limit, we take the pion mass \( m_\pi = 139 \text{ MeV} \) into account. The self-consistent profile function obtained by the Kahana-Ripka method has a good behavior in the solitonic region, but the tail of the pion field is spoiled a little due to the finite size of the radial box when we take into account the pion mass. Hence, we shall make a parametrization in order to have the correct tail of the pion field using the Yukawa-type profile function:
\begin{equation}
\begin{aligned}
P(r) &= \alpha \exp \left( -m_x r \right) \frac{1 + m_x r}{r^2},
\end{aligned}
\end{equation}

where $\alpha$ is a constant governing the strength of the pion field. $\alpha$ is determined by the parametrization written in Eq. (57). Note that the parametrization is only considered in the tail, i.e. from about 4 fm. It satisfies the asymptotic behavior of the profile function $P(r) \to 0$ when $r \to \infty$.

Figure 1 shows the electric form factor of the proton while Figure 2 draws that of the neutron as a function of $Q^2$ with the constituent quark mass $370$ MeV, $420$ MeV and $450$ MeV. The empirical data are provided by Ref. [38]. From Figures 1-2, we can easily find that the nucleon electric form factors increase as the constituent quark mass does. For the best fit, we choose the constituent quark mass $M = 420$ MeV as usually done for the other observables. The contribution of strange quark mass correction is displayed in Figures 3-4. As indicated in the figures, the $m_s$ correction diminishes the electric form factors. This can be understood by examining Eq. (54). The $m_s$ term includes $(I_i K_i (\bar{Q}^2) - \bar{I}_i (\bar{Q}^2) K_i)$ which vanish at $Q^2 = 0$. However, when the momentum transfer $Q^2$ turns on, $I_i (\bar{Q}^2) K_i$ is getting greater than $I_i K_i (\bar{Q}^2)$, so that the whole $m_s$ correction weakens the electric form factors. In particular, it pulls the neutron electric form factor sizably down and as a result some disagreement with the empirical data occurs in. However, the experimental uncertainty in the neutron electric form factor should be taken into account. Some recent experiments give rather large errors in extracting the neutron electric form factor as shown in Ref. [39]–[41]. Moreover, one should keep in mind that the neutron electric form factor is a very small quantity, compared to that of the proton. It could be very sensitive to a minute numerical uncertainty. More important observables for us are probably electric charge radii which are determined by the behavior of the electric form factors near $Q^2 = 0$ which are defined by

\begin{equation}
\begin{aligned}
\langle r^2 \rangle_E^B &= -6 \left. \frac{dG_E^B(Q^2)}{dQ^2} \right|_{Q^2=0}.
\end{aligned}
\end{equation}

Using Eq. (58), we obtain the electric charge radii of the proton and the neutron $\langle r^2 \rangle_p^{th} = 0.88$ and $\langle r^2 \rangle_n^{th} = -0.11$, respectively. The experimental data are $\langle r^2 \rangle_p = 0.74$ and $\langle r^2 \rangle_n = -0.12$. We can see that our results are in a good agreement with experimental ones within about 10%.

In dotted curves in Figures 3-4 we show the prediction of the SU(2) model [12]. As for the proton electric form factor, it is comparable to the SU(3), whereas a great discrepancy is observed in case of the neutron electric form factor. This discrepancy can be understood by looking into the electric isospin form factors. Figure 5 shows differences in the electric isospin form factors between the SU(2) and SU(3) model. From Figure 5, we can find that in case of the SU(3) the difference between the isoscalar and isovector form factors are quite small while their sum is comparable. The discrepancy in the neutron form factors lies in this difference between electric isospin form factors. It is partly due to the absence of $m_s$ and terms depending
on the $I_2$ in the SU(2) model and partly due to the different expectation values of the collective operators. In particular, the terms with the $I_2$ in Eq. (51) can be understood as kaonic contributions in the mesonic language [43]. They are relevant to the hidden strangeness having an effect on the nucleon.

We now turn our attention to the other SU(3) hyperons. In Figures 6-7 we present the electric form factors for the SU(3) octet hyperons. Figure 6 draws those of charged hyperons while Figure 7 displays those of neutral ones. Without $m_s$ correction, we could observe $U$-spin symmetry expressed by

$$\begin{align*}
C_E^{\Sigma^+} &= C_E^{\Xi^+}, \\
C_E^{\Xi^-} &= C_E^{\Xi^0}, \\
C_E^{\Xi^-} &= C_E^{\Xi^0}.
\end{align*}$$

Figures 6-7 show us SU(3) symmetry breaking arising from the $m_s$ correction. In case of the charged octet baryons the SU(3) splitting of the electric form factors are rather small while it is quite visible for the neutral ones. The predicted electric charge radii for different baryons are listed in table 1, compared with the SU(3) Skyrme model with pseudoscalar vector mesons [24].

**IV. THE MAGNETIC PROPERTIES OF THE SU(3) OCTET BARYONS**

The space components of the electromagnetic current is responsible for the magnetic form factors. As used in case of the electric form factor, we again make use of the projection operators given in Eq. (52) and SU(2)$_T \times$ U(1)$_V$ invariant tensors, so that we obtain the expression of $G_M^B(Q^2)$:

$$G_M^B(Q^2) = \frac{M_N}{|Q|} \left[ \langle D_{Q_3}^{[8]} \rangle_B \left( Q_0(Q^2) + \frac{Q_1(Q^2)}{I_1} + \frac{Q_2(Q^2)}{I_2} \right) \right. \right.$$

$$\left. - \langle D_{Q_8}^{[8]} \rangle_B \frac{X_1(Q^2)}{\sqrt{3} I_1} \right. \right.$$

$$\left. - \langle d_{s_3}^{[8]} D_{Q_8}^{[8]} \rangle_B \delta_{p_3} \frac{X_2(Q^2)}{I_2} \right. \right.$$

$$\left. + 2m_s \langle (D_{Q_3}^{[8]} - 1) D_{Q_3}^{[8]} \rangle_B \mathcal{M}_0(Q^2) \right. \right.$$

$$\left. + m_s \langle D_{Q_8}^{[8]} D_{Q_8}^{[8]} \rangle_B \left( \frac{2}{3} \mathcal{M}_1(Q^2) - \frac{2}{3} r_1 \mathcal{X}_1(Q^2) \right) \right. \right.$$

$$\left. + m_s \sqrt{3} \langle d_{s_3}^{[8]} D_{Q_8}^{[8]} D_{Q_8}^{[8]} \rangle_B \delta_{p_3} \left( \frac{2}{3} \mathcal{M}_2(Q^2) - \frac{2}{3} r_2 \mathcal{X}_2(Q^2) \right) \right],$$

where

$$Q_0(Q^2) = N_c \int d^3x \bar{j}(qr) \left[ \Psi_{vad}^+(x) \gamma_5 \{ \vec{r} \times \vec{\sigma} \} \cdot \vec{\tau} \Psi_{vad}(x) \right.$$\n
$$\left. - \frac{1}{2} \sum_n \text{sign}(E_n) \Psi_{n}^+(x) \gamma_5 \{ \vec{r} \times \vec{\sigma} \} \cdot \vec{\tau} \Psi_{n}(x) \mathcal{R}(E_n) \right],$$

$$Q_1(Q^2) = \frac{i N_c}{2} \sum_n \int d^3x \bar{j}(qr) \int d^3y$$

16
\[ Q_2(\bar{Q}^2) = \frac{N_c}{2} \sum_{m_0} \int d^3 x j_1(qr) \int d^3 y \left( \frac{\Psi_m(x) \Psi_n(y)}{E_n - E_m} \right) \]

\[ \times \left[ \text{sign}(E_m) \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right] \]

\[ + \frac{1}{2} \sum_{m} \psi_m(x) \beta \psi_n(y) \mathcal{R}_f(E_n, E_m) \]

\[ + \frac{1}{2} \sum_{m} \psi_n(x) \beta \psi_m(y) \mathcal{R}_f(E_n, E_m) \]}

\[ X_1(\bar{Q}^2) = N_c \sum_{m} \int d^3 x j_1(qr) \int d^3 y \left[ \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right] \]

\[ \times \left( \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right) \]

\[ + \frac{1}{2} \sum_{m} \psi_m(x) \beta \psi_n(y) \mathcal{R}_f(E_n, E_m) \]

\[ + \frac{1}{2} \sum_{m} \psi_n(x) \beta \psi_m(y) \mathcal{R}_f(E_n, E_m) \]}

\[ X_2(\bar{Q}^2) = N_c \sum_{m} \int d^3 x j_1(qr) \int d^3 y \left[ \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right] \]

\[ \times \left( \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right) \]

\[ + \frac{1}{2} \sum_{m} \psi_m(x) \beta \psi_n(y) \mathcal{R}_f(E_n, E_m) \]

\[ + \frac{1}{2} \sum_{m} \psi_n(x) \beta \psi_m(y) \mathcal{R}_f(E_n, E_m) \]}

\[ M_0(\bar{Q}^2) = N_c \sum_{m} \int d^3 x j_1(qr) \int d^3 y \left[ \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right] \]

\[ \times \left( \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right) \]

\[ + \frac{1}{2} \sum_{m} \psi_m(x) \beta \psi_n(y) \mathcal{R}_f(E_n, E_m) \]

\[ + \frac{1}{2} \sum_{m} \psi_n(x) \beta \psi_m(y) \mathcal{R}_f(E_n, E_m) \]}

\[ M_1(\bar{Q}^2) = N_c \sum_{m} \int d^3 x j_1(qr) \int d^3 y \left[ \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right] \]

\[ \times \left( \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right) \]

\[ + \frac{1}{2} \sum_{m} \psi_m(x) \beta \psi_n(y) \mathcal{R}_f(E_n, E_m) \]

\[ + \frac{1}{2} \sum_{m} \psi_n(x) \beta \psi_m(y) \mathcal{R}_f(E_n, E_m) \]}

\[ M_2(\bar{Q}^2) = N_c \sum_{m} \int d^3 x j_1(qr) \int d^3 y \left[ \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right] \]

\[ \times \left( \frac{\psi_m(x) \beta \psi_n(y)}{E_m - E_n} \right) \]

\[ + \frac{1}{2} \sum_{m} \psi_m(x) \beta \psi_n(y) \mathcal{R}_f(E_n, E_m) \]

\[ + \frac{1}{2} \sum_{m} \psi_n(x) \beta \psi_m(y) \mathcal{R}_f(E_n, E_m) \]}

\[ \text{The regularization functions } \mathcal{R}, \mathcal{R}_Q, \mathcal{R}_\mathcal{M} \text{ and } \mathcal{R}_\beta \text{ are defined in Eq. (49). The subscripts } p \text{ and } q \text{ in Eq. (60) designate flavor indices from 4 to 7. The } m^0 \text{ in the summation stands for the vacuum states with the SU(2) flavor. } r_i \text{ is } K_i/I_i \text{ for short.} \]
As we can see from the densities for the magnetic form factors in Eq. (61), they are pure SU(2) quantities. The SU(3) components are only found in the collective operators in Eq. (60). Therefore, it is straightforward to calculate Eq. (60) numerically. To make sure, we have compared the density of each contribution with the corresponding density in the gradient expansion given in appendix B. As the soliton size increases, our expressions converge to those of the gradient expansion.

The nucleon magnetic form factors are displayed in Figures 8-9, as the constituent quark mass is varied from $M = 370$ to $M = 450$ MeV. In contrast to the case of the electric form factors, the dependence of the magnetic form factors on the constituent quark mass is not linear. Up to around $Q^2 = 0.2$ GeV$^2$ in case of the proton ($Q^2 = 0.4$ GeV$^2$ for the neutron), smaller constituent quark masses are more contributive to the magnetic form factors. However, as $Q^2$ increases, the dependence on the constituent quark mass undergoes a change, i.e., the greater constituent quark masses contribute more to the magnetic form factors. In fact, we can reach the empirical data in the vicinity of $Q^2 = 0$ with $M = 370$ MeV, we reproduce roughly the correct momentum-dependence. We select $M = 420$ MeV for the best fit to be consistent with all observables in this paper.

Figures 10-11 present the contribution of the strange quark mass. On the contrary to the electric form factors, the $m_s$ correction enhances the magnetic form factors about 5% to 10%. In particular, it is of great significance for the neutron magnetic form factor in fitting the empirical data as shown in Figure 11. Our theoretical magnetic form factors are in a good agreement with the empirical data within about 15% like the other quantities. We note that our SU(3) model has more predictive power than the SU(2) model, in particular, in case of the magnetic form factors, as we can see in Figures 10-11 (the dotted curve draws the SU(2) prediction). Although the SU(2) model describes almost all static properties of the nucleon quantitatively, it produces the magnetic form factors only within about 30%, compared to the empirical data. This is due to the absence of the strangeness on the one hand and of the Wess-Zumino terms on the other hand. As we have seen in Figures 10-11, the strange quark mass should not be neglected. Moreover, the Wess-Zumino terms arising from the SU(3) quantization contribute notably to the magnetic form factors of the nucleon.

Table 2 shows each contribution of the rotational $1/N_c$ and $m_s$ corrections to the magnetic moments, i.e. $G^B_M(Q^2)$ at $Q^2 = 0$ (in Ref. [29] the magnetic moments are discussed in detail.) Figures 12-13 display the magnetic form factors of the charged and neutral octet baryons, respectively. The explicit breaking of $U$ spin symmetry in the magnetic form factors are shown. The corresponding magnetic charge radii are defined by

$$\langle r^2 \rangle^B_M = -\frac{6}{\mu_B} \frac{dG^B_M(Q^2)}{dQ^2} \bigg|_{Q^2=0}. \quad (62)$$

Their numerical results are listed in table 3. The results for the nucleon are in a good agreement with the experimental data.
V. SUMMARY AND CONCLUSION

The aim of this work has been to investigate the electromagnetic form factors of the SU(3) octet baryons and related quantities such as electromagnetic charge radii and magnetic moments in the SU(3) semibosonized NJL model (CQSM). Starting from the effective chiral action, we have expressed the matrix elements of electromagnetic current in the model. When quantizing the soliton, the time-ordering arising from the non-commutativity of collective operators was considered. It gives a non-zero contribution of the rotational $1/N_c$ corrections. $m_s$ corrections are treated perturbatively, the collective wave function correction being taken heed of. The octet states of the baryon are mixed with higher irreducible representations due to $m_s$.

Since the Dirac sea polarization is not finite, it is inevitable to introduce the regularization. We have chosen the proper-time regularization scheme as ours so that we may include the regularization in a manifest way. The parameters of the model, including the cut-off, are adjusted to $m_\pi = 139$ MeV and $f_\pi = 93$ MeV. The constituent quark mass is varied between 370 MeV and 450 MeV with $M = 420$ MeV being considered as the best value.

The electric form factor of the proton is in an excellent agreement, whereas that of the neutron is by a factor of two smaller than the empirical data [38]. However, it is well known that there are large uncertainties in extracting it from experiments [39]. Since it is a very tiny quantity, it is extremely sensitive to details of the model. Though the neutron form factor rather deviates from the empirical data, the electric charge radii of the nucleon are obtained in a good agreement with the experimental result within about 10%.

As was shown in case of axial constants [27], the rotational $1/N_c$ contribution is of great importance to fit the experimental data. In particular, the Wess-Zumino anomalous term originating from the imaginary part of the effective chiral action plays extremely an important role in enhancing the magnetic moments i.e., the magnetic form factors at $Q^2 = 0$. In fact, since the WZ term is absent in the SU(2) model, our SU(3) model yields noticeably better results for the magnetic moments, compared to the SU(2). The $m_s$ corrections improve the results by about 10% of the whole contribution. Altogether the absolute values of the magnetic moments deviate less than 20% from the experimental data. Actually the $Q^2$-dependence of the magnetic form factors of the nucleon is reproduced well.

We also evaluated electric and magnetic form factors of all other members of the SU(3) baryon octet. The magnetic moments are in a good agreement with the experimental data. As far as the $Q^2$-dependence is concerned, since there are no experimental data available, these numbers are predictions. In all cases the $m_s$ corrections are about 10%.

Electromagnetic form factors of the baryons are used in order to extract strange form factors from the experimental data. The evaluation of these quantities and of semileptonic and mesonic decays of hyperons will be the next steps in our research.
ACKNOWLEDGEMENT

We would like to thank Ch. Christov, P. Pobylitsa, M. Praszałowicz, and T. Watabe for fruitful discussions and critical comments. This work has partly been supported by the BMFT, the DFG and the COSY–Project (Jülich).

APPENDIX A: THE DERIVATION OF THE REGULARIZATION

In this appendix, we shall give an explicit derivation of the regularized $\Omega^0$ and $\Omega^1$ contributions to the electromagnetic form factors. We make use of the proper-time regularization scheme. We can see that the procedure is very similar to the case of the axial constants Ref. [19]. Note that the non-anomalous part is regularized. As is written in Eq. (44), the regularized effective action is expressed as

$$\text{Re} S^\text{eff} = S_p \int \frac{du}{u} \phi(u) \exp(-uDD^1),$$ (A1)

where

$$D = \partial_r + H + +i\Omega_E + \beta A^i \dot{Q} A - iA_4 A^i \dot{Q} A - \alpha_k A_k A^i \dot{Q} A$$

$$D^1 = -\partial_r + H + -i\Omega_E + \beta A^i \dot{Q} A + iA_4 A^i \dot{Q} A - \alpha_k A_k A^i \dot{Q} A.$$ (A2)

Hence,

$$DD^1 = W_0(A_\mu^0, \Omega^0, m^0) + W_1(A_\mu^1, \Omega^0, m^0) + W_2(A_\mu^1, \Omega^1) + W_3(A_\mu^0, \Omega^1) + W_4(m^1) + O(\Omega^1, m^1) + O(\Omega^2) + O(m^2)$$ (A3)

with

$$W_0 = -\partial^2_r + H_E^2$$

$$W_1 = i\{A_4 A^i \dot{Q} A, \partial_r\} - [\alpha_k A_k A^i \dot{Q} A, \partial_r]$$

$$- i[H_E, A_4 A^i \dot{Q} A] - \{H_E, \alpha_k A_k A^i \dot{Q} A\}$$

$$W_2 = -\{\Omega_E, A_4 A^i \dot{Q} A\} + i[\Omega_E, \alpha_k A_k A^i \dot{Q} A]$$

$$W_3 = -i\{\Omega_E, \partial_r\} + i[H_E, \Omega_E]$$

$$W_4 = [\beta A^i \dot{m} A, \partial_r] + \{H_E, \beta A^i \dot{m} A\}$$

$$- i[\beta A^i \dot{m} A, A_4 A^i \dot{Q} A] + \{\beta A^i \dot{m} A, \alpha_k A_k A^i \dot{Q} A\}.$$ (A4)

The terms of higher orders in $\Omega$ and $\dot{m}$ and of $\Omega \cdot \dot{m}$ are neglected, since they are very tiny.

Taking advantage of the Feynman-Schwinger-Dyson formula, we can expand $\exp(-uW)$ around $W_0$:
\[ \exp(-uW) = \exp(-uW_0) \]
\[ - u \int_0^1 d\alpha \exp(-u\alpha W_0)[W - W_0] \exp(-u(1-\alpha)W_0) \]
\[ + u^2 \int_0^1 d\beta \int_0^1 \exp(-u\alpha W_0)[W - W_0] \times \]
\[ \times \exp(-u\beta W_0)[W - W_0] \exp(-u(1-\alpha-\beta)W_0) \]
\[ + \cdots \] (A5)

First, we shall consider in case of the electric form factor. The lowest order contribution of \( \Omega^0_E \) vanishes. The sea contribution of \( \Omega^0_E \) comes only from the imaginary part of the effective action. As for the next order of \( \Omega_E \), we need \( W_2 \) and \( W_1 \cdot W_3 \). After some manipulations, we obtain

\[ \langle B, p'|v_0(0)|B, p \rangle^{\Omega} = \frac{N_c}{4} \sum_{nm} \frac{\delta}{\delta A_i} \frac{1}{\delta A_i} \int d\phi(u) \int_0^1 d\alpha \exp(-u\alpha W_0) \]
\[ \times A_k \{ H_E, \alpha_k A^i \hat{Q} \} \exp(-u(1-\alpha)W_0) \]
\[ = \frac{N_c}{2} D_{Q_2}^{(8)} \sum_n \langle n | \alpha_i \lambda^a | n \rangle R(E_n), \] (A6)

The \( m_s \) correction due to \( W_4 \) and \( W_1 \cdot W_1 \) vanishes like \( \Omega^0 \) contribution. The \( m_s \) correction arises only from the quantization of \( i\Omega_E \) [22].

The regularization of the magnetic form factor is more involved due to the time-ordering of collective operators. Here, we need only the term \(-A_k \{ H_E, \alpha_k A^i \hat{Q} \} \) for the lowest order contribution:

\[ \langle B, p'|v_1(0)|B, p \rangle^{\Omega_1} = \frac{\delta}{\delta A_i} \frac{1}{\delta A_i} \int d\phi(u) \int_0^1 d\alpha \exp(-u\alpha W_0) \]
\[ \times A_k \{ H_E, \alpha_k A^i \hat{Q} \} \exp(-u(1-\alpha)W_0) \]
\[ = \frac{N_c}{2} D_{Q_2}^{(8)} \sum_n \langle n | \alpha_i \lambda^a | n \rangle R(E_n), \] (A7)

where \( R(E_n) \) is defined in Eq. (49).

As a next step, we proceed to evaluate the \( \Omega_1^E \) correction to the magnetic form factor. It is tedious but straightforward:

\[ \langle B, p'|v_1(0)|B, p \rangle^{\Omega_1} = \frac{\delta}{\delta A_i} \frac{1}{\delta A_i} (X_1[A_k] + X_2[A_k]) |_{A_k=0}, \] (A8)

where

\[ X_1[A_k] = \frac{1}{2} \int d\phi(u) \int_0^1 d\alpha \exp(-u\alpha W_0) \]
\[ \times W_1[A_k] \exp(-u(1-\alpha)W_0), \] (A9)

\[ X_2[A_k] = \frac{1}{2} \int d\phi(u) \int_0^1 d\beta \int_0^{1-\beta} d\alpha \exp(-u\alpha W_0) \]
\[ \times (W_1[A_k] + W_2[A_k]) \exp(-u\beta W_0) (W_1[A_k] + W_3[A_k]) \]
\[ \exp(-u(1-\alpha-\beta)W_0). \] (A10)
The terms including $W_1 \cdot W_1$ and $W_3 \cdot W_3$ vanish. The first term $\frac{\delta}{\delta A_i} X_1[A_k]$ is obtained to be
\[
\frac{\delta}{\delta A_i} X_1[A_k] = -\frac{i}{16} N_c \sum_{n,m} \sqrt{\frac{m}{\pi}} (e^{-aE_m^n} - e^{-aE_m^n}) \{ i\Omega_E^n, D_{Q_b}^{[s]} \} \\
\langle n|\lambda^a|m\rangle\langle m|\alpha;\lambda^b|n\rangle.
\]  
(A11)

The second term is as follows:
\[
\frac{\delta}{\delta A_i} X_2[A_k] = -u \frac{iN_c}{8\pi} \sum_{n,m} \int_0^1 d\beta e^{-u[\beta E_m^n + (1-\beta)E_m^n]} \\
[\beta E_m - (1 - \beta) E_n] \frac{1}{\sqrt{\beta(1-\beta)}} [i\Omega_E^n, D_{Q_b}^{[s]}] \\
\langle n|\lambda^a|m\rangle\langle m|\alpha;\lambda^b|n\rangle \\
+ \frac{i}{16} N_c \sum_{n,m} \sqrt{\frac{m}{\pi}} (e^{-aE_m^n} - e^{-aE_m^n}) \{ i\Omega_E^n, D_{Q_b}^{[s]} \} \\
\langle n|\lambda^a|m\rangle\langle m|\alpha;\lambda^b|n\rangle.
\]  
(A12)

The second part of Eq. (A12) is cancelled by $\frac{\delta}{\delta A_i} X_1[A_k]$, so that we have
\[
\langle B, p'|V_i(0)|B, p\rangle^{\Omega^l} = -u \frac{iN_c}{8\pi} \sum_{n,m} \int_0^1 d\beta e^{-u[\beta E_m^n + (1-\beta)E_m^n]} \\
[\beta E_m - (1 - \beta) E_n] \frac{1}{\sqrt{\beta(1-\beta)}} [i\Omega_E^n, D_{Q_b}^{[s]}] \\
\langle n|\lambda^a|m\rangle\langle m|\alpha;\lambda^b|n\rangle.
\]  
(A13)

Having integrated over $\beta$, we obtain
\[
\langle B, p'|V_i(0)|B, p\rangle^{\Omega^l} = -\frac{N_c}{4} \sum_m \langle[D_{Q_a}^{[s]}, J_i_B]|n \rangle \langle n|\lambda^a|m\rangle\langle m|\alpha;\lambda^b|n\rangle R_Q(E_n, E_m), \tag{A14}
\]
where $R_Q$ is defined in Eq. (49).

**APPENDIX B: THE GRADIENT EXPANSION OF THE MAGNETIC MOMENTS**

It is well known that the exact expressions for the magnetic moments can be expanded in powers of gradients of the chiral fields [45]. In this way the quark determinant gives terms, which are quite similar to the Skyrme model expressions [24]. An important difference is however the contributions of order $\Omega^l$ from the real part of the action. In the present case we obtain
\[ \mu_B = -2M_n \int dr \ r^2 \sin^2 \theta \langle D_{Q3} \rangle_B \left[ \frac{8\pi}{3} f_\pi^2 + \frac{1}{3} \frac{M_n}{4 I_1} + \frac{1}{3} \frac{M_u}{8 I_2} \right] \\
+ \frac{4}{9\pi} \int dr \ r^2 \sin^2 \theta \theta' \left[ -\frac{\langle d_{3\gamma} D_{Q3} J_\gamma \rangle_B}{I_2} - \frac{\langle D_{Q8} J_3 \rangle_B}{I_1 \sqrt{3}} \right]. \] (B1)

Our numerical densities for the electromagnetic form factors are compared with those obtained from the gradient expansion in order to warrant the calculation.
REFERENCES

TABLE I. The electric charge radii of the SU(3) octet baryons predicted by our model are compared with the evaluation from the Skyrme model of Park and Weigel [24] and the experimental values.

<table>
<thead>
<tr>
<th>Baryons</th>
<th>Our model</th>
<th>Park &amp; Weigel</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>0.88</td>
<td>1.20</td>
<td>0.74</td>
</tr>
<tr>
<td>$n$</td>
<td>−0.11</td>
<td>−0.15</td>
<td>−0.12</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>0.003</td>
<td>−0.06</td>
<td>−</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>0.95</td>
<td>1.20</td>
<td>−</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
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<td>−0.01</td>
<td>−</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>−0.73</td>
<td>−1.21</td>
<td>−</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>0.01</td>
<td>−0.10</td>
<td>−</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>−0.67</td>
<td>−1.21</td>
<td>−</td>
</tr>
</tbody>
</table>

TABLE II. The magnetic moments of the SU(3) octet baryons predicted by our model. Each contribution is listed from the leading order. The results are also compared with the Skyrme model of Park and Weigel [24]. The experimental data for the magnetic moments are taken from Ref.[44]. Our final values are given by $\mu_B(\Omega^1, m^1)$.

<table>
<thead>
<tr>
<th>Baryons</th>
<th>$\mu_B(\Omega^0, m^0)$</th>
<th>$\mu_B(\Omega^1, m^0)$</th>
<th>$\mu_B(\Omega^1, m^1)$</th>
<th>Park &amp; Weigel</th>
<th>Exp.</th>
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<td>$p$</td>
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<td>2.79</td>
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<td>$n$</td>
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<td>−1.91</td>
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<tr>
<td>$\Lambda$</td>
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</tr>
<tr>
<td>$\Sigma^+$</td>
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<td>2.34</td>
<td>2.41</td>
<td>2.46</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
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<td>0.70</td>
<td>0.74</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^-$</td>
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<td>−1.16</td>
</tr>
<tr>
<td>$\Xi^0$</td>
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<td>−1.59</td>
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<td>−1.25</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>−0.21</td>
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<td>−0.67</td>
<td>−0.84</td>
<td>−0.65</td>
</tr>
<tr>
<td>$[\Sigma^0 \to \Lambda]$</td>
<td>0.66</td>
<td>1.35</td>
<td>1.44</td>
<td>1.74</td>
<td>1.61</td>
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</table>
TABLE III. The magnetic charge radii of the SU(3) octet baryons predicted by our model are compared with the Skyrme model of Park and Weigel [24].

<table>
<thead>
<tr>
<th>Baryons</th>
<th>Our model</th>
<th>Park &amp; Weigel</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>1.03</td>
<td>0.94</td>
<td>0.74</td>
</tr>
<tr>
<td>$n$</td>
<td>1.06</td>
<td>0.94</td>
<td>0.77</td>
</tr>
<tr>
<td>$\Lambda$</td>
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<td>0.78</td>
<td>–</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>1.02</td>
<td>0.96</td>
<td>–</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>1.01</td>
<td>0.86</td>
<td>–</td>
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<td>1.07</td>
<td>–</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>1.02</td>
<td>0.90</td>
<td>–</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>1.06</td>
<td>0.84</td>
<td>–</td>
</tr>
</tbody>
</table>