5A

CERN LIBRARIES, GENEVA

SCAN-9504116

Sc 3518

ISSN 0106 - 2646

DYNAMO GENERATED TURBULENCE IN DISCS

NORDITA - 95/19 A

A. Brandenburg¹ A. Nordlund² R.F. Stein³ U. Torkelson⁴

1. Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark.

2. TAC, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark; and
Copenhagen University Observatory.

3. Dept. of Physics and Astronomy, Michigan State Univ,
East Lansing, MI 48824, USA.

4. Sterrenkundig Instituut, Postbus 80000, 3508 TA Utrecht,
the Netherlands.

To appear in: Small-scale structures in three-dimensional hydro
and magnetohydrodynamic turbulence, eds. M. Meneguzzi,
A. Pouquet, & P.-L. Sulem. Lecture Notes in Physics, Springer-Verlag.

NORDITA · Nordisk Institut for Teoretisk Fysik

Blegdamsvej 17 DK-2100 København Ø Danmark

Dynamo Generated Turbulence in Discs

A. Brandenburg¹, Å. Nordlund², R.F. Stein³, U. Torkelsson⁴

¹Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

²TAC, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark; and
Copenhagen University Observatory

³Dept. of Physics and Astronomy, Michigan State University, East
Lansing, MI 48824, USA

⁴Sterrenkundig Instituut, Postbus 80000, 3508 TA Utrecht, The
Netherlands

Abstract: The magnetic shear instability appears to be a workable mechanism for generating turbulence in accretion discs. The magnetic field, in turn, is generated by a dynamo process that taps energy from the Keplerian shear flow. Large scale magnetic fields are generated, whose strength is comparable with, or in excess of, the turbulent kinetic energy. Such models enable us to investigate the detailed nature of turbulence in discs. We discuss in particular the possibility of generating convection, where the heat source is viscous and magnetic heating in the bulk of the disc.

1 Introduction

The origin of turbulence in accretion discs has been a long standing problem. For Keplerian rotation ($\Omega \sim R^{-3/2}$), the Rayleigh criterion predicts stability, because the angular momentum per unit mass increases outwards, i.e. $\partial(R^2\Omega)/\partial R > 0$. However, if accretion discs are magnetized, then the Rayleigh criterion is replaced by the much weaker condition $\partial\Omega/\partial R < 0$ for instability (Balbus & Hawley 1991). This magnetic shear instability was first studied by Velikhov (1959) and Chandrasekhar (1960, 1961), and it is believed to be responsible for driving turbulence in accretion discs. Numerical simulations have now also been carried out in three dimensions (Hawley et al. 1995, Brandenburg et al. 1995, Stone & Hawley 1995). Hawley et al. adopted an initially uniform magnetic field that cannot die out, but other simulations with vanishing initial net flux show that the magnetic field required to drive the turbulent motions can in turn be regenerated by the turbulent motions through dynamo action (Brandenburg et al. 1995, Stone & Hawley 1995).

We are now in a situation where we can try to understand a variety of properties of turbulence in accretion discs. In the past this was not possible because

the origin of the turbulence was unknown. In the present paper we focus attention on selected topics using data from a simulation similar to that presented by Brandenburg et al. (1995), hereafter referred to as BNST. In that simulation the magnetic field has a strong large scale component in the toroidal direction. We first address the question of the nature of the magnetic field generation in that case. We then investigate in more detail the resulting vertical stratification in the disc and address the possibility of convection in some layers of the disc.

2 Numerical Simulations

In order to resolve turbulent motions in an accretion disc it is advantageous to restrict attention to a small subvolume of the disc. This is accomplished by adopting periodic boundary conditions in the toroidal or streamwise direction, and quasi-periodic boundary conditions in the radial or spanwise direction. The quasi-periodic condition is periodic with respect to a toroidal position that is advected with the flow (Wisdom & Tremaine 1988). The governing equations are then solved in a Cartesian geometry using a linear approximation for the gravitational acceleration, i.e. $\mathbf{g} = -(GM/R^3)(R - 2x, 0, z)$. This leads to a linear shear flow $u_y^{(0)}(x) = -\frac{3}{2}\Omega_0 x$, and the equations for the *deviations* from this shear flow are

$$\frac{\mathcal{D}\mathbf{u}}{\mathcal{D}t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} + \mathbf{g} - \frac{1}{\rho}\nabla p + \mathbf{f}(\mathbf{u}) + \frac{1}{\rho}\mathbf{J} \times \mathbf{B} + \frac{1}{\rho}\nabla \cdot (2\nu\rho\mathbf{S}), \quad (1)$$

where $\mathcal{D}/\mathcal{D}t = \partial/\partial t + u_y^{(0)}\partial/\partial y$ is the time derivative that includes the transport by the Keplerian shear flow, $\mathbf{f}(\mathbf{u}) = \Omega_0(2u_y, -\frac{1}{2}u_x, 0)$ describes epicyclic deviations from purely circular rotation, and $\mathbf{J} \times \mathbf{B}$ is the Lorentz force, which plays the quintessential role in driving the turbulence. The magnetic field is written as $\mathbf{B} = \nabla \times \mathbf{A}$, where $\mathbf{A}$ is the magnetic vector potential governed by

$$\frac{\mathcal{D}\mathbf{A}}{\mathcal{D}t} = \mathbf{u} \times \mathbf{B} + \frac{3}{2}\Omega_0 A_y \hat{\mathbf{x}} - \eta\mu_0\mathbf{J}. \quad (2)$$

The system of equations is supplemented by energy and continuity equations

$$\frac{\mathcal{D}e}{\mathcal{D}t} = -(\mathbf{u} \cdot \nabla)e - \frac{p}{\rho}\nabla \cdot \mathbf{u} + \frac{1}{\rho}\nabla(\chi\rho\nabla e) + 2\nu\mathbf{S}^2 + \frac{\eta\mu_0}{\rho}\mathbf{J}^2 + Q, \quad (3)$$

$$\frac{\mathcal{D}\ln\rho}{\mathcal{D}t} = -(\mathbf{u} \cdot \nabla)\ln\rho - \nabla \cdot \mathbf{u}, \quad (4)$$

where ρ is density and e the internal energy (which is proportional to the temperature, i.e. $e = c_v T$). The important sources in the energy equations are viscous and Joule heating, where $S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} - \frac{2}{3}\delta_{ij}u_{k,k})$ is a modified rate of strain tensor, and $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$ is the current density. (The magnetic permeability μ_0 is put to unity.) The radiative heating and cooling term, Q , is neglected in the first calculations, but we return to it in Sect. 4.

3 Magnetic field generation

The Balbus-Hawley instability operates on a scale that is proportional to the magnetic field strength. Thus, in order to get the instability going, we need to adopt an initial magnetic field that is strong enough so that the excited scales are well above the dissipation scale. For modest mesh resolution the plasma beta is therefore restricted to a value around one hundred. In order to address a dynamo problem, we have to make sure that the initial and boundary conditions do not prevent the field from decaying to zero. In the simulations of BNST the initial field was vertical, but with vanishing flux, so the field could indeed decay to zero. However, given that the initial field had appreciable strength, one would like to be assured that any memory of the initial field has been lost at later times. In order to preclude this possibility, we plot in Fig. 1 power spectra of B_z for 4 different times, covering basic stages of the run presented in Figs. 1-3 of BNST. Since the system is highly anisotropic, it would not make sense to compute isotropic spectra. Therefore we consider spectra taken separately in the three directions. In contrast to the spectra presented in BNST, where we considered all three components of $\mathbf{B}$ simultaneously, we take here only spectra of the vertical magnetic field component, because this allows us to detect spurious traces of the initial field.

Fig. 1. Power spectra of B_z in all three directions for different times (in units of $T_{\text{rot}} = 2\pi/\Omega_0$). The solid line is the spectrum in the y direction, the dotted line in the x direction, and the dashed line in the z direction.

Between the first and second panels we see a clear decay in the large scale component of the vertical field, indicating that possible relics of the initial field tend to decay away. At later times, however, there is evidence for a continuous build-up of a large scale component of the vertical magnetic field. This indicates that a large scale field is generated, independently of the initial conditions.

The question of the origin of large scale fields in other rotating astrophysical bodies such as the sun or the galaxy is a matter of debate (Vainshtein & Cattaneo 1992), that has recently even reached the pages of Scientific American (January

1995). The debate was initiated by the fact that kinematic dynamos tend to generate fields preferentially at small scales. At those small scales the magnetic field saturates very early, when the magnetic energy becomes comparable to the kinetic energy *at very small scales*, as was pointed out already by Moffatt (1961). At this point, something interesting is expected to happen: either the dynamo process stops, as suggested in a number of papers with more pessimistic views regarding dynamos, or the growth of magnetic energy is redirected into the larger scales, where saturation is more and more postponed until large field strengths are reached. The latter viewpoint is motivated by the idea of an inverse magnetic cascade leading to a growth of magnetic energy at large length scales, which was first demonstrated by Pouquet, Frisch & Lorat (1976) using the eddy-damped quasinormal Markovian (or EDQNM) approximation. Our present simulations show that large scale fields can indeed be generated (Fig. 1), and that their strength $\langle \mathbf{B} \rangle$ can exceed equipartition with the turbulent motions, i.e. $\langle \mathbf{B} \rangle^2 = O(\langle \mu_0 \rho \mathbf{u}^2 \rangle)$.

4 Convection

In most of the models of BNST the cooling function Q was set to zero. Thus, the gas in the disc was continuously heated by viscous and Joule heating. Moreover, since the heating per unit mass is proportional to $1/\rho$, see Eq. (3), the outer layers are heated more efficiently than the layers near the midplane. Thus, the entropy increased monotonically outwards. In reality, radiative cooling acts mostly in the outer layers where the density is low and the gas optically thin. We have modeled this by allowing for a cooling function that is nonvanishing only in the outer layers, i.e.

$$Q = -f(z)(e - e_0)/\tau, \quad (5)$$

where $f(z) = 1$ for $|z| > 1.8$, and 0 elsewhere. In Fig. 2 we plot horizontal averages of density, temperature, and specific entropy.

A new feature resulting from this cooling term is the reversal in the entropy gradient, i.e. the formation of layers with $z \cdot \nabla s < 0$ at $1 < |z| < 1.8$. These layers are unstable to convection according to the Schwarzschild criterion. To verify the presence of convection in parts of the disc, we plot in Fig. 3 the convective flux, $F_{\text{conv}} = \langle \rho u'_z T' \rangle$, where primes denote fluctuations about the horizontal averages. Note that in the convectively unstable layers the convective flux is comparable to $\frac{1}{2} \langle \mathbf{B}^2 \rangle \Omega_0$, which is the natural unit for the flux generated by the Balbus-Hawley instability.

Finally, we wish to know whether the efficiency of turbulence is affected by the presence or absence of convection in our accretion disc simulations. It is the (x, y) -components of the Reynolds and Maxwell stress tensors that govern the rate of turbulent heating of the disc, and that would control the transport of mass and angular momentum in a real disc. Their mean values relative to the gas pressure, $\alpha_{\text{SS}}^{\text{kin}}$ and $\alpha_{\text{SS}}^{\text{mag}}$ (cf. Shakura & Sunyaev 1972), turn out to be rather small ($\alpha_{\text{SS}}^{\text{kin}} \sim 0.001$ and $\alpha_{\text{SS}}^{\text{mag}} \sim 0.004$; see BNST). We find similar values even

Fig. 2. Horizontal averages of density, temperature, and specific entropy at different times (in units of $T_{\text{rot}} = 2\pi/\Omega_0$). Convection zones (where $z \cdot \nabla s < 0$) have developed around $1 < |z| < 1.8$.

Fig. 3. The convective flux in units of $\langle \rho \rangle c_s^3$, where c_s is the (isothermal) speed of sound (scale on the left hand side) and in units of $\frac{1}{2} \langle B^2 \rangle \Omega_0$ (scale on the right hand side). On the right hand panel we plot the average toroidal magnetic field together with $\langle B^2 \rangle^{1/2}$ (dotted) and $\langle \rho u^2 \rangle^{1/2}$ (dashed line) at $t = 43$ orbits.

in the presence of convection. Thus, although convection significantly changes the vertical energy transport, it does not seem to affect the values of α_{SS} . We should point out that in the presence of an imposed magnetic field Hawley et al. (1995) have obtained much larger values of α_{SS} .

5 Future developments

The simulations discussed in the present paper open paths to understanding a variety of problems of great current interest. Here we have only touched upon two of them, namely the origin of large scale magnetic fields and properties of the turbulent flows generated in discs. One avenue of future research is to include the effect of linear shear in other simulations of dynamo action (e.g. solar and galactic dynamos). Another important extension of accretion disc simulations is to allow for a net transport of matter in the radial direction which is currently not allowed because of the symmetry of the Cartesian box. This may affect the values of α_{SS} . The type of accretion discs most amenable to comparison with observations are those responsible for dwarf novae outbursts. Their temperature and density regimes are similar to those of stars. It appears therefore feasible to include radiation transfer in such models using methods developed in the context of three-dimensional simulations of stellar convection.

References

- Balbus, S. A., & Hawley, J. F. (1991): "A powerful local shear instability in weakly magnetized disks. I. Linear analysis", *Astrophys. J.* **376**, 214–222
- Brandenburg, A., Nordlund, Å., Stein, R. F., Torkelson, U. (1995): "Dynamo generated turbulence and large scale magnetic fields in a Keplerian shear flow", *Astrophys. J.* (June 20)
- Chandrasekhar, S. (1960): "The stability of non-dissipative Couette flow in hydromagnetics", *Proc. Nat. Acad. Sci.* **46**, 253–257
- Chandrasekhar, S. (1961): *Hydrodynamic and Hydromagnetic Stability*, (Dover Publications, Inc., New York), p. 384
- Hawley, J. F., Gammie, C. F., & Balbus, S. A. (1995): "Local three-dimensional magnetohydrodynamic simulations of accretion discs", *Astrophys. J.* **440**, 742–763
- Moffatt, H. K. (1961): "The amplification of a weak applied magnetic field by turbulence in fluids of moderate conductivity", *J. Fluid Mech.* **11**, 625–635
- Pouquet, A., Frisch, U., Léorat, J. (1976): "Strong MHD helical turbulence and the nonlinear dynamo effect", *J. Fluid Mech.* **77**, 321–354
- Stone, J. M., & Hawley, J. F. (1995): "Dynamo action in weakly magnetized accretion disks" (in preparation)
- Vainshtein, S. I., Cattaneo, F. (1992): "Nonlinear restrictions on dynamo action", *Astrophys. J.* **393**, 165–171
- Velikhov, E. P. (1959): "Stability of an ideally conducting liquid flowing between cylinders rotating in a magnetic field", *Sov. Phys. JETP* **9**, 995–1404
- Wisdom, J., & Tremaine, S. (1988): "Local simulations of planetary rings", *Astron. J.* **95**, 925–940