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On the Random Cascading Model Study of Anomalous Scaling in Multiparticle Production with Continuously Diminishing Scale

Liu Lianshou Zhang Yang Wu Yuanfang

Institute of particle physics, Hua-Zhong Normal University, Wuhan 430070, China

ABSTRACT

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The anomalous scaling of factorial moments with continuously diminishing scale is studied using a random cascading model. It is shown that the model currently used have the property of anomalous scaling only for discrete values of elementary cell size. A revised model is proposed which can give good scaling property also for continuously varying scale. It turns out that the strip integral has good scaling property provided the integral regions are chosen correctly, and that this property is insensitive to the concrete way of self-similar subdivision of phase space in the models.

INSTITUTE OF PARTICLE PHYSICS

HUA-ZHONG NORMAL UNIVERSITY

WUHAN CHINA

Since the pioneer work of A. Bialas and R. Peschansky^[1] the possible existence of anomalous scaling in multiparticle final states in high energy collisions has attracted much attention. Almost all of the experimental collaborations concerning multiparticle production have analysed the proposed anomalous scaling of factorial moments (FM), whose definition in one-dimension (1-D) is^[1]

$$F_q(\delta x) = \frac{1}{M} \sum_{m=1}^M \frac{\langle n_m(n_m-1) \cdots (n_m-q+1) \rangle}{\langle n_m \rangle^q}, \quad (1)$$

where q is the order of moment, $\delta x = \Delta x/M$ is the size of the elementary cell, n_m is the multiplicity in the m th cell, M is the number of elementary cells within the total region Δx used in the "horizontal average" of FM, x is the phase space variable in consideration. Plenty of interesting experimental results appeared in a few years^[2]. It turns out that all of the present available data show a common tendency of upward bending for the log-log plot of 3-D factorial moment (FM) and saturation for that of 1-D FM. The 2-D cases are more complicated but also show systematic regularity. To understand the physics behind these experimental phenomena and to clear up the debate about anomalous scaling in multiparticle final states is necessary and worthwhile.

In studying the anomalous scaling in multiparticle production a simple model — the random cascading model — is commonly used. It has the property of anomalous scaling when the scale δ is diminished according to the following rule

$$\delta \propto \lambda^{-\nu}, \quad \nu = 1, 2, \dots, \quad (2)$$

where λ is an positive integer denoting the number of elementary partition. This is a discrete sequence of number.

However, to study the anomalous scaling with only discretely varying scale is insufficient, especially when the self-affinity of higher dimensional fluctuation is taken into account. The aim of this short paper is to examine the property of random cascading model when the scale is diminished continuously. A revision of the model will be proposed to get a good scaling also for continuously varying scale.

Firstly, let us recall the procedure of self-affine analysis of FM and show that in this kind of analysis the size of elementary cell is necessarily to be able to vary continuously. Consider the two-dimensional case as example. Denoting the two phase space variables as x_1, x_2 , we have

$$F_q(\delta x_1 \delta x_2) = \frac{1}{M^{(2)}} \sum_{m=1}^{M^{(2)}} \frac{\langle n_m(n_m-1) \cdots (n_m-q+1) \rangle}{\langle n_m \rangle^q}, \quad (3)$$

where $\delta x_1 \delta x_2$ is the size of a 2-D cell, n_m is the multiplicity in the m th cell, $M^{(2)}$ is the number of the 2-D cells used in horizontal average.

A problem is: How to determine $\delta x_1, \delta x_2$ and $M^{(2)}$? As the starting point we can fix a 2-D region $\Delta x_1 \Delta x_2$ and divide it into sub-cells with widths

$$\begin{cases} \delta x_1 = \Delta x_1 / M_1 \\ \delta x_2 = \Delta x_2 / M_2 \end{cases} \quad (4)$$

in the x_1 and x_2 directions respectively. There are two possibilities: $M_1 = M_2$ or $M_1 \neq M_2$. In the former case, we say that the phase space is diminished "self-similarly", while in the latter case the phase space is said to be diminished "self-affinely". In view of the highly anisotropy of phase space^[3], it is most probable that the dynamical fluctuations, if exist, should also be anisotropic, and so in studying these fluctuations

the phase space should be diminished self-affinely^[4]. In this case, the scale factors M_1 and M_2 satisfy the condition

$$M_1 = M_2^H, \quad (5)$$

where $0 < H < 1$, called Hurst exponent^[5], is a parameter characterizing the self-affine property of dynamical fluctuations. From Eq.(5) it is clear that, in general, the scale factors M_1 and M_2 can not simultaneously be integer, so that the size of elementary phase space cell should be able to take continuously varying values.

We use the following method to calculate FM for continuously varying scale, taking notice on a common assumption that both the particle distribution and the dynamical fluctuation in phase space are uniform†, which means that not only the probability of particle distribution does not change with the translation in phase space, but also the displacement of elementary cells and the change of number of cells in the horizontal average of FM do not influence the anomalous scaling property of FM. In particular, taking M cells to do the average, or taking less cells, or even using only one cell in the calculation of FM, the final result is the same, provided the number of event is large enough to guarantee the necessary statistics.

Taking the uniformity of phase space into account, it is easily seen that the scale factor M can be any positive real number. For example, let

$$M = N + a \quad (N \text{ is an integer, } 0 \leq a < 1). \quad (6)$$

When we use the elementary cells of width $\delta y = \Delta y/M$ as "scale" to "measure" the region Δy we get N of them and a smaller cell of width $a\Delta y/M$ left. Putting the smaller cell at the last (or first) place of the region and doing average with only the first (or last) N cells we have

$$F_q(\delta x) = \frac{1}{N} \sum_{m=1}^N \frac{\langle n_m(n_m-1) \cdots (n_m-q+1) \rangle}{\langle n_m \rangle^q}. \quad (7)$$

The width $\delta x = \Delta x/M$ of elementary cell in Eq.(7) with M determined by Eq.(6) can be any positive real number and so can vary continuously.

Let us apply Eq's.(6),(7) to study the random cascading model with continuously varying scale. In this model the probability of particle located in the phase space region Δy is taken to be unity. Dividing this region into λ cells, the probability in the m th ($m = i_1 = 1, \dots, \lambda$) cell is $p_m = \omega_{i_1}^{(1)}$, where the subscript 1 of i and the superscript (1) of ω denote that it is the first generation of phase space partition. Then divide each sub-cells into λ even smaller sub-cells, and continue the cascade partition up to ν th generation. The corresponding probability in the m th cell ($m = \{i_1, i_2, \dots, i_\nu\}$) is

$$p_m = \omega_{i_1}^{(1)} \omega_{i_2}^{(2)} \cdots \omega_{i_\nu}^{(\nu)}, \quad i_1, i_2, \dots, i_\nu = 1, 2, \dots, \lambda. \quad (8)$$

The normalized moment of p_m is

$$C_q(\delta x) \equiv \frac{\langle p_m^q \rangle}{\langle p_m \rangle^q} = \frac{\langle \omega_{i_k}^{(k)q} \rangle^\nu}{\langle \omega_{i_k}^{(k)} \rangle^{q\nu}} = (\lambda^q \langle \omega^q \rangle)^\nu = M^{\frac{\ln(\lambda^q \langle \omega^q \rangle)}{\ln \lambda}} = \left(\frac{\Delta x}{\delta x} \right)^{\frac{\ln(\lambda^q \langle \omega^q \rangle)}{\ln \lambda}}, \quad (9)$$

where $k = 1, 2, \dots, \nu$, $\langle \omega_{i_k}^{(k)} \rangle = 1/\lambda$. Assuming the statistical fluctuation to be of the form of Bernouli or Poisson distribution, the probability moment C_q is equal to the factorial moment F_q ^[1]. Thus we see that C_q (or F_q) obeys the anomalous scaling law provided

$$\delta x = \frac{\Delta x}{M} \quad \text{with} \quad M = \lambda^\nu. \quad (10)$$

† In case this uniform assumption does not hold, we can turn to a new variable^[7] to make it hold.

This is just Eq.(2).

Now, what is the behaviour of C_q (or F_q) in random cascading model with δx beyond the restriction Eq.(10)? Let us turn to the answer of this question.

For definiteness the random number $\omega_{i_k}^{(k)}$ in the model is taken as^[4]

$$\omega_{i_k}^{(k)} = \frac{1 + \alpha r_{i_k}}{\lambda + \alpha \sum_{i_k=1}^{\lambda} r_{i_k}} \quad i_k = 1, 2, \dots, \lambda, \quad (11)$$

where r_{i_k} is a random number distributed uniformly in the region $[-1, 1]$, α is a model parameter characterizing the strength of dynamical fluctuation. Taking $\alpha = 0.5$, the elementary partition number $\lambda = 2$, the total multiplicity $n = 20$ and doing cascade partition up to $\nu = 13$ th generation, the result of Monte Carlo simulation with 10000 events is shown in Fig.1. It can be seen that the points with δx and M satisfying the condition Eq.(10) (solid circles in the figure) lie on a straight line, while those points violating this condition do not lie on this line but form a series of "chains" hanging on the solid circles.

This "chain effect" is a property of the random cascading model and does not depend on the concrete form of the probability distribution of $\omega_{i_k}^{(k)}$. For showing this, we have tried the forms used in Ref's.[1] and [8] and the results turn out to be qualitatively the same. Therefore, we conclude that the random cascading model can give an anomalous scaling of moments only for discrete values of the elementary cell size δx given in Eq.(10).

In order to get a model that has an anomalous scaling of moments for continuously varying cell size, we have tried to revise the random cascading model to eliminate the specialness of the points satisfying Eq.(10). A simple way is to take a random cascading region to be larger than the region for calculating the moments and to let the size and place of this region vary from event to event. This procedure is not unreasonable if we notice that in real experiments the collision parameter is different for different event, resulting in a region of particle production with fluctuating place and size, while the region for the calculation of moments is usually fixed.

Fig.1 The results of Monte Carlo simulation for the second order factorial moment with continuously diminishing scale in the original random cascading model.

For concreteness let us take the total size of the random cascading region to be fluctuating from 1 to $1 + f$, with f satisfying $0 \leq f < 1$, and calculate the moments within a fixed range of width $1 - f$ located randomly inside the cascading region. The results for six values of f are shown in Fig.2. It can be seen from the figure that as the increasing of f the "chain effect" becomes weaker and weaker and an approximately straight line is obtained with $f \geq 0.25$. This result turns out to be insensitive with respect to the value of α .

Fig.2 The same as Fig.1 in the revised model for different values of the fluctuation parameter f of cascading region.

Before closing the paper let us examine the scaling property of the strip integral (ST) in the above models. As an example, let us consider the second order ST, whose definition is^[9]

$$F_2^{\text{ST}} = \frac{\int_{\Omega} dx_1 dx_2 \rho_2(x_1, x_2)}{\int_{\Omega} dx_1 dx_2 \rho_1(x_1) \rho_1(x_2)}, \quad (12)$$

where the integration domain Ω is the strip region consisting of all the points inside the investigating region $\Delta x_1 \Delta x_2$ whose distance to the axis $x_1 = x_2$ is less than some scale δ , cf. the netted region in Fig.3.

In Fig.4 is shown the results of F_2^{ST} for $f = 0$ (the original model) and $f = 0.1, 0.2, 0.3$ (the revised model) respectively. It can be seen from the figure that the log-log plot of F_2^{ST} versus the inverse scale is straight only for small values of δ and the shape is almost independent of the value of f .

Fig.3(a) The integration region of the strip integral for large scale δ . In the triangle regions the scale is decreasing as the approaching to the apices.

Fig.3(b) The same as (a) for small scale δ .

In Fig.4 is shown the results of F_2^{ST} for $f = 0$ (the original model) and $f = 0.1, 0.2, 0.3$ (the revised model) respectively. It can be seen from the figure that the log-log plot of F_2^{ST} versus the inverse scale is straight only for small values of δ and the shape is almost independent of the value of f .

Fig.4 The strip integral for the original ($f = 0$) and the revised model. The integration region consists of a rectangle and two triangles, cf. Fig.3

The reason why the scaling is not good at the lower end of log-log plot is due to the improper choice of integration region^[9]. As can be seen from Fig.3, this region consists of one rectangle and two triangles. The rectangle has a fixed scale δ , so that all the points in it satisfy the condition $d \leq \delta$, where d is the distance from the point to the diagonal $x_1 = x_2$. In the triangles, however, the scale is varying and becomes smaller and smaller with the approaching to the apices, cf. Fig.3a. This violates the scaling of ST. When δ is large, the area of triangles dominate the integration region, cf. Fig.3a, and therefore there is almost no scaling. While, when δ is small, the area of triangles are small with respect to that of the rectangle, cf. Fig.3b, and the scaling becomes better. This is just the case, cf. Fig.4.

In order to eliminate this effect we have recalculated the ST using only the rectangle as integration region, cf. the straightly netted region in Fig.3. The results are shown in Fig.5. The scaling is good for all scales.

Fig.5 The same as Fig.4 with only the rectangle as integration region.

In this short paper, for the convenience of the study of anisotropic (self-affine) dynamical fluctuations, the anomalous scaling of factorial moments with continuously diminishing scale is studied using a random cascading model. It is shown that, for the random cascading model currently used, the probability moments (FM's) have the property of anomalous scaling only when the subdividing intervals (sub-bins) in the analysis coincide with those in the model. A revised model is proposed which can give good scaling property of FM for arbitrary sub-bin size. The strip integral is also studied in these models. It turns out that good scaling holds for ST provided the integration region are properly chosen, and that this property is insensitive to the concrete way of self-similar subdivision of phase space in the models. Finally, let us point out in passing that the factorial moment (FM) is equal to the bin-wise (BW) integral of the density function, while ST is the strip integral of the same function. The different behaviour of these two integrals is a further evidence for their inequivalence^[10].

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