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Quark Binding Potential and Debye Screening Length *

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Abstract

The binding and dissolution for the $c\bar{c}$ and $b\bar{b}$ bound states are studied by using different quark binding potentials in a non-relativistic approximation. We calculate the critical value of the screening length by considering the Debye screening effect. The critical value r_D^c of the Debye screening length is highly sensitive to the form of the quark binding potential. We also estimate the critical temperature T_c of medium. The influence of the form of the quark binding potentials on dissolution for $c\bar{c}$ and $b\bar{b}$ bound states is discussed and compared to the previous results.

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1. Introduction

It was theoretically predicted that the formation of a deconfining medium in high energy heavy ion collisions would lead to the suppression of the J/ψ production[1]. The NA38 collaboration has observed the J/ψ suppression in high energy nucleus-nucleus collisions[2] after the theoretical prediction. The J/ψ suppression as one of the quark-gluon plasma(QGP) signals in high energy heavy ion collisions has generated much theoretical and experimental interest. The goal of relativistic heavy ion collisions is to try to observe the properties of the hot and dense medium produced in collisions and the transition from the normal hadron matter to the QGP.

The confining force between a quark and an antiquark at temperature higher than the deconfinement temperature T_c becomes screened. The screening length $r_D(T)$ decreases rapidly with increasing temperature. When r_D becomes smaller than the size of bound state of heavy quarks, the binding of bound state of heavy quarks in the QGP will be broken. The bound states of heavy quarks, such as the J/ψ , are dominantly produced in the very stage of the whole collision process. In the QGP, quarks and gluons are deconfined and the string tension between a charm c and a charm antiquark $\bar{c}$ vanishes, the only interaction between the c and the $\bar{c}$ is the Coulomb-type color interaction. The color charge of the charm quark c will be screened by the quarks, antiquarks and gluons in the plasma. The Debye screening will weaken the interaction between c and $\bar{c}$ and the binding interaction between c and $\bar{c}$ would be modified. The final yield of the J/ψ resonance will be suppressed due to the Debye screening if a quark-gluon plasma is formed in high energy heavy ion collisions. So the J/ψ is good for testing the Debye screening in the QGP. Once the J/ψ is dissolved it is hardly created by the hadronization of plasma. Therefore, the suppression of the J/ψ production in high energy heavy ion collisions may be used as a signature for the presence of the quark-gluon plasma[1,3-8] that has attracted much attention[9-22].

The study of the dissolution of heavy quark resonances by the Debye screening in a thermal environment is helpful for the understanding of the J/ψ suppression

and the experimental observation in high energy heavy ion collisions. How can one describe the interaction between a quark q and an antiquark $\bar{q}$ at finite temperature T ? We try to describe a $q\bar{q}$ system at finite temperature by a new type of the binding force.

To study the deconfinement of bound states of heavy quarks the temperature-dependent potential has been suggested[23] and the critical value of the screening length r_D for each bound state of the heavy quarks is calculated. In order to clarify whether the calculated critical value of r_D depends on the parametrized form of the potential, in this paper, we use different quark binding potentials at $T=0$ to study the binding of charmonium($c\bar{c}$) and bottomium($b\bar{b}$) and suggest another form of a temperature-dependent quark binding potential to study the deconfinement of bound states of the heavy quark system in a non-relativistic approximation according to the procedure given by ref.[23].

This paper is organized as follows. In Sec. 2 we use different quark binding potentials to calculate the mass spectra and r.m.s. radii of $c\bar{c}$ and $b\bar{b}$ bound states and propose another temperature-dependent quark binding potential for studying the dissolution of the heavy quark resonances. We calculate the critical value of r_D and compare to the previous results given by ref.[23]. In Sec. 3 we estimate the critical temperature T_c of plasma. In final section, we make discussions and conclusions.

2. Quark binding potential and Debye screening length

Due to the fact that no body knows what the right binding potential between a quark q and an antiquark $\bar{q}$ is, a lot of the potential models have been proposed for describing the $q\bar{q}$ system. We want to study the dissolution of charmonium($c\bar{c}$) and bottomium($b\bar{b}$) in a non-relativistic potential model. The Hamiltonian for a $q\bar{q}$ system can be described by

$$H(r, T) = \frac{\vec{p}^2}{2m_{re}} + V_{q\bar{q}}(r, T), \quad (1)$$

where m_{re} is the reduced mass of the $q\bar{q}$ system, p is the relative momentum, $V_{q\bar{q}}(r, T)$ is the binding potential between a quark q and an antiquark $\bar{q}$, r is the relative coordinate of a quark q and an antiquark $\bar{q}$, T is the temperature of thermodynamic environment. To get the static properties of the $q\bar{q}$ system the binding potential between a quark q and an antiquark $\bar{q}$ at $T=0$ has been proposed, which is given in a different form [24-26]:

$$V_{q\bar{q}}(r, 0) = \sigma r^\beta - \frac{\alpha}{r^\beta} + V_0, \quad (2)$$

where σ is the string tension coefficient, α is the coupling constant, V_0 is the additive constant which is used to fit the mass spectra of a heavy quark system, and β is the parameter of the potential type, the different β in eq.(2) denotes the different quark binding potential; $\beta = 1$ stands for the Cornell form[24], $\beta = 1/2$ is the Song-Lin form[25], $\beta = 3/4$ is the Turin potential[26].

In a thermodynamic environment of quarks and gluons the binding potential between a quark q and an antiquark $\bar{q}$ will be modified due to the Debye screening effects. So far little is known what the right form of the temperature-dependent quark binding potential is ? The appearance of the temperature-dependent potential model is natural and expected. The temperature-dependent quark binding potential has been parametrized in the form [23] (thereafter we call model I)

$$V_{q\bar{q}}^I(r, \mu(T)) = \frac{\sigma}{\mu(T)^\beta} (1 - e^{-(\mu(T)r)^\beta}) - \frac{\alpha}{r^\beta} e^{-(\mu(T)r)^\beta} + V_0, \quad (3)$$

Here $\mu(T)$ is the Debye screening mass, $r_D(T)=1/\mu(T)$ is the Debye screening length. The β is introduced in eq.(3) due to we want to know the effect of screening

form on dissolution for bound states of heavy quarks. As well known, the Debye screened potential has been approximated in a form[1]

$$V_{sc}(r, \mu) = -\frac{\alpha}{r} e^{-\mu(T)r} \quad (4)$$

The second term in eq.(3) will recover to eq.(4) if putting $\beta = 1$. We also want to know whether the dissolution for each bound state of a heavy quark system depends on the form of the temperature-dependent quark binding potential, so we here present another parametrized form to describe the quark binding potential at finite temperature T (thereafter we call model (I))

$$V_{II}^H(r, \mu(T)) = \frac{2\alpha}{\mu(T)^\beta} \left(1 - \frac{(\mu(T)r)^\beta}{e^{(\mu(T)r)^\beta} - 1}\right) - \frac{\alpha}{r^\beta} e^{-(\mu(T)r)^\beta} + V_0. \quad (5)$$

In the absence of the color screening, $\mu(T) = 0$ and $r_D(T) = \infty$ at $T=0$, thus eq.(3) and (5) recover to the eq.(2).

The numerical solution of the Schrödinger equation for the potential (3) and (5) leads to the temperature-dependent binding energies which vanish at a critical temperature value T_c .

We want to consider the numerical solution of bound states of $c\bar{c}$ and $b\bar{b}$ quark-antiquark pairs. We here follow ref.[23] and solve the Schrödinger equation

$$H(r, \mu(T))\psi_{nI}(r, \mu(T)) = \epsilon_{nI}(\mu(T))\psi_{nI}(r, \mu(T)). \quad (6)$$

The mass $M_{nI}(\mu)$ of the each bound state is related to the energy eigenvalue

$$M_{nI}(\mu) = m_q + m_{\bar{q}} + \epsilon_{nI}(\mu). \quad (7)$$

The interesting quantity is the energy needed for the dissolution of bound states of heavy quarks at the critical temperature T_c . The dissolution energy of the bound state at a fixed μ is defined as

$$E_{nI}^{nl}(\mu) = m_q + m_{\bar{q}} + V_{q\bar{q}}^i(r \rightarrow \infty, \mu) - \epsilon_{nI}(\mu), \quad i = I, II. \quad (8)$$

Question is how one can find what value of r_D at which there are no bound states. We follow the procedure given by ref.[23] and find the critical value of μ

$$E_{nI}^{nl}(\mu_c) = 0. \quad (9)$$

The magnitude value of $r_D' = 1/\mu_c$ can be calculated after finding μ_c from eq.(9).

We want to compare the Debye screening lengths calculated by the two kinds of the temperature-dependent binding potentials for each bound state of a heavy quark system and to know whether they depend on the form of the potential. The parameters in our calculations for the different quark binding potentials are given in table 1.

The static properties of $c\bar{c}$ and $b\bar{b}$ bound states have been studied by the various quark potentials[24-27]. We first calculate the masses of bound states and the corresponding bound state radii at $T=0$ by using the different quark binding potentials. The results are given in table 2.

We calculate the dissolution energy $E_{nI}^{nl}(\mu)$ as a function of μ for $c\bar{c}$ and $b\bar{b}$ states by using the model II with $\beta = 1$. The results are shown in figs.1 and 2. In

order to compare to the model [23], the results of the model I are also shown in figs.1 and 2, respectively. The solid lines are our results in figs.1 and 2. In fig.1 the dash-dotted line is for the J/ψ , dotted line for the χ_c and the dashed line for the ψ' given by the model I. The only dotted line in fig.2 is given by the model I for the Υ resonance. The figs. 1 and 2 show that our results are shifted to large μ comparing to the corresponding results given by the model I.

We calculate the μ_c and r_D^c for charmonium ($c\bar{c}$) and bottonium ($b\bar{b}$) by using the model I, which are given in table 3. The table 3 shows that our calculations by $\beta = 1$ for the μ_c are consistent with that given by ref.[23], but the results by $\beta = 1/2$ and $3/4$ for the μ_c and r_D^c are very different from the results by $\beta = 1$, especially, the μ_c given by $\beta = 1/2$ is very large for each bound state of the heavy quarks. Our calculations show that the μ_c and r_D^c depend sensitively on the form of the quark binding potential. In order to compare to the model I, We calculate the same quantities by using the model II. The calculated μ_c and r_D^c for charmonium ($c\bar{c}$) and bottonium ($b\bar{b}$) are given in table 4 by using the model II. The table 4 shows that the calculated μ_c is larger than the result given by the model I for corresponding bound state by the same β , the r_D^c is smaller than that given by the model I for corresponding bound state by the same β .

The μ_c and r_D^c calculated by the model II with the different β for each bound state are also very different. The results in tables 3 and 4 show that the choice of the form of the quark binding potential leads a great effect on the μ_c and r_D^c . In calculating the magnitude of the μ_c and r_D^c by a non-relativistic approximation the form of the quark binding potential is very important. The accuracy of calculating the μ_c and r_D^c depends on the right choice of the quark binding potential.

3. The critical temperature T_c

We calculate the r_D^c and μ_c values for the charmonium $c\bar{c}$ and bottonium $b\bar{b}$ in a non-relativistic potential model. What plasma temperature T_c does a obtained

r_D^c correspond? The critical temperature T_c can be calculated from the relation between μ and T given by perturbative QCD or lattice gauge theory. The lowest order perturbation QCD gives [3,8,23,28]

$$m_E^2 = (1 + \frac{N_f}{6})g^2(T)T^2, \quad (10)$$

where $m_E = \mu/2$, N_f is the number of light quark flavors, $g^2(T)$ is the temperature-dependent running coupling constant which is given for $N_f = 0$ [23] as

$$g^2(T) = \frac{24\pi^2}{33\ln(19T/\Lambda_{\overline{MS}})}. \quad (11)$$

The Lattice calculation gives[28-29]

$$T_c/\Lambda_{\overline{MS}} = 1.79. \quad (12)$$

Inserting eq.(12) in eq. (11) we can obtain

$$G_c^I = g(T_c) = 1.427. \quad (13)$$

The strong coupling constant at T_c is also given by author[29]

$$\frac{g(T_c)^2}{4\pi} = 0.29. \quad (14)$$

Then we can obtain from eq.(14)

$$G_c^{II} = g(T_c) = 1.909. \quad (15)$$

The T_c and r_D values of each quarkonium (J/ψ , ψ' and Υ) are given in table 2. The estimated T_c values from the present work are larger than those from r_D and μ assumptions are given in table 3. Table 4 shows the r_D values from the present work for the each resonance in comparison with those given by the model 1. The reason is very simple: that the μ provided by our model is larger than that by the model 1 and the r_D provided by the model 1 is smaller than that by the model 2. Due to the temperature T is inversely proportional to the screening length r_D and the coupling constant $g(T)$, the critical temperature T_c is related to each J/ψ , ψ' and Υ resonances are highly sensitive to the r_D and $g(T)$. Though the r_D and T_c values for the each resonance the good agreement of the r_D and the correct adoption of the $g(r)$ value are important.

3. Discussion and conclusions

We have studied the quarkonium quark interaction and the Debye screening effect on dissociation of the bound state bound states by the two parameterized form of the temperature-dependent static binding potential. We calculate the critical values of the screening mass μ and the screening length r_D by the different quark binding potential. The calculated critical values of the screening mass μ and the screening length r_D depend on the form of the quark binding potential. So the study of the quark binding potential is still needed in the future.

Given a relation between a temperature T and the screening mass μ (or the screening length r_D) by perturbative QCD or lattice QCD, the temperature T_c of the medium at which the bound state disappears can be estimated. Due to the obtained screening length r_D depends on the form of the binding potential, though we only estimated the T_c values for the J/ψ , ψ' and Υ resonances, the results show that T_c value for each bound state depends on the form of the potential. The calculated $r_D(\psi')$ is larger than the $r_D(J/\psi)$ and the $r_D(J/\psi)$ is larger than the $r_D(\Upsilon)$ by the both models. The magnitude values of the r_D given by the two models for each bound state are dramatically different. Our results indicate obviously that the ψ'

first dissolve, then the J/ψ , and finally the Υ if the QGP is produced at T_c , which is consistent with the statement[22].

Due to some uncertainties our results for T_c are rough estimations we can do. The fine calculations for T_c from given μ_c or r_D' are needed by a good relation between T_c and μ_c (or r_D') that are useful for the understanding of the J/ψ suppression as one of the QGP signals and experimental observation in high energy nucleus-nucleus collisions, especially at RHIC.

References

- [1] T. Matsui and H. Satz, Phys. Lett. B178(1986), 416.
- [2] M. C. Abtru et al. (NA38 Collab.), Z. Phys. C38 (1988), 117; C. Baqlin, A. Bussiere, J. P. Guillaud et al. (NA38 Collab.), Phys. Lett. B220 (1988), 471; B255 (1991), 459; Nucl. Phys. A544 (1992), 209c.
- [3] T. Matsui, Z. Phys. C38 (1988), 245.
- [4] H. Satz, Nucl. Phys. A488 (1988), 511c; Nucl. Phys. A498 (1989), 495c.
- [5] T. Matsui, Nucl. Phys. A488 (1988), 535c.
- [6] P.V. Ruuskanen et al., Z. Phys. C37 (1988), 623.
- [7] F. Karsch et al., Phys. Lett. B212 (1988), 255.
- [8] F. Karsch et al., Z. Phys. C37 (1988), 627.
- [9] J. P. Blaizot et al., Phys. Lett. B199 (1987), 499.
- [10] R. Lietarva et al., Z. Phys. C54 (1992), 449.
- [11] S. Nagamiya, Nucl. Phys. A488 (1988), 3c.
- [12] G. Röpke et al., Phys. Lett. B202 (1988), 479.
- [13] F. Karsch et al., Phys. Lett. B212 (1988), 255.
- [14] M. C. Chu, Nucl. Phys. A498 (1989), 471c.
- [15] R. Lietawa, Z. Phys. C50 (1991), 107.
- [16] R.V. Gavai et al., Z. Phys. C49 (1991), 663.
- [17] B. Petersson, Nucl. Phys. A525 (1991), 237c.
- [18] F. Karsch, Nucl. Phys. A498 (1989), 489.
- [19] R. V. Gavai et al., Nucl. Phys. A525 (1991) 479c.

- [20] J.P. Blaizot, Nucl. Phys. A555 (1993), 73c.
- [21] C.Y. Wong, "Introduction to High-Energy Heavy Ion Collisions", World Scientific Publishing Co. Ltd, Singapore, 1994.
- [22] S. Nagamia, Nucl. Phys. A566 (1994), 287c
- [23] F. Karsch, M.T. Mehr and H. Satz, Z. Phys. C37 (1988), 617.
- [24] E. Eichten et al., Phys. Rev. D17 (1978), 3090; Phys. Rev D21(1980), 203.
- [25] X.T. Song and H. Lin, Z. Phys. C34 (1987), 223.
- [26] D.B. Lichtenberg et al., Z. Phys. C41 (1989), 615; D. B. Lichtenberg et al., Z. Phys. C46 (1990), 75.
- [27] G. Piller et al., Nucl Phys. A560 (1993), 437.
- [28] A. Ukawa, Nucl. Phys. A496 (1989), 227c.
- [29] F. Karsch, Z. Phys. C38 (1988), 147.

Figure Caption

Fig.1. Our model by $\beta = 1$ is compared to the model I [23] in dissociation energies for $c\bar{c}$ bound states. The solid lines are our results. The dash-dotted line is for the J/ψ , dotted line for the χ_c and the dashed line for the ψ' given by the model I.

Fig.2. Our model by $\beta = 1$ is compared to the model I [23] in dissociation energies for $b\bar{b}$ bound states. The solid lines are our results. The dotted line is given by the model I for the Υ resonance.

Table Caption

Table 1. The parameters for the various binding potentials

Table 2. Mass spectrum and r.m.s of $c\bar{c}$ and $b\bar{b}$ bound states at $T=0$ in a non-relativistic potential model

Table 3. calculated μ_c and r_D^c for charmonium ($c\bar{c}$) and bottonium ($b\bar{b}$) by using the model I

Table 4. calculated μ_c and r_D^c for charmonium ($c\bar{c}$) and bottonium ($b\bar{b}$) by using the model II

Table 5. The estimated deconfinement temperature T_c .

fig. 1

Table 1.

	β	$a(\text{GeV}^{1+\beta})$	$b(\text{GeV}^{1-\beta})$	$V_0(\text{GeV})$	$m_c(\text{GeV})$	$m_b(\text{GeV})$
Ref. [24]	1	0.192	0.471	0	1.32	4.746
Ref. [25]	1/2	0.877	0.375	-0.908	1.81	5.20
Ref. [26]	3/4	0.39	0.644	-0.78	1.79	5.18

Table 2.

n^2	State		$\beta = 1$	$\beta = 1/2$	$\beta = 3/4$
	J/ψ	$M(\text{GeV})$	3.070	3.065	3.110
1S	(3.097)	$\langle r^2 \rangle^{1/2} (\text{fm})$	0.446	0.374	0.402
	ψ'	$M(\text{GeV})$	3.698	3.668	3.678
2S	(3.686)	$\langle r^2 \rangle^{1/2} (\text{fm})$	0.863	0.796	0.875
	ψ''	$M(\text{GeV})$	4.016	4.036	4.068
3S	(4.040)	$\langle r^2 \rangle^{1/2} (\text{fm})$	1.343	1.189	1.324
	χ_c	$M(\text{GeV})$	3.500	3.506	3.514
1P	(3.500)	$\langle r^2 \rangle^{1/2} (\text{fm})$	0.686	0.616	0.607
	Υ	$M(\text{GeV})$	9.445	9.450	9.462
1S	(9.460)	$\langle r^2 \rangle^{1/2} (\text{fm})$	0.223	0.218	0.214
	Υ'	$M(\text{GeV})$	10.004	10.040	10.040
2S	(10.023)	$\langle r^2 \rangle^{1/2} (\text{fm})$	0.503	0.482	0.485
	Υ''	$M(\text{GeV})$	10.035	10.378	10.375
3S	(10.255)	$\langle r^2 \rangle^{1/2} (\text{fm})$	0.732	0.734	0.728
	χ_b	$M(\text{GeV})$	9.897	9.898	9.919
1P	(9.890)	$\langle r^2 \rangle^{1/2} (\text{fm})$	0.402	0.367	0.477

Table 2.

State		$\beta = 1$	$\beta = 3/2$	$\beta = 3/4$
π	$\mu_c(\text{GeV})$	0.70	2.50	1.20
(0.704)	$r_D^2(\text{fm})$	0.282	0.0783	0.164
ρ	$\mu_c(\text{GeV})$	0.36	1.50	0.48
(0.364)	$r_D^2(\text{fm})$	0.548	0.132	0.340
ρ'	$\mu_c(\text{GeV})$	0.32	1.00	0.40
(0.323)	$r_D^2(\text{fm})$	0.617	0.197	0.493
τ_0	$\mu_c(\text{GeV})$	0.54	1.30	0.55
(0.530)	$r_D^2(\text{fm})$	0.580	0.182	0.359
τ	$\mu_c(\text{GeV})$	1.55	5.00	2.35
(0.460)	$r_D^2(\text{fm})$	0.127	0.8395	0.0847
τ'	$\mu_c(\text{GeV})$	0.56	2.60	1.00
(0.623)	$r_D^2(\text{fm})$	0.290	0.0759	0.197
τ''	$\mu_c(\text{GeV})$	0.42	1.70	0.650
(0.350)	$r_D^2(\text{fm})$	0.470	0.115	0.304
λ_1	$\mu_c(\text{GeV})$	0.57	2.30	0.90
(0.490)	$r_D^2(\text{fm})$	0.346	0.0868	0.219

Table 4.

States		$\beta = 1$	$\beta = 3/2$	$\beta = 3/4$
J/ψ	$\mu_c(\text{GeV})$	1.022	5.75	1.93
(3.097)	$r_D^c(\text{fm})$	0.193	0.0343	0.102
ψ'	$\mu_c(\text{GeV})$	0.565	3.05	1.025
(3.686)	$r_D^c(\text{fm})$	0.349	0.0647	0.193
ψ''	$\mu_c(\text{GeV})$	0.425	2.15	0.725
(4.040)	$r_D^c(\text{fm})$	0.464	0.0918	0.272
χ_c	$\mu_c(\text{GeV})$	0.580	2.85	0.985
(3.500)	$r_D^c(\text{fm})$	0.340	0.0692	0.200
Υ	$\mu_c(\text{GeV})$	1.95	9.25	3.20
(9.460)	$r_D^c(\text{fm})$	0.101	0.0213	0.0617
Υ'	$\mu_c(\text{GeV})$	0.95	4.75	1.625
(10.023)	$r_D^c(\text{fm})$	0.208	0.0415	0.121
Υ''	$\mu_c(\text{GeV})$	0.65	3.50	1.125
(10.355)	$r_D^c(\text{fm})$	0.304	0.0564	0.175
χ_b	$\mu_c(\text{GeV})$	0.925	4.35	1.525
(9.890)	$r_D^c(\text{fm})$	0.213	0.0454	0.129

Table 5.

	Model I			Model II		
	$\beta = 1$	$T_c(\text{MeV})$		$\beta = 1$	$T_c(\text{MeV})$	
State	$r_D^c(\text{fm})$	G_c^I	G_c^{II}	$r_D^c(\text{fm})$	G_c^I	G_c^{II}
$J/\psi(3.097)$	0.282	245	183	0.193	358	268
$\psi'(3.686)$	0.548	126	94	0.349	198	148
$\Upsilon(9.460)$	0.127	544	407	0.101	685	512