Chaotic Inflation based on an Abelian D-flat Direction

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Abstract
We study the inflation due to the D-flat direction of an extra $U(1)$. This scenario is a hybrid of a right-handed sneutrino inflaton scenario and a gauge non-singlet inflaton scenario. The inflaton is a gauge non-singlet field which induces a right-handed neutrino mass spontaneously through an extra $U(1)$ D-flat direction. This right-handed neutrino mass can explain the solar neutrino problem. The reheating temperature resulting from the decay of the coherent oscillation of the right-handed sneutrino is sufficiently high so that the baryogenesis based on the lepton number asymmetry can be applicable. We also discuss the realistic model building.

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The inflation is now believed as a basic idea to resolve various cosmological problems such as the flatness problem, the horizon problem and so on[1, 2]. It also seems to be supported by the observation of an anisotropy of the cosmic microwave background by the Cosmic Background Explorer(COBE) satellite[3]. However, as is often referred, no satisfactory candidates of an inflaton are not known within particle physics models still now. They are usually introduced only for the inflation without any substantial motivations from particle physics. And they are considered to be very weakly interacting to have a flat potential. This brings various difficulties from the view points of particle physics.

The basic features required for the inflaton may be summarized as :
(i) to give a sufficiently long exponential expansion period for the explanation of the flatness and the horizon problem and also produce the sufficient entropy,
(ii) to bring a suitable reheating of the universe to make the certain baryogenesis scenario applicable after the inflation,
(iii) to explain the microwave background anisotropy \((\delta T/T)_Q \sim 6 \times 10^{-6}\), which has been recently detected by the COBE.

Recently some candidates of the inflaton have been proposed on the basis of the motivations coming from particle physics. Here we want to quote two interesting candidates. One is the right-handed sneutrino scenario proposed by Murayama et al.[4]. This is mainly motivated by the supersymmetry and also the explanation of the small neutrino mass, which can resolve the solar neutrino problem. The other ones are gauge non-singlet fields which have a D-flat direction of an extra \(U(1)\)[5]. Such fields often appear in the models inspired by superstring, which is now considered as the most promising unified theory including gravity. Although these have sufficient motivations and are also interesting enough, there seem to remain some unsatisfactory features. The right-handed sneutrino scenario satisfies the above three conditions. In the model of ref.[4], however, the bare mass whose origin is not known is introduced for the right-handed neutrino by hand. This seems to be unsatisfactory from the view point of particle physics, where all masses of particles are usually considered to be generated spontaneously. On the other hand, the gauge non-singlet inflation scenario can not satisfy the condition (ii) unless we introduce a special kind of baryon number violating interactions. Although these usually exist in superstring-inspired models, they are dangerous for the phenomena such as the proton
decay. This is caused by the fact that the gauge non-singlet inflaton is too light to produce sufficiently high reheating temperature.

In this brief report we propose an inflation scenario in the supersymmetric model. It looks like a hybrid of these two scenarios and also resolves their faults. We study it in detail and also discuss the realistic model building. In order to make our scenario definite, firstly we want to make some basic assumptions and explain the features of our model. We will return to these assumptions later in relation to the model building.

A part of the superpotential relevant to the inflation is assumed to be [6, 7]

$$ W = \sum_{i,j} h^{ij} L_i H N_j + \sum_i \frac{a_0}{M_{pl}} \tilde{N}^2 N_i^2 + \frac{a_n}{M_{pl}^{2n+3}} (\tilde{N} \tilde{N})^n, $$

(1)

where $M_{pl} \sim 1.2 \times 10^{19}$ GeV is the Planck mass and $a_0$ and $a_n$ are constants, which will be determined in the following analysis. An integer $n$ is assumed to satisfy $n \geq 3$. The chiral superfields $N_i$ and $\tilde{N}$ have the similar quantum number as a right-handed neutrino except for having a non-trivial extra $U(1)$ charge $y_N$. Because of its Yukawa coupling $h^{ij} L_i H N_j$ with the lepton doublets $L_i$, $N_j$ is regarded as a right-handed neutrino superfield. The suffices $i$ and $j$ represent the generation. And also $\tilde{N}$ is a chiral superfield which has an opposite charge of the extra $U(1)$ against $N^*$.

The structure of non-renormalizable parts of the superpotential is similar to that of ref.[5]. It is also assumed that there are no other higher order terms for $N_i$, $\tilde{N}$ and $\tilde{N}$ in eq.(1). This is a very non-trivial assumption and we shall come back to this point later.

The scalar potential for these fields is derived from eq.(1) as

$$ V = \sum_i \frac{4a_0^2}{M_{pl}^2} |\tilde{N}^*|^2 |\tilde{N}|^2 + \left( \frac{na_n}{M_{pl}^{2n+3}} \right)^2 |\tilde{N}^*|^{2n} |\tilde{N}|^{2n} + \left| \sum_i \frac{2a_0}{M_{pl}^2} \tilde{N} \tilde{N}^* + \frac{na_n}{M_{pl}^{2n+3}} \tilde{N}^n \tilde{N}^{n-1} \right|^2 $$

$$ + \sum_i m_N^2 |\tilde{N}|^2 - m_N^2 (|\tilde{N}|^2 + |\tilde{N}^*|^2) $$

$$ + \frac{1}{2} g^2 y_N^2 \sum_i |\tilde{N}|^2 + |\tilde{N}^*|^2 - (|\tilde{N}|^2)^2, $$

(2)

where $\tilde{N}_i$, $\tilde{N}$ and $\tilde{N}^*$ represent scalar components of each superfield, respectively. The scalar masses in the second line are assumed to appear as a result of the supersymmetry breaking. The last line represents the D-term contribution of an extra $U(1)$. As easily seen from eq.(2), this scalar potential is D-flat along the direction of $\sum_i |\tilde{N}^*|^2 + |\tilde{N}|^2 = |\tilde{N}|^2$.

\footnote{For simplicity, we consider a model including only one pair of $(\tilde{N}, \tilde{N}^*)$. We also do not consider the generation mixing of $N_j$ in the second term of eq.(1).}
The right-handed neutrino $\tilde{N}_i$ can have the supersymmetric large mass because of this D-flatness. In fact, this scalar potential has a nontrivial minimum at

$$|\tilde{N}_i| = 0, \quad |\tilde{N}| = |\tilde{N}'| = \left[ \frac{m_N^2 M_{pl}^{2(2n-3)}}{(2n-1)n^2a_n^2} \right]^{\frac{1}{2n-2}}. \quad (3)$$

Because of these large vacuum expectation values of $\tilde{N}$ and $\tilde{N}'$, the lepton number violation occurs\(^2\) and the mass of a right-handed neutrino is produced spontaneously through the second term of $W$ as\(^6\)

$$M_N = a_0 \left[ \frac{1}{(2n-1)n^2a_n^2} \right]^{\frac{1}{2n-2}} (M_{pl}^{-2} m_N)^{\frac{1}{2n-2}}. \quad (4)$$

This relation constrains the coefficients $a_0$ and $a_n$ as

$$a_0 a_n^{-\frac{1}{2n-2}} = \left[ (2n-1)n^2 \right]^{\frac{1}{2n-2}} \frac{M_N}{(M_{pl}^{-2} m_N)^{\frac{1}{2n-2}}}. \quad (5)$$

If we require $M_N \sim 10^{11}$ GeV, the neutrino can get the Majorana mass about $\sim 10^{-3}$ eV via the seesaw mechanism for a Dirac mass $\sim 1$ GeV[8]. This is suitable for the MSW solution of the solar neutrino problem[9]. Taking $n = 3$ and the soft scalar mass as $m_N \sim 100$ GeV, from eq.(5) we get

$$a_0 \sim 7.5 a_3^{1/2}. \quad (6)$$

If $\tilde{N}$ and $\tilde{N}'$ quickly damp to the value in eq.(3) along the D-flat direction, our model is expected to reduce to that of ref.[4] after their damping.

Now we consider the chaotic inflation of this model. At the Planck epoch there is not enough time to realize the thermal equilibrium and then every field is expected to be out of equilibrium. Some of them can have larger values than $M_{pl}$ under the condition that each term in the Lagrangian is $\lesssim O(M_{pl}^4)$[10]. In the present model, if the coefficients $a_0$ and $a_n$ are extremely small, $\tilde{N}_i$ and $\tilde{N}'$ can have the large field values larger than $O(M_{pl})$. This is because these have a D-flat direction and a renormalizable interaction of $\tilde{N}_i$ in the superpotential is only a Yukawa coupling $h^{ij} L_i H N_j$, whose effect is sufficiently small as discussed in ref.[4]. Among the scalar components $\tilde{N}_i$, $\tilde{N}_1$ which has the smallest Yukawa coupling can have the largest field value. Then we only consider its time evolution among

\(^2\)There appears no massless Majoron associated to this lepton number violation because it is eaten by the extra $U(1)$ gauge boson.
\( \tilde{N}_i \) hereafter. In the following study we confine ourselves on the \( n = 3 \) case. And \( \tilde{N}_1, \tilde{N}_i, \tilde{N}_j \) and \( \tilde{N}_k \) are assumed to evolve along the D-flat direction \(|\tilde{N}_1|^2 + |\tilde{N}_i|^2 = |\tilde{N}_j|^2|\).

For the successful inflation scenario our model should satisfy the above mentioned three conditions (i)\(~\) (iii). We also need to require the condition (6) to make the MSW mechanism applicable to the solar neutrino problem. These requirements constrain parameters \( a_0 \) and \( a_3 \) in the superpotential. If we put \(|\tilde{N}_1| = u\) and \(|\tilde{N}_i| = v\), the scalar potential along the D-flat direction becomes

\[
V = V_{a_0} + V_{a_3} + m_N^2 u^2 - m_X^2 (u^2 + 2v^2),
\]

\[
V_{a_0} = 4 \frac{a_0^2}{M^2_{pl}} (2u^6 + 3u^4v^2 + u^2v^4),
\]

\[
V_{a_3} = 9 \frac{a_3^2}{M^2_{pl}} v^4 (u^2 + v^2)^2 (u^2 + 2v^2) + 12 \frac{a_0 a_3}{M^2_{pl}} u^2 v^3 (u^2 + v^2)^2.
\]

As noticed above, the coefficient \( a_0 \) and \( a_3 \) are required to be extremely small in the successful chaotic inflation. And we find from eq.(6) that \( a_0 \gg a_3 \) should be satisfied. Thus \( V_{a_0} \) is expected to be a dominant part of the scalar potential \( V \) during inflation era. In the followings we shall determine \( a_0 \) and \( a_3 \) under such a situation.

At first we consider the condition (i). The time evolution equations of the field \( \phi \) are

\[
\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi \dot{\phi} = - \frac{\partial V}{\partial \phi} \quad (\phi = u, v),
\]

where \( H \) is the Hubble parameter which is now approximated as \( H = \sqrt{\frac{8\pi V_{a_0}}{3M^4_{pl}}} \). During this period the effective masses \( M^2_\phi \equiv (\frac{\partial^2 V}{\partial \phi^2})/\phi \) are estimated as

\[
M^2_u \sim \frac{8a_0^2}{M^2_{pl}} (6u^4 + 6u^2v^2 + v^4), \quad M^2_v \sim \frac{8a_0^2}{M^2_{pl}} (3u^4 + 2u^2v^2).
\]

From the form of \( V_{a_0} \) we can take \( u \gg v > M_{pl} \) as the initial values for these fields. When \( u \gg v > M_{pl} \), both \( u \) and \( v \) slowly damp. Once \( H \sim M_u \) is realized, \( u \) critically damps with a small e-folding and begins to oscillate with a sufficiently large amplitude. At this time \( u \) becomes smaller than \( M_{pl} \) but \( v \) still remains larger than \( M_{pl} \). This results in \( M_v < H < M_u \) and causes the above phenomenon. After \( u \) starting the oscillation, \( v \) continues the slow rolling.\(^3\) When \( H \sim M_v \), \( v \) begins to oscillate and damps to its vacuum

\(^3\)The condition \( a_0 \gg a_3 \) coming from the right-handed neutrino mass makes \( \tilde{N} \) inflaton generally. In the present scenario a right-handed neutrino \( \tilde{N}_i \) seems to be difficult to play a role of inflaton.
value rapidly.\footnote{This comes from the features of $H$ and $M_{\phi}$. It is recently used to solve the cosmological moduli problem\cite{11}.} Through this era, $u$ continues oscillating without any effective damping.\footnote{These pictures have been checked by the numerical calculation.}

After $v$ conversing its vacuum value, our model reduces to the one of ref.[4].

The validity of the slow-roll approximation for $v$ after $u$ starting the oscillation is justified under the conditions\cite{12}

$$\epsilon(v) = \left(\frac{M_{pl}^2}{16\pi}\right) \left(\frac{V_{a0}'(v)}{V_{a0}(v)}\right)^2 < 1, \quad \eta(v) = \frac{M_{pl}^2 V_{a0}''(v)}{8\pi V_{a0}(v)} < 1. \tag{10}$$

where $V_{a0}(v) \sim \frac{4\pi^2}{M_{pl}^2} u_{av}^2 v^4$ with the averaged value of $u^2$.\footnote{To estimate the averaged value of $u^2$ we used the numerical analysis in the self-consistent way. In the following arguments we use that result $u_{av} \sim 10^{-2} M_{pl}$.}

These parameters can be estimated as

$$\epsilon(v) \sim \frac{2}{3} \eta(v), \quad \eta(v) \sim \frac{3}{2\pi} \left(\frac{M_{pl}}{v}\right)^2. \tag{11}$$

Therefore, the slow-roll approximation is valid as far as $v_{end} \gtrsim 0.7 M_{pl}$.

Eq.(8) is reduced to $3H\dot{v} = -V_{a0}'$ in a region where the above slow roll condition is satisfied. Thus the $e$-folding of the expansion between $v_{in}$ and $v_{end}$ can be expressed as

$$N(v_{in}, v_{end}) = -\int_{v_{in}}^{v_{end}} \frac{8\pi}{M_{pl}^2 V_{a0}} dv \sim \frac{\pi}{M_{pl}^2} (v_{in}^2 - v_{end}^2). \tag{12}$$

The condition for the sufficient inflation is given as $N(v_{in}, v_{end}) \gtrsim 60$. This requires $v_{in} \gtrsim 4.4 M_{pl}$ when $v_{end} \gtrsim 0.7 M_{pl}$.

Next we impose the condition (iii). Following the usual argument on the scalar density perturbation, the microwave background quadrupole anisotropy $(\delta T/T)_Q$ is expressed as

$$\left|\left(\frac{\delta T}{T}\right)_Q\right| \sim \sqrt{\frac{32\pi}{45}} \left(\frac{V_{a0}^3}{4 V_{a0}'^2 M_{pl}^5}\right)_{k \sim H} = \frac{2}{3} \sqrt{\frac{2\pi}{5}} \frac{a_0 u_{av}}{M_{pl}} \left(\frac{v}{M_{pl}}\right)^3 \sim 0.13 \left(\frac{a_0 u_{av}}{M_{pl}}\right) N_H^2 \tag{13}$$

where $N_H \equiv \pi (\frac{v}{M_{pl}})^2 k_H$. This value is estimated when the scale $k^{-1}$, which corresponds to the present horizon size, crossed inside the horizon during inflation\cite{2}. If we put $N_H \sim 50$ to realize $(\delta T/T)_Q \sim 6 \times 10^{-6}$, we find that we should take $a_0 \sim 1.3 \times 10^{-5}$. Assuming this value for $a_0$, the requirement eq.(6) from the right-handed neutrino mass determines $a_3$ as $a_3 = 2.9 \times 10^{-12}$. For these values the above estimated condition of the sufficient inflation is trivially satisfied. From eq.(3), for these parameters the true minimum is realized at $[\tilde{V}] = [\tilde{V}] \sim 10^{-1.6} M_{pl}$. As explained above, $v$ quickly damps to its vacuum values after
the end of inflation. When \( v \) reaches to its true vacuum, \( u \) still continues oscillating with the sufficiently large amplitude of order \( 10^{-2} M_{pl} \) and dominates the energy density. This coherent oscillation of \( u \) decays to the light particles through the Yukawa couplings satisfying the D-flat condition. Thus our model is expected to be equivalent to one of ref.[4] after the inflation.

The reheating temperature \( T_{RH} \sim 0.1 \sqrt{M_{pl} \Gamma_{N_1}} \) is crucial to consider what kind of baryogenesis scenario can work. \( \Gamma_{N_1} \) is the decay ratio of the field \( \tilde{N}_1 \). In the present model the reheating is expected to occur due to the decay of the oscillation of \( \tilde{N}_1 \) to the light particles as ref.[4]. In this process, if CP violation exists, the lepton number asymmetry is produced. This asymmetry will be converted to the baryon number asymmetry through the electroweak anomalous process if the reheating temperature is high enough. As seen in the previous part, \( \tilde{N} \) reaches its true vacuum soon after the end of the inflation because of its critical damping. Therefore \( \tilde{N} \) is irrelevant to the production of the lepton number asymmetry, which occurs at the later stage. The decay of \( \tilde{N}_1 \) is mediated by the Yukawa coupling \( h^{i1} L_1 H \tilde{N}_1 \) and the decay ratio is estimated as \( \Gamma_{N_1} \sim \frac{|h^{i1}|^2}{4\pi} M_N \). Here \( h^{i1} \) is the largest Yukawa coupling of \( \tilde{N}_1 \) to the charged lepton and we take it as \( h^{i1} \sim 10^{-4} \) to \( 10^{-5} \). The right-handed neutrino mass \( M_N \) is \( \sim 10^{11} \) GeV so that the reheating temperature can be estimated as \( T_{RH} \sim 10^8 \) GeV to \( 10^9 \) GeV. This reheating temperature seems to be suitable for the baryogenesis based on the lepton number asymmetry[13]. In fact, following the detailed discussion of ref.[4] the expected maximum baryon number to entropy ratio \( Y_B(\equiv n_B/s) \) is estimated as[14]

\[
Y_B = \frac{8n_g + 4}{22n_g + 13} Y_{L_{max}}^{max}
\]

by using the generated maximum lepton number to entropy ratio \( Y_{L_{max}}^{max} \) and the generation number \( n_g = 3 \). If we take \( \epsilon \) as the asymmetry in the decay of \( \tilde{N}_1 \) into leptons and antileptons, \( Y_{L_{max}}^{max} \) is expressed as \( Y_{L_{max}}^{max} \sim \frac{2 \pi}{4 M_{pl}^2} \). In order to realize the correct baryon number asymmetry \( Y_B \sim 10^{-10} \), we need \( \epsilon \sim 10^{-7} \) to \( 10^{-8} \) for the above mentioned reheating temperature. This value of \( \epsilon \) is somehow smaller than the one of ref.[4] mainly coming from our smaller setting of \( M_N \). To make \( \epsilon \) larger we can consider the dilution effect discussed in ref.[4].

Here we summarize the features of our model again. In the present model the field \( \tilde{N} \) plays the role of inflaton due to its slow-roll property as ref.[5] and produces the right-handed neutrino mass spontaneously through its abelian D-flat direction. The reheating
occurs by the decay of $\bar{N}_1$ with the large mass, which can explain the solar neutrino problem. We can get the sufficiently high temperature which is convenient for the baryogenesis. In the present model the gravitino mass is considered as $O(1)$ TeV as usual and the reheating temperature is $T_{RH} \sim 10^8$ GeV to $10^9$ GeV. These values make our model free from the gravitino problem[15].

Finally we discuss the construction of particle physics models which have the features discussed in this brief report. It is very interesting that our model seems to be naturally embedded in the certain class of superstring-inspired models. There often appear the extra $U(1)$ symmetries accompanied with the singlet fields as $N_i$, $\mathcal{N}$ and $\bar{\mathcal{N}}$. Such a concrete model is discussed in ref.[7]. In the above arguments we require the particular type of the superpotential $W$ and also the extremely small coefficients $a_0$ and $a_3$. As is well-known, in the superstring theory there are a lot of discrete symmetries which can constrain the form of the superpotential $W$. We can expect that it may at least explain the non-existence of the lower order terms in the non-renormalizable ones in $W$ due to such discrete symmetries[16]. Moreover, it is shown that in a certain type of superstring the non-renormalizable terms are produced only through the non-perturbative world sheet instanton effects. There appears an exponentially small suppression factor $\exp(-c/g^2)$, where $c > 0$ and $g$ is a world sheet coupling constant[17]. This may explain the reason of the smallness of coefficients $a_0$ and $a_3$ as suggested in ref.[5]. These facts are favorable for the present scenario to be realized in realistic particle physics models. However, there remains a problem on supergravity corrections to the scalar potential. This problem has been seriously discussed in the inflation scenario within the supergravity framework[18]. In the present stage we can not say anything about this problem and also why the higher order non-renormalizable terms do not appear in $W$. A special form of the Kähler potential derived from superstring and stringy symmetry may be relevant to such problems as suggested in ref.[19]. Anyway, this is a crucial problem when we consider the inflation within the superstring framework.

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