Kaon production cross sections from baryon-baryon interactions

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Abstract

In a one-pion plus one-kaon exchange model, we calculate the kaon production cross sections in nucleon-nucleon, nucleon-delta and delta-delta interactions. We find that this model describes reasonably well the experimental data on kaon production in the proton-proton interaction. Near the kaon production threshold, the cross section obtained from this model is smaller than that from the linear parameterization of Randrup and Ko. For kaon production cross sections from the nucleon-delta and delta-delta interactions, the cross sections are singular in free space, so we calculate them in a nuclear medium by including the (complex) pion self-energy. The results are compared with the scaling ansatz of Randrup and Ko. The theoretical cross sections are then used in a transport model to study kaon production from Au+Au collisions at 1 GeV/nucleon.
I. INTRODUCTION

Kaon production in heavy-ion collisions has been extensively studied both experimentally [1–4] and theoretically [5–15,?,17–21] at various incident energies. Its interest stems from the fact that because of strangeness conservation, kaons will not be absorbed in nuclear medium once they are produced in the collisions. They are thus expected to carry useful information about the early stage of heavy-ion collisions. Indeed, at incident energies below the kaon production threshold in the nucleon-nucleon interaction in free space (∼1.58 GeV), the kaon yield has been shown in transport models to be sensitive to both the nuclear equation of state at high densities [7,10,11,15–17] and the kaon scalar potential in nuclear medium [13]. The latter is related to the partial restoration of chiral symmetry [22,23]. From comparing theoretical results with experimental data measured in Au+Au collisions at 1 GeV/nucleon, it has been concluded that the attractive kaon scalar potential in dense nuclear matter is appreciable and the nuclear equation of state at high densities is soft.

For heavy ion collisions at incident energies of around 1 GeV/nucleon, the colliding system consists mainly of nucleons, deltas and pions. Kaons can be produced from baryon-baryon (nucleon-nucleon, nucleon-delta, and delta-delta) and pion-baryon (pion-nucleon and pion-delta) interactions. Kaon production cross sections in these elementary processes are thus needed in transport models in order to evaluate the kaon yield in heavy-ion collisions. The most straightforward way is to use experimentally measured cross sections. There exist some experimental data for kaon production cross sections [24] from the pion-proton and proton-proton interactions. For the proton-proton interaction, there are no data near the the kaon production threshold of about 1.58 GeV, which are the energies at which kaons are mostly produced in heavy ion collisions at around 1 GeV/nucleon. The extrapolation from the available experimental data at high energies to near the threshold thus depends on the parameterization used for the cross sections. There are two popular parameterizations of the experimental data; the linear parameterization of Randrup and Ko [5] and the quartic parameterization of Zwermann [9]. Although both fit the experimental data at the lowest
available incident energy, they differ considerably near the kaon production threshold.

Furthermore, the kaon production cross sections in nucleon-delta and delta-delta interactions are not available experimentally. In the work of Randrup and Ko [5], these cross sections are related to that from the nucleon-nucleon interaction mainly through arguments based on isospin symmetry. This leads to a simple scaling ansatz: 

\[ \sigma_{NN-NYK}(\sqrt{s}) = (4/3)\sigma_{\Delta-\Lambda_{NYK}}(\sqrt{s}) = 2\sigma_{\Delta-\Lambda_{NYK}}(\sqrt{s}) \]

No detailed theoretical study has been carried out to examine its accuracy. All existing calculations based on transport models have shown that deltas play a significant role in subthreshold kaon production due to their larger masses than the nucleon mass [7,10,11,13]. In these calculations, the scaling ansatz of Ref. [5] have usually been used. At higher incident energies in AGS experiments, deltas have also been found to be important for kaon production. In microscopic models such as the ARC [19] and the RQMD [20], the kaon production cross sections from nucleon-delta and delta-delta interactions are assumed, however, to be the same as that from the nucleon-nucleon interaction.

The above discussions thus point to the need for a theoretical model in evaluating the elementary kaon production cross sections in hadron-hadron interactions. In Ref. [25], a resonance model has been used to study kaon production from the pion-nucleon interaction and has been shown to reproduce rather well the available experimental data. The resonance model has also been extended to kaon production from the pion-delta interaction. For kaon production in the nucleon-nucleon interaction, a one-pion exchange model has been used in Ref. [26] by including also medium modifications of the pion propagation in the delta-hole model. The contribution of one-kaon exchange to kaon production in proton-proton interaction has been studied in Ref. [27], and has been found to be as important as the one-pion exchange contribution if off-shell effects are included. No detailed calculation has been carried out for kaon production from the nucleon-delta and delta-delta interactions.

In this paper, we shall calculate kaon production cross sections in nucleon-nucleon, nucleon-delta and delta-delta interactions based on a one-pion plus one-kaon exchange model. In Section 2, we present the formalism for these cross sections. The results are
presented in Sections 3 and compared with available experimental data and existing parameterizations. In particular, we discuss kaon production cross sections from the nucleon-delta and delta-delta interactions and compare them with the scaling ansatz of Ref. [5]. In section 4, we use the theoretical elementary kaon production cross sections in a transport model to study kaon production from Au+Au collisions at 1 GeV/nucleon. Finally, a summary is given in Section 5.

II. KAON PRODUCTION IN BARYON-BARYON INTERACTIONS: FORMALISM

In this section, we present the formalism for kaon production cross sections in nucleon-nucleon, nucleon-delta and delta-delta interactions. Only final states without a delta will be considered as the threshold energies for final states with a delta are much higher than the average energy of two interacting baryons in heavy ion collisions at 1 GeV/nucleon.

A. $NN \rightarrow NYK$

The relevant pion and kaon exchange diagrams are shown in Fig. 1, where $Y$, denoted by thick solid lines, represents either a $\Lambda$ or a $\Sigma$ hyperon. Let us first consider the case in which the kaon is produced in association with a $\Lambda$ hyperon. To fix the relative sign between the pion and kaon exchange amplitudes requires a well defined interacting Lagrangian. Since we shall treat the $\pi NYK$ and $KNNK$ interactions phenomenologically using their cross sections determined from the resonance model [25], their relative sign can not be fixed. Instead choosing the sign to maximize the cross sections as in Ref. [27], we shall simply neglect the interference between the two amplitudes and fit the experimental cross sections by adjusting other parameters in the model. The isospin-averaged cross section is then given by [26,28]

$$\sigma_{NN\rightarrow NAK}(\sqrt{s}) =$$
\[
\frac{3m_N^2}{2\pi^2p^2s} \int_{(w_\pi)_{\text{min}}}^{(w_\pi)_{\text{max}}} dw_\pi w_\pi^2 k_\pi \int_{(q_\pi^2)^-}^{(q_\pi^2)^+} dq_\pi^2 \frac{f_{\pi NN}^2}{m_N^2} F^4(q_\pi^2) q_\pi^2 |D_\pi(q_\pi^2)|^2 \sigma_{\pi N -*K}(w_\pi) \\
+ \frac{m_N m_A}{2\pi^2p^2s} \int_{(w_K)_{\text{min}}}^{(w_K)_{\text{max}}} dw_K w_K^2 k_K \int_{(q_K^2)^-}^{(q_K^2)^+} dq_K^2 \frac{f_{K\pi A}^2}{m_K^2} F^4(q_K^2) q_K^2 |D_K(q_K^2)|^2 \sigma_{K \pi N -*K}(w_K).
\]

(1)

In the above, masses of the nucleon, pion, kaon, and A hyperon are denoted by \(m_N\), \(m_\pi\), \(m_K\), and \(m_A\), respectively. \(f_{\pi NN}\) and \(f_{K\pi A}\) are the pseudovector \(\pi NN\) and \(K\pi A\) coupling constants. The four-momenta of two initial nucleons are denoted by \(p_1\) and \(p_2\), respectively. Their total energy in the center-of-mass system is denoted by \(\sqrt{s}\), while the magnitude of the three-momentum of each nucleon in the center-of-mass frame is \(p\). \(w_\pi\) denotes the total energy of the pion-nucleon system in the center-of-mass frame, with \((w_\pi)_{\text{min}} = m_K + m_A\) and \((w_\pi)_{\text{max}} = \sqrt{s} - m_N\). Similarly, \(w_K\) is the total energy of the kaon-nucleon system, with \((w_K)_{\text{min}} = m_K + m_N\) and \((w_K)_{\text{max}} = \sqrt{s} - m_A\). The magnitude of the three-momentum of the exchanged pion, \(k_\pi\), is related to \(w_\pi\) by

\[
k_\pi = \frac{1}{2w_\pi} \sqrt{[w_\pi^2 - (m_N + m_\pi)^2][w_\pi^2 - (m_N - m_\pi)^2]},
\]

(2)

and a similar expression for \(k_K\), with \(m_\pi\) and \(w_\pi\) in Eq. (2) replaced by \(m_K\) and \(w_K\), respectively.

In Eq. (1), the four-momentum of the exchanged pion is denoted by \(q_\pi\). In the center-of-mass frame, we have

\[
(q_\pi^2)^- = 2m_N^2 - 2(m_N^2 + p^2)^{1/2}(m_N^2 + p^2)^{1/2} + 2pp',
\]

\[
(q_\pi^2)^+ = 2m_N^2 - 2(m_N^2 + p^2)^{1/2}(m_N^2 + p^2)^{1/2} - 2pp',
\]

(3)

where \(p'\) is the magnitude of the three-momentum of the final nucleon and is related to \(\sqrt{s}\) and \(w_\pi\) by

\[
p' = \frac{1}{2} \sqrt{[s - (w_\pi + m_N)^2][s - (w_\pi - m_N)^2]/s}.
\]

(4)

Similarly, for \((q_K^2)_{\pm}\) we have

\[
(q_K^2)^- = m_N^2 + m_A^2 - 2(m_N^2 + p^2)^{1/2}(m_A^2 + p^2)^{1/2} + 2pp',
\]

\[
(q_K^2)^+ = m_N^2 + m_A^2 - 2(m_N^2 + p^2)^{1/2}(m_A^2 + p^2)^{1/2} - 2pp',
\]

(5)
with \( p' \) calculated from Eq. (4) by replacing \( m_N \) with \( m_\Lambda \).

The pion propagator \( D_\pi(q_\pi^2) \) and the kaon propagator \( D_K(q_K^2) \) in free space are given, respectively, by

\[
D_\pi(q_\pi^2) = \frac{1}{q_\pi^2 - m_\pi^2},
\]

and

\[
D_K(q_K^2) = \frac{1}{q_K^2 - m_K^2}.
\]

In Eq. (1), \( \bar{\sigma}_{\pi N \rightarrow \Lambda K} \) should be the isospin-averaged \( \pi N \rightarrow \Lambda K \) cross section for an off-shell pion, but we replace it by an on-shell one. It can then be obtained from the experimental data via

\[
\bar{\sigma}_{\pi N \rightarrow \Lambda K} = \frac{1}{2} \sigma_{\pi^- p \rightarrow \Lambda K^0}.
\]

It has been parameterized by Cugnon et al. [6] and used in Ref. [26]. It can also be calculated from theoretical models. We use here the results of Ref. [25] based on a resonance model which describes rather well the experimental data. To take into account the off-shell nature of the exchanged pion, we introduce as in Ref. [27] at the \( \pi NN \) vertex a pion form factor similar to the one used in the \( \pi NN \) vertex, i.e.,

\[
F(q_\pi^2) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q_\pi^4},
\]

with \( \Lambda_\pi \) being the cut-off parameter.

In the kaon exchange contribution, \( \bar{\sigma}_{KNN \rightarrow KNN} \) is the isospin-averaged kaon-nucleon scattering cross section, and can be determined from experiments by

\[
\bar{\sigma}_{KNN \rightarrow KNN} = \frac{1}{2}(\sigma_{K^+ p \rightarrow K^- p} + \sigma_{K^0 p \rightarrow K^0 p} + \sigma_{K^0 p \rightarrow K^+ n}),
\]

where we have made use of the following relations based on the isospin symmetry:

\[\sigma_{K^0 n \rightarrow K^0 n} = \sigma_{K^+ p \rightarrow K^- p}, \quad \sigma_{K^+ n \rightarrow K^+ n} = \sigma_{K^0 p \rightarrow K^0 p}, \quad \text{and} \quad \sigma_{K^0 n \rightarrow K^0 p} = \sigma_{K^0 p \rightarrow K^+ n} \].

These cross sections are taken from the phase shift analysis of Martin [29]. The off-shell effect is again included by the following kaon form factor at both vertices,

\[
F(q_K^2) = \frac{\Lambda_K^2 - m_K^2}{\Lambda_K^2 - q_K^4},
\]
with $\Lambda_K$ being the cut-off parameter.

For the reaction $pp \to p\Lambda K^+$, where experimental data exist, the cross section is similar to Eq. (1) except that the first term on the right hand side is reduced by a factor of 3 due to the isospin and $\bar{\sigma}_{KN-N\Lambda}$ in the second term replaced by $\bar{\sigma}_{K^+p-K^+p}$. If only the pion exchange is included, the isospin-averaged cross section is three times the $pp \to p\Lambda K^+$ cross section as in Refs. [5,26]. Since the isospin factor for the kaon exchange is one, the isospin-averaged cross section including both pion and kaon exchanges is thus less than three times the $pp \to p\Lambda K^+$ cross section.

Next, let us consider the reaction where a kaon is produced in association with a $\Sigma$ hyperon. Since the $\Sigma$ hyperon has three charge states, there are more final states than in the case for the $\Lambda$ hyperon. The isospin-averaged cross section is given by

$$\sigma_{N-N\Sigma\Lambda}(\sqrt{s}) =$$

$$\frac{3m_T^2}{2\pi^2p^2s} \int_{(w_\pi)_{\text{min}}}^{(w_\pi)_{\text{max}}} dw_\pi w_\pi^2 k_\pi \int_{(q_\pi^2)_{-}}^{(q_\pi^2)_{+}} dq_\pi^2 \frac{f_{\pi NN}^2}{m_\pi^2} F^4(q_\pi^2) q_\pi^2 |D_\pi(q_\pi^2)|^2 \bar{\sigma}_{N-N\Lambda}(w_\pi)$$

$$+ \frac{3m_{NN}^2}{2\pi^2p^2s} \int_{(w_K)_{\text{min}}}^{(w_K)_{\text{max}}} dw_K w_K^2 k_K \int_{(q_K^2)_{-}}^{(q_K^2)_{+}} dq_K^2 \frac{f_{K\Sigma N}^2}{m_K^2} F^4(q_K^2) q_K^2 |D_K(q_K^2)|^2 \bar{\sigma}_{N-N\Lambda}(w_K). \quad (6)$$

The notations are similar to those in Eq. (1). The mass of the $\Sigma$ hyperon and the $KN\Sigma$ coupling constant are denoted by $m_{\Sigma}$ and $f_{KN\Sigma}$, respectively. The isospin-averaged cross section for the $\pi N \to \Sigma K$ reaction is denoted by $\bar{\sigma}_{\pi N-\Sigma K}$ and is calculated from

$$\bar{\sigma}_{\pi N-\Sigma K} = \frac{1}{3} (\sigma_{\pi^+ p-\Sigma^+ K^+} + 2\sigma_{\pi^+ n-\Sigma^0 K^+} + \sigma_{\pi^- p-\Sigma^- K^+} + \sigma_{\pi^- n-\Sigma^- K^+}).$$

For the cross sections on the right hand side, we use again the results of Ref. [25] based on the resonance model which has been shown to fit the experimental data satisfactorily.

For $pp \to n\Sigma^+ K^+$, $pp \to p\Sigma^0 K^+$, and $pp \to p\Sigma^+ K^0$, there are experimental data available, and their cross sections can be obtained from Eq. (6) by replacing $3\bar{\sigma}_{N-N\Lambda}$ in the first term by $2\bar{\sigma}_{\pi^+ p-\Sigma^+ K^+}$, $\bar{\sigma}_{\pi^+ p-\Sigma^0 K^+}$, and $\bar{\sigma}_{\pi^- p-\Sigma^- K^+}$, respectively, and $3\bar{\sigma}_{KN-N\Lambda}$ in the second term by $2\bar{\sigma}_{K^0 p-\Sigma^+ K^+}$, $\bar{\sigma}_{K^0 p-\Sigma^0 K^+}$, and $2\bar{\sigma}_{K^0 p-\Sigma^- K^+}$, respectively.
B. $N\Delta \rightarrow NYK$

As mentioned in the Introduction, deltas play an important role in subthreshold kaon production from heavy-ion collisions. Unfortunately, there are no experimental data for kaon production cross sections from the nucleon-delta and delta-delta interactions. A theoretical model is therefore needed to determine these cross sections. In this and next subsections we extend the pion and kaon exchange model to calculate kaon production cross sections in these reactions.

For the $N\Delta \rightarrow NYK$ reaction, the relevant diagrams for pion and kaon exchanges are shown in Fig. 2. In each case, there are two different diagrams; one with a kaon produced from the meson-nucleon interaction and the other from the meson-delta interaction. For $N\Delta \rightarrow NAK$ reaction, diagram (d) does not exist as there is no $K\Delta\Lambda$ coupling due to isospin conservation. The isospin-averaged cross section for this reaction is

$$\sigma_{N\Delta-N\Lambda K}(\sqrt{s}) =$$

$$\frac{3m_N^2}{4\pi^2p^2s} \int_{(w_\pi)_{\text{min}}}^{(w_\pi)_{\text{max}}} dw_\pi w_\pi^- k'_\pi \int_{(q^2_\pi)_-}^{(q^2_\pi)_+} dq^2_\pi \frac{F^2_\pi NN}{m_\pi^2} F^4(q^2_\pi) q^2_\pi D_{\pi}(q^2_\pi) |\bar{\sigma}_{\pi\Delta-\Lambda K}(w_\pi)|^2$$

$$+ \frac{m_N m_\Delta}{4\pi^2p^2s} \int_{(w_\pi)_{\text{min}}}^{(w_\pi)_{\text{max}}} dw_\pi w_\pi^- k_- \int_{(q^2_\pi)_-}^{(q^2_\pi)_+} dq^2_\pi \frac{F^2_\pi N\Delta}{m_\pi^2} F^4(q^2_\pi) A(q^2_\pi) D_{\pi}(q^2_\pi) |\bar{\sigma}_{\pi\Delta-\Lambda K}(w_\pi)|^2$$

$$+ \frac{m_N m_\Delta}{4\pi^2p^2s} \int_{(w_K)_{\text{min}}}^{(w_K)_{\text{max}}} dw_K w_K^- k'_K \int_{(q^2_K)_-}^{(q^2_K)_+} dq^2_K \frac{F^2_{K\Lambda N}}{m_K^2} F^4(q^2_K) q^2_K D_{\Lambda}(q^2_K) |\bar{\sigma}_{K\Lambda\Delta-KN}(w_K)|^2.$$

The notations are again similar to those in Eq. (1). In the first term, corresponding to Fig. 2(a), $k'_\pi$ is calculated from Eq. (2) with $m_N$ replaced by the mass of the delta resonance, $m_\Delta$. $\bar{\sigma}_{\pi\Delta-\Lambda K}$ is the isospin-averaged cross section for the $\pi\Delta \rightarrow \Lambda K$ reaction which is determined from

$$\bar{\sigma}_{\pi\Delta-\Lambda K} = \frac{1}{3} \sigma_{\pi-\Delta++-\Lambda K^+},$$

with $\sigma_{\pi-\Delta++-\Lambda K^+}$ taken from Ref. [25].

In the second term, corresponding to Fig. 2(b), $f_{\pi N\Delta}$ is the $\pi N\Delta$ coupling constant, and $(q^2_\pi)_\pm$ are given by

$$(q^2_\pi)_\pm = m_N^2 + m_\Delta^2 - 2(m_N^2 + p^2)^{1/2}(m_\Delta^2 + p^2)^{1/2} \pm 2pp',$$
\[(q_\pi^2)_+ = m_N^2 + m_\Delta^2 - 2(m_\Delta^2 + p^2)^{1/2}(m_N^2 + p^2)^{1/2} - 2pp'. \tag{8}\]

\(A(q_\pi^2)\) is from the \(\pi N\Delta\) vertex and is given by

\[A(q_\pi^2) = \frac{1}{48m_N m_\Delta} [q_\pi^2 - (m_\Delta + m_N)^2][(m_\Delta^2 - m_N^2 + q_\pi^2)^2 - 4m_\Delta^2 q_\pi^2].\]

In the third term, corresponding to Fig. 2(c), \(k'_K\) is calculated from Eq. (2) with \(m_N\) and \(m_\pi\) replaced by \(m_\Delta\) and \(m_K\), respectively. \(\bar{\sigma}_{K\Delta-KN}\) is the isospin-averaged cross section for the \(K\Delta \to KN\) reaction. Since there is neither experimental nor theoretical information on this cross section, we assume that it is the same as the isospin-averaged cross section \(\bar{\sigma}_{KN-KN}\) for the \(KN \to KN\) reaction.

Since a delta can decay into a physical pion and a nucleon, the exchanged pion in Fig. 2(b) can be on shell, which leads to a singularity in the pion propagator. However, a pion in nuclear matter acquires a (complex) self-energy due to the strong pion-nucleon interaction. The finite imaginary part of the pion self-energy then makes the contribution from the on-shell pion finite. Since we are interested in kaon production in nuclear medium, we replace the free space pion propagator by an in-medium one,

\[D(q_\pi^2) = \frac{1}{q_\pi^2 - m_\pi^2 - \Pi}, \tag{9}\]

with \(\Pi\) being the pion self-energy to be specified later.

The \(\Delta N \to NYK\) reaction includes contributions from both on-shell and off-shell pions. In transport models, the on-shell pion contribution is usually treated as a two step process, i.e., a delta decaying into a physical pion and a nucleon, and the subsequent production of a kaon from the pion-nucleon interaction [30]. In this case, we should include only the off-shell pion contribution in \(\Delta N \to NYK\). Since it is not clear how one can properly separate the on-shell and the off-shell contribution in \(\Delta N \to NYK\), we shall use in the transport model the total \(\Delta N \to NYK\) cross section and neglect kaon production from explicit pion-nucleon interactions. A similar strategy has been adopted in Ref. [31] where the eta (\(\eta\)) production cross section from the nucleon-delta interaction has been considered.

For \(N\Delta \to N\Sigma K\), the isospin-averaged cross section is given by
The isospin/averaged cross section given by reaction. The isospin/averaged cross section is also produced from the last term in Eq. (10) where calculated from Eq. (9) with \( m_k \) kaon is produced in association with a hyperon the exchange is not allowed because of isospin conservation. The isospin/averaged cross section is given by

\[
\sigma_{N\Delta - N\Sigma K}(\sqrt{s}) = \frac{3m_N^2}{4\pi^2p^2s} f_{2NN}(q_x^2) q_x^2 D_{\pi}(q_x^2) |D_{\pi}(q_x^2)|^2 \bar{\sigma}_{\pi\Delta - \Sigma K}(w_x) \\
+ \frac{m_N m_\Delta}{4\pi^2p^2s} \int_{(w_x)_{\text{min}}}^{(w_x)_{\text{max}}} \int_{(q_x^2)_{\text{min}}}^{(q_x^2)_{\text{max}}} dK_{\pi} dw_x w_x^2 k_x^2 \int_{(q_x^2)_{\text{min}}}^{(q_x^2)_{\text{max}}} dK_{\pi} m_N^2 \frac{f_{2NN}(q_x^2) q_x^2 D_{\pi}(q_x^2) |D_{\pi}(q_x^2)|^2 \bar{\sigma}_{\pi\Delta - \Sigma K}(w_x)}{m_N^2}.
\]

Similar notations as those in Eq. (11) are used. \( A(q_k^2) \), obtained from the \( K\Delta\Sigma \) vertex, is given by

\[
A(q_k^2) = \frac{1}{48m_\Sigma m_\Delta} [q_k^2 - (m_\Delta + m_\Sigma)^2] [(m_\Delta^2 - m_\Sigma^2 + q_k^2)^2 - 4m_\Delta^2 q_k^2].
\]

The isospin-averaged cross section \( \bar{\sigma}_{\pi\Delta - \Sigma K} \) is determined from

\[
\bar{\sigma}_{\pi\Delta - \Sigma K} = \frac{1}{6}(\sigma_{\pi+\Delta^0 - \Sigma^0 K^+} + \frac{4}{3}\sigma_{\pi+\Delta^- - \Sigma^- K^+} + \sigma_{\pi^0\Delta^0 - \Sigma^- K^+} + \frac{5}{3}\sigma_{\pi-\Delta^+ - \Sigma^0 K^+}).
\]

The cross sections on the right hand side are again taken from Ref. [25]. For \( N\Delta \rightarrow N\Sigma K \), a kaon can also be produced from the the \( KN \) vertex (Fig. 2(d)). This contribution is given by the last term in Eq. (10), where \( f_{K\Delta\Sigma} \) is the \( K\Delta\Sigma \) coupling constant, and \( (q_k^2)_{\pm} \) are calculated from Eq. (5) with \( m_N \) and \( m_\Delta \) replaced by \( m_\Delta \) and \( m_\Sigma \), respectively.

**C. \( \Delta\Delta \rightarrow NYK \)**

The pion and kaon exchange diagrams for this process are shown in Fig. 3. In the case a kaon is produced in association with a \( \Lambda \) hyperon, the kaon exchange is not allowed because of isospin conservation. The isospin-averaged cross section is given by

\[
\frac{\sigma_{\Delta\Delta - N\Lambda K}(\sqrt{s})}{\sqrt{s}} = \frac{m_N m_\Delta}{2\pi^2p^2s} \int_{(w_x)_{\text{min}}}^{(w_x)_{\text{max}}} \int_{(q_x^2)_{\text{min}}}^{(q_x^2)_{\text{max}}} dK_{\pi} dw_x w_x^2 k_x^2 \int_{(q_x^2)_{\text{min}}}^{(q_x^2)_{\text{max}}} dK_{\pi} m_N^2 \frac{f_{2NN}(q_x^2) q_x^2 D_{\pi}(q_x^2) |D_{\pi}(q_x^2)|^2 \bar{\sigma}_{\pi\Delta - \Lambda K}(w_x)}{m_N^2}.
\]

On the other hand, both pion and kaon exchanges contribute to the \( \Delta\Delta \rightarrow N\Sigma K \) reaction. The isospin-averaged cross section is
\[
\sigma_{\Delta\Delta-N\Sigma K}(\sqrt{s}) = \\
\frac{m_N m_\Delta}{2\pi^2 p^2 s} \int_{(w_\pi)_{\text{max}}}^{(w_\pi)_{\text{min}}} w_\pi^2 w_\pi^4 \frac{f_\pi^2 m_\Delta}{m_\pi^2} F^4(q_\pi^2) A(q_\pi^2) \left| D_\pi(q_\pi^2) \right|^2 \bar{\sigma}_{\pi \Delta-SK}(w_\pi) \\
+ \frac{m_\Delta m_S}{2\pi^2 p^2 s} \int_{(w_K)_{\text{max}}}^{(w_K)_{\text{min}}} dK w_K^2 k'_K \frac{f_K^2 m_K^2}{m_K^2} F^4(q_K^2) A(q_K^2) \left| D_K(q_K^2) \right|^2 \bar{\sigma}_{K \Delta-SN}(w_K). \tag{12}
\]

**D. The pion self-energy in nuclear matter**

As mentioned earlier, to calculate the pion exchange diagrams for kaon production in nucleon-delta and delta-delta interactions requires the inclusion of a (complex) pion self-energy. Following Refs. [26,32–35], we calculate the self-energy of a pion with energy \(\omega\) and momentum \(k\) in nuclear matter by taking into account the delta-hole polarization in the random-phase approximation, i.e.,

\[
\Pi(\omega, k) = \frac{k^2 \chi(\omega, k)}{1 - g' \chi(\omega, k)}, \tag{13}
\]

where \(g' \approx 0.6\) is the Migdal parameter due to the short-range repulsion. The pion susceptibility \(\chi\) is given by

\[
\chi(\omega, k) \approx \frac{8}{9} \left( \frac{f_{\pi N \Delta}}{m_\pi} \right)^2 \frac{\omega_\Delta}{\omega^2 - \omega_\Delta^2} \exp \left( - \frac{2k^2/b^2}{\rho} \right), \tag{14}
\]

where \(\rho\) is the nuclear matter density and \(b \approx 7m_\pi\) is the width of the form factor [33]. Including the delta width \(\Gamma_\Delta\), \(\omega_\Delta\) is approximately given by

\[
\omega_\Delta \approx \frac{k^2}{2m_\Delta} + m_\Delta - m_N - \frac{i}{2} \Gamma_\Delta.
\]

The delta width in nuclear matter can be determined by extending the delta-hole model to include effects of the in-medium pion dispersion relation, i.e.,

\[
\Gamma_\Delta(\omega) = -2 \int k^4 e^{\frac{k^2}{2m_\pi}} \left( \frac{f_{\pi N \Delta}}{m_\pi} \right)^2 \exp \left( - \frac{2k^2/b^2}{\rho} \right) \cdot \frac{1}{3} \frac{1}{1-g' \chi} D_\pi(\omega, k') \frac{1}{1-g' \chi} + g'^2 \Pi_{Kz}, \tag{15}
\]

where \(D_\pi(\omega, k)\) is the in-medium pion propagator given by Eq. (9).
The delta decay width and the pion self-energy are then calculated self-consistently. We show in Fig. 4 the real (left panel) and imaginary (right panel) parts of the pion self-energy as functions of its momentum for a number of energies. The nuclear matter density is taken to be $2\rho_0$, with $\rho_0 = 0.17 \text{ fm}^{-3}$. We see that with increasing pion energy, the real part decreases, while the imaginary part first increases and then decreases.

The pion self-energy is also affected by temperature. For heavy-ion collisions at 1 GeV/nucleon, the temperature reached in the collisions is below 100 MeV, and according to Ref. [36] its effect is not appreciable. Thus, we shall not consider the temperature effect in present study.

**III. KAON PRODUCTION IN BARYON-BARYON INTERACTIONS: RESULTS AND DISCUSSIONS**

Our model for kaon production in baryon-baryon interactions involves five coupling constants and two cut-off parameters. We use the following values for these parameters,

$$\frac{f_{\pi NN}^2}{4\pi} = 0.08, \quad \frac{f_{\pi N\Delta}^2}{4\pi} = 0.37, \quad \frac{f_{\bar{K}N\Lambda}^2}{4\pi} = 0.97, \quad \frac{f_{\bar{K}N\Sigma}^2}{4\pi} = 0.07, \quad \frac{f_{\bar{K}\Delta\Sigma}^2}{4\pi} = 0.23,$$

$$\Lambda_{\pi} = 1.2 \text{ GeV}, \quad \Lambda_{\bar{K}} = 0.9 \text{ GeV}.$$

The $\pi NN$ coupling constant and the pion cut-off mass are typical values used in the Bonn model for the nucleon-nucleon interaction [37]. The $\pi N\Delta$ coupling constant are taken from Ref. [38] and is consistent with that determined from the $\Delta$ decay width. The $KN\Lambda$ and $KN\Sigma$ coupling constants are the same as those used in Ref. [27], and are close to the values used in the Nijmegen [39] as well as the Bonn-Jülich [40] models for the hyperon-nucleon interaction. The $K\Delta\Sigma$ coupling constant is taken from Ref. [40]. Finally, the kaon cut-off mass is similar to that used in Ref. [27]. The two cut-off parameters have been slightly adjusted in order to obtain good agreements with available experimental data on kaon production.
The comparison of our model prediction with the experimental data for $pp \rightarrow p\Lambda K^+$ is shown in Fig. 5, where the solid curve gives the result including both the pion and kaon exchange contributions, while the dash-dotted curve gives only the pion-exchange contribution. We find as in Ref. [27] that once the off-shell effect at the $\pi N\Lambda K$ vertex is included, the pion exchange alone underestimates appreciably the experimental data. Thus, the kaon exchange mechanism is essential for a correct account of the data. As already pointed out in Refs. [13,16], the fact that the pion exchange alone also fits the experimental data in Ref. [26] is attributed to the neglect of off-shell effects at the $\pi N\Lambda K$ vertex.

In Fig. 5, we also show the predictions from the linear parameterization of Ref. [5] (dashed curve) and the quartic parameterization of Ref. [9] (dotted curve), which are given, respectively, by

$$\sigma_{\text{linear}} = 24 \frac{p_{\text{max}}}{m_K^2} \mu b, \quad \sigma_{\text{quartic}} = 800 \left( \frac{p_{\text{max}}}{m_K} \right)^4 \mu b,$$

with

$$p_{\text{max}} = \frac{1}{2\sqrt{s}} \left[ (s - (m_N + m_\Lambda + m_K)^2)(s - (m_N + m_\Lambda - m_K)^2) \right]^{1/2}. \quad (16)$$

At low energies (below the lowest experimental point of $\sqrt{s} \approx 2.7$ GeV) our prediction is close to the quartic parameterization, and is considerably smaller than the linear parameterization. Therefore, the use of the linear parameterization for subthreshold kaon production in heavy-ion collisions probably overestimates the contribution from the nucleon-nucleon interaction. On the other hand, the quartic parameterization greatly overestimates the cross section for $\sqrt{s} \geq 2.7$ GeV.

Figs. 6-8 show comparisons of our results with the experimental data from reactions in which the kaon is produced in association with a $\Sigma$ hyperon. In this case, the pion exchange alone can explain the data as the kaon exchange contribution is very small due to a small $KN\Sigma$ coupling. This is consistent with the observations in Ref. [27].

In transport models, isospin-averaged cross sections are usually used. We denote, by $\sigma_{BB-NYK}$, the isospin-averaged kaon production cross section in baryon-baryon interactions
including both the $N\Lambda K$ and $N\Sigma K$ final states, i.e., $\sigma_{BB-NYK} = \sigma_{BB-N\Lambda K} + \sigma_{BB-N\Sigma K}$.

In Fig. 9, we compare $\sigma_{NN-NYK}$ based on the present model (Eqs. (1) and (6)) with the parameterization of Ref. [5] (dashed curve)

$$\sigma_{RK} = 72 \frac{p_{\text{max}}}{m_K} + 72 \frac{p_{\text{max}}}{m_K} \mu b,$$  \hspace{1cm} (17)

where $p'_{\text{max}}$ is calculated from Eq. (16) with $m_\Lambda$ replaced by $m_\Sigma$.

The theoretical results for $\sigma_{NN-NYK}$ are shown for nuclear matter at densities $\rho = 0$, $\rho_0$ and $2\rho_0$. In general, our results are smaller than the linear parameterization of Ref. [5], especially near the threshold. The sudden change of the kaon cross section at $\sqrt{s} \sim 2.63\text{GeV}$ in the linear parameterization is due to the onset of the $NN \rightarrow N\Sigma K$ channel. With increasing density, the kaon production cross section is seen to increase. This is consistent with the findings of Ref. [26], and is mainly due to the softening of the pion dispersion relation, which tends to enhance the cross section.

Our results for the kaon production cross section in the $N\Delta$ interaction is shown in Fig. 10. The result from the scaling ansatz of Ref. [5] (i.e., three-fourth of Eq. (17)) is also shown in the figure by the dashed curve. Our results near the threshold are close to the scaling ansatz. The density dependence of $\sigma_{N\Delta-NYK}$ is, however, different from that of $\sigma_{NN-NYK}$. While the latter increases with density because of the softening of the pion dispersion relation, the former decreases with density. This is largely due to the strong suppression of the singular on-shell pion contribution at high densities. A similar observation has been found in Ref. [31] where the eta production cross section in the nucleon-delta interaction is seen to also decrease with increasing density.

Finally, our results for the kaon production cross section in the delta-delta interaction are shown in Fig. 11. Near the threshold they are smaller than the scaling ansatz of Ref. [5] (i.e., half of Eq. (17)). Also, the kaon production cross section in the delta-delta interaction is considerably smaller than that in the nucleon-delta interaction as a result of the smaller $\pi\Delta \rightarrow YK$ cross section than the $\pi N \rightarrow YK$ cross section [25]. Again, we see that the kaon production cross section decreases with increasing density.
IV. APPLICATIONS TO AU+AU COLLISIONS AT 1 GEV/NUCLEON

In this section, we shall use the theoretical kaon production cross sections from baryon-baryon interactions in a transport model to study kaon production in Au+Au collisions at 1 GeV/nucleon. This reaction has been previously studied by many groups [12-17] based on various transport models and using mainly the linear parameterization of Ref. [5] for the elementary kaon production cross sections in baryon-baryon interactions.

Ideally, we would like to carry out the calculation in the relativistic transport model as in our previous studies [13,17]. In this way, we can treat properly the medium modification of hadron properties [13]. Since the kaon production cross sections from baryon-baryon interactions determined in the present work have been obtained without these medium effects, we shall use instead the non-relativistic transport model in which hadron masses are taken to be free ones. The difference between the relativistic and the non-relativistic transport model for kaon production is, however, insignificant, as already discussed in Ref. [17]. The reason is simple: in the reaction $BB \rightarrow NYK$, both scalar and vector potentials in the initial and final states cancel in leading order. The kaon production threshold in the medium is thus not affected by medium effects, which are neglected in the non-relativistic transport model. The situation is certainly different for $BB \rightarrow N N K \bar{K}$ and $BB \rightarrow N N p \bar{p}$ where a net scalar potential is present in the final state. Including the change of the production threshold in the medium, it has been shown that the relativistic transport model leads to almost two order-of-magnitude enhancement of the antiproton yield [42] and a factor of 4 enhancement of the antikaon yield [43] as compared to those from the non-relativistic transport model. In the future, we plan to carry out a more complete study of subthreshold kaon production in heavy ion collisions based on the relativistic transport model that includes consistently medium effects on the elementary kaon production cross sections and the collision dynamics.

With a soft Skyrme equation of state, corresponding to a compressibility $K = 200$ MeV at normal nuclear matter density, we show in Fig. 12 the energy distribution of baryon-baryon interactions that lead to kaon production in central Au+Au collisions at 1
GeV/nucleon. There are about 112 baryon-baryon collisions with available energies above the respective kaon production thresholds. The average energy in these collisions is about 2.64 GeV. Since the threshold energy for the $BB \rightarrow \Delta Y K$ reaction with a delta in the final state is about 2.84 GeV, our neglect of its contribution to the kaon yield is therefore justified. We would like to point out that an average energy of 2.64 GeV corresponds to a kaon maximum momentum of 0.275 GeV, which is very close to the average kaon maximum momentum of 0.272 GeV obtained in Ref. [17] based on the relativistic transport model and a soft (relativistic) equation of state. This is an indication of the similarity between the relativistic and non-relativistic transport model descriptions of kaon production in heavy-ion collisions.

The kaon production probabilities from different reactions are shown in Fig. 13. The results based on the theoretical elementary cross sections are shown in the left panel. The contribution from the nucleon-delta interaction is most important and accounts for about 65%. As in previous studies, deltas play the most important role as reactions involving delta resonances produce about 90% of the kaons.

We have also carried out a VUU calculation using the Randrup-Ko parameterization for kaon production in baryon-baryon interactions. The results are shown in the middle panel of Fig. 13. We see that the kaon yield in this case is only about 20% less than that obtained with the theoretical cross sections calculated in the present paper.

To see quantitatively the difference (or similarity) between the relativistic and non-relativistic descriptions of kaon production in heavy-ion collisions, we reproduce the Fig. 8 of Ref. [13] in the right panel of Fig. 12. The relativistic transport model is seen to lead to a reduction of about 30% in the total kaon yield. This reduction is mainly due to the momentum-dependent nuclear mean-field potential included in the relativistic transport model. This momentum-dependent potential gives rise to an additional repulsion besides that from the compressional pressure, and a smaller maximum density is thus reached in the relativistic transport model. This reduction in kaon yield is much smaller than that found in Ref. [8] based on the non-relativistic QMD model with a momentum-dependent Skyrme-
type potential, where the reduction factor is about 3. The reason for this difference between these two calculations has been discussed in Ref. [17]. We believe that the treatment of the momentum-dependent mean-field potential in Ref. [8] is incomplete as it has not taken into account the difference in the initial and final potential energies in the $BB \rightarrow NYK$ reaction. Since baryons in the initial state have larger momenta, the intial potential energy in the above reaction is larger than that of the final state and can thus be used for kaon production. If this potential difference is properly included, as in the relativistic transport model, the difference between the kaon yields obtained in QMD calculations with and without the momentum-dependent potential should be small.

Finally, we show in Fig. 14 the kaon momentum spectrum obtained in this study. The theoretical result is seen to overestimate the experimental data [3] by about 30%. As mentioned earlier, the relativistic transport model would lead to a reduction of a similar magnitude in the kaon yield as compared to the non-relativistic one. We thus expect that a consistent calculation based on the relativistic transport model and using the theoretical elementary cross sections which properly include the medium effects, will give a better account of the experimental data.

V. SUMMARY

In this work, we have extended our previous studies of kaon production in Au+Au collisions at 1 GeV/nucleon by using in the transport model the elementary kaon production cross sections calculated from a theoretical model. The main purpose of this study is to examine our previous conclusions concerning the nuclear equation of state at high densities [17] and the kaon scalar potential [13] in dense matter. A particular concern is the possible underestimate of the kaon production cross section from the nucleon-delta interaction in the scaling ansatz of Randrup and Ko. For this purpose, we have constructed a one-pion plus one-kaon exchange model for kaon production in baryon-baryon interactions. The parameters of the model are fitted to available experimental data in the proton-proton
interaction. This model is then extended to the nucleon-delta and delta-delta interactions. We have found that near the kaon production threshold, which are relevant for subthreshold kaon production, the cross sections in the Randrup-Ko parameterization and scaling ansatz that have been used in previous studies by us [13,17] and by other groups [12,15,16] are overestimated for the nucleon-nucleon interaction but underestimated for the nucleon-delta interaction.

We have then used these theoretical cross sections to calculate in a non-relativistic transport model the kaon yield in Au+Au collisions at 1 GeV/nucleon. The results are found in reasonable agreement with experimental data. We expect that a consistent relativistic transport model calculation will lead to an even better agreement with the data. The conclusion of this extensive study is thus clear: we need both an attractive scalar potential and a soft nuclear equation of state to account for the experimental data from the Kaos collaboration at the GSI.

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Figure Captions

Fig. 1: One-pion and one-kaon exchange diagrams for the $NN \rightarrow NYK$ reaction.

Fig. 2: Same as Fig. 1, for the $N\Delta \rightarrow NYK$ reaction.

Fig. 3: Same as Fig. 1, for the $\Delta\Delta \rightarrow NYK$ reaction.

Fig. 4: Real (left panel) and imaginary (right panel) parts of the pion self-energy in nuclear matter with density $\rho = 2\rho_0$.

Fig. 5: Comparisons of model predictions with experimental data for the kaon production cross section in $pp \rightarrow p\Lambda K^+$. The dashed-dotted curve gives the one-pion exchange contribution, while the solid curve includes both the one-pion and one-kaon exchange contributions. The dashed curve is based on the linear parameterization of Randrup and Ko [5], while the dotted curve is based on the quartic parameterization of Zwermann [9]. The experimental data are from Ref. [24].

Fig. 6: Same as Fig. 4, for $pp \rightarrow p\Sigma^0 K^+$.

Fig. 7: Same as Fig. 4, for $pp \rightarrow p\Sigma^+ K^0$.

Fig. 8: Same as Fig. 4, for $pp \rightarrow n\Sigma^+ K^+$.

Fig. 9: Isospin-averaged cross section $\sigma_{NN-NYK}(= \sigma_{NN-\Lambda K} + \sigma_{NN-\Sigma K})$. The solid curve is our model prediction and the dashed curve is from the parameterization of Ref. [5].

Fig. 10: Same as Fig. 9 for $\sigma_{N\Delta-NYK}(= \sigma_{N\Delta-\Lambda K} + \sigma_{N\Delta-\Sigma K})$.

Fig. 11: Same as Fig. 9 for $\sigma_{\Delta\Delta-NYK}(= \sigma_{\Delta\Delta-\Lambda K} + \sigma_{\Delta\Delta-\Sigma K})$.

Fig. 12: The energy distribution of baryon-baryon collisions that are above the kaon production threshold.
Fig. 13: Kaon production probabilities from different channels obtained in three calculations.

Fig. 14: Comparisons of kaon momentum spectra obtained in this work using the theoretical kaon production cross sections (solid curve) with the experimental data (open squares) from Ref. [3].