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Magnus Force and Chern-Simons Vortices

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Abstract

By employing Manton's method, we show that Chern-Simons vortices in the presence of a background density experience the Magnus force.

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In a recent letter, Barashenkov and Harin studied a non-relativistic Chern-Simons theory in the presence of a background density. A similar model was also studied by Lozano [1]. As found by Barashenkov and Harin, this model admits vortex solutions with a non-vanishing field configuration at infinity, in contrast to the theory without the background density [2]. In this brief report, we show that the background density induces a Magnus force [3] on moving Chern-Simons vortices, a force that vanishes when the background density is absent [4].

We start by reminding the reader of the conclusions of Ref. [1] that are essential to the present discussion. First, the authors of Ref. [1] have formulated a $2+1$ dimensional Chern-Simons theory in which the matter density is finite at infinity, and have found that the Euler-Lagrange equations corresponding to that theory admit vortex solutions. These equations can be derived from a Lagrangian written solely in terms of the matter field ψ . The Lagrangian can be put in the form

$$L = \int d^2\mathbf{r} (i\psi^* \partial_t \psi - \mathcal{H}(\psi)) \quad (1)$$

where $\mathcal{H}(\psi)$ is the energy density of the system and has no explicit time dependence. Second, the one-vortex solution centered at $\mathbf{R}$ has the following form

$$\psi_1(\mathbf{r}; \mathbf{R}) = \rho^{\frac{1}{2}} e^{i\theta} \quad (2)$$

with $\mathbf{r} = (x, y)$ and $\mathbf{R} = (X, Y)$. Here ρ and θ are functions of $\mathbf{r} - \mathbf{R}$ where the latter is given by $\theta = \arctan(\frac{y-Y}{x-X})$. The density ρ vanishes continuously at $\mathbf{r} = \mathbf{R}$ and approaches the background density ρ_0 as $|\mathbf{r} - \mathbf{R}| \rightarrow \infty$.

Following the standard Manton procedure [5], we reduce the Lagrangian (1) to a Lagrangian of a single degree of freedom, that of the vortex' center, R . This is done by regarding R as time dependent and substituting (2) into (1). The Lagrangian thus obtained effectively describes the dynamics of the vortex center. For convenience, we consider the two terms in (1) separately. The second term, when integrated, gives the rest energy of the vortex and is irrelevant to our discussion of the Magnus force. On the other hand, the first term leads to

$$L_{eff}^1 = \int d^2\mathbf{r} \left[\left(\frac{i}{2} \partial_t \rho(\mathbf{r} - \mathbf{R}(t)) - \rho(\mathbf{r} - \mathbf{R}(t)) \partial_t \theta(\mathbf{r} - \mathbf{R}(t)) \right) \right] \quad (3)$$

Since $\frac{d}{dt} \int d^2\mathbf{r} \rho = 0$, (3) can be simplified to

$$\begin{aligned} L_{eff}^1 &= - \int d^2\mathbf{r} \rho(\mathbf{r} - \mathbf{R}(t)) \partial_t \theta(\mathbf{r} - \mathbf{R}(t)) \\ &= \dot{\mathbf{R}} \cdot \int d^2\mathbf{r} \rho(\mathbf{r} - \mathbf{R}(t)) \nabla_{\mathbf{r}} \theta(\mathbf{r} - \mathbf{R}(t)) \end{aligned} \quad (4)$$

here we have used the fact that θ only depends on $\mathbf{r} - \mathbf{R}$. The subscripts $\mathbf{r}$ and $\mathbf{R}$ denote the argument of the differentiation. Recalling that $\dot{\mathbf{R}}$ is the velocity of the vortex, we observe that L_{eff}^1 describes an interaction of the vortex with a vector potential. The force $\mathbf{F}$ experienced by the vortex due to this interaction can then be obtained by varying (4) with respect to $\mathbf{R}$

$$F_i = \epsilon_{ij} \dot{R}_j B \quad (5)$$

The field strength B is given by

$$B = \nabla_{\mathbf{R}} \times \int d^2\mathbf{r} \rho(\mathbf{r} - \mathbf{R}(t)) \nabla_{\mathbf{r}} \theta(\mathbf{r} - \mathbf{R}(t)) \quad (6)$$

The evaluation of B is straightforward. Since the integrand only depends on $\mathbf{r} - \mathbf{R}$, we write

$$B = - \int d^2\mathbf{r} \nabla_{\mathbf{r}} \times (\rho(\mathbf{r} - \mathbf{R}(t)) \nabla_{\mathbf{r}} \theta(\mathbf{r} - \mathbf{R}(t))) \quad (7)$$

Using Stoke's theorem and the asymptotic behavior of ρ , we finally have

$$B = -2\pi \rho_0 \quad (8)$$

Thus, the force exerted on the vortex when it moves relative to the background density ρ_0 is just the Magnus force. As a final remark, we point out that in our derivation the essential inputs are the asymptotic behavior of the vortex at infinity and the non-relativistic nature of the theory. This suggests that the existence of the Magnus force is a universal feature for non-relativistic vortices with non-vanishing field configuration at large distances. Also note that the effective action obtained by integrating (4) can be obtained by a Berry phase analysis as carried out for the case of vortices in superconductors in Ref. [3].

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