88

DIPOLE POLARIZABILITY OF NEUTRON-RICH NUCLEI*

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We calculate the dynamic polarization potential for neutron-rich nuclei arising from the Coulomb coupling of the elastic channel to: i) the break-up channel; ii) the low-lying dipole mode. These potentials are found to be different only by a numerical factor. The effect of these polarization potentials on the elastic scattering of $^{11}\text{Li} + ^{208}\text{Pb}$ at $E_{CM} = 50$ MeV is calculated.

I. INTRODUCTION

The Coulomb dipole excitation in heavy-ion scattering involving neutron-rich nuclei has recently been studied within the polarization potential approach [1]. What prompted the study of ref. [1] was the realization that whereas Coulomb quadrupole excitation is a common phenomenon which has been extensively discussed, low energy Coulomb dipole excitation only becomes important in cases involving loosely bound nuclei that exhibit the so called soft giant dipole vibrational mode [2].

A very important aspect of these modes, which was not considered in ref. [1], is the fact that these resonances have very short lifetimes and, therefore, a conventional treatment [3] of the Coulomb polarizability of these nuclei must be carefully assessed. A more natural way of treating this problem, is to consider explicitly the effect of the width of these soft resonances. Short of doing this, one should consider the extreme case of Coulomb polarizability in the form of the coupling of the elastic channel to the break-up channel containing, at least, three particles. We should mention here, that a similar dipole polarization effect also occurs in deuteron-nucleus scattering [4].

The main purpose of this paper is to calculate the Coulomb-induced dynamic polarization potential in the elastic channel involving a neutron-rich light projectile, such as ^{11}Li or ^{11}Be , from a very heavy target nucleus. Our aim is to obtain wave-function-equivalent polarization potentials, in contrast to the phase-shift equivalent potentials derived by Andres *et al.* [1]. As will be seen, our potentials are orbital angular momentum and energy-dependent while those of ref. [1] are not. The second aim of this paper is to clarify the differences between Coulomb excitation of a dipole state in the projectile, and Coulomb break-up. The paper is organized as follows: in Section II, the Coulomb break-up polarization potential is calculated for the two-cluster nucleus ^{11}Li . In Section III the corresponding potential arising from the coupling to a zero-width soft giant dipole mode in the ^{11}Li projectile is calculated. In Section IV, an angular momentum independent approximation for the potentials of the previous sections is discussed. In Section V, the effect of a finite width for the soft giant mode on the polarization potentials is considered. In Section VI, the theory is applied to a detailed study of the $^{11}\text{Li} + ^{208}\text{Pb}$ collision. Finally, in Section VII, concluding remarks are made.

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II. THE POLARIZATION POTENTIAL ARISING FROM DIRECT COULOMB DISSOCIATION

In previous works, we have calculated polarization potentials due to nuclear dissociation of ^{11}Li (refs. [5] and [6]) and their effects on the fusion cross-section (refs. [7] and [8]). In those cases, we were concerned with collisions of ^{11}Li with light targets, where the main contribution to the cross sections arises from nuclear forces. However, for collisions with very heavy targets (like ^{208}Pb) the situation is quite different and Coulomb forces dominate. In this section, we evaluate the polarization potential arising from the Coulomb break-up of ^{11}Li .

Let us consider the collision of ^{11}Li , treated within the dineutron approximation, with a heavy target. The Coulomb coupling can be written

$$v_c(\mathbf{r}, \mathbf{x}) = \frac{Z_P Z_T e^2}{|\mathbf{R}_{T-^9\text{Li}}|} \equiv \frac{Z_P Z_T e^2}{|\mathbf{r} + \frac{2}{11}\mathbf{x}|}, \quad (1)$$

where $\mathbf{r}$ is the vector joining the centers of mass of projectile and target and $\mathbf{x}$ is the vector from the dineutron to the ^9Li cluster.

Using the multipole expansion for the denominator of eq. (1) (see, e.g., ref. [9], with $\mathbf{R}_> = \mathbf{r}$ and $\mathbf{R}_< = -\frac{2}{11}\mathbf{x}$) and taking the dipole approximation, we get

$$v_c(\mathbf{r}, \mathbf{x}) = \frac{8\pi}{33} \frac{Z_P Z_T e^2}{r^2} \frac{x}{r^2} \sum_m Y_{1m}^*(\hat{x}) Y_{1m}(\hat{r}), \quad (2)$$

where hats stand for unitary vectors. In the sudden limit, the polarization potential is given as (eq. (6) of ref. [6])

$$V(\mathbf{r}, \mathbf{r}') = G^{(+)}(\mathbf{r}, \mathbf{r}') \int |\phi_0(\mathbf{x})|^2 v_c(\mathbf{r}, \mathbf{x}) v_c(\mathbf{r}', \mathbf{x}) d^3\mathbf{x} \equiv G^{(+)}(\mathbf{r}, \mathbf{r}') \cdot F(\mathbf{r}, \mathbf{r}'), \quad (3)$$

where

$$\phi_0(\mathbf{x}) = (2\pi\alpha)^{-1/2} \frac{e^{-x/\alpha}}{x}, \quad \alpha = \frac{\hbar}{\sqrt{2B\mu(^{11}\text{Li})}}, \quad (4)$$

B is the dineutron binding energy in ^{11}Li and $\mu(^{11}\text{Li}) = \frac{18}{11}m_0$, where m_0 is the nucleon mass, is the reduced mass of the ^{11}Li system. Using the explicit form of v_c (eq. (2)) in the integrand of eq. (3), we get

$$F(\mathbf{r}, \mathbf{r}') = \left[\frac{8\pi}{33} \frac{Z_P Z_T e^2}{r^2} \right]^2 \frac{1}{r^2 r'^2} \sum_m Y_{1m}^*(\hat{r}) Y_{1m}(\hat{r}'). \quad (5)$$

For the calculation of cross sections, one usually carries out angular momentum projections. The l -component of the polarization potential is obtained through the integral

$$V_l(r, r') = rr' \int d\Omega_r d\Omega_{r'} Y_{l0}(\hat{r}) V(\mathbf{r}, \mathbf{r}') Y_{l0}(\hat{r}'). \quad (6)$$

Using the partial-wave decomposition of the Green's function,

$$G^{(+)}(\mathbf{r}, \mathbf{r}') = \frac{1}{rr'} \sum_{lm} Y_{lm}(\hat{r}) G_l^{(+)}(r, r') Y_{lm}^*(\hat{r}'), \quad (7)$$

together with eqs. (2), (5) and (7) in eq. (6), and carrying out the lengthy angular momentum algebra, we get

$$V_l(r, r') = \frac{2\alpha^2}{3} \left[\frac{Z_P Z_T e^2}{11} \right]^2 \left\{ \frac{l+1}{2l+1} G_{l+1}^{(+)}(r, r') + \frac{l}{2l+1} G_{l-1}^{(+)}(r, r') \right\} \frac{1}{r^2 r'^2}. \quad (8)$$

Following ref. [5], we calculate $G_{l\pm 1}^{(+)}(r, r')$ within the on-shell approximation and replace the radial wave functions by $\simeq \sqrt{|S_{l\pm 1}^{opt}|} F_{l\pm 1}(kr)$, where $S_{l\pm 1}^{opt}$ is the partial-wave components of the optical S-matrix and $F_{l\pm 1}$ are the regular Coulomb wave functions. It takes the form

$$G_{i\pm 1}^{(+)}(r, r') = -i \frac{2\mu}{\hbar^2 k} |S_{i\pm 1}^{opt}| F_{i\pm 1}(kr) F_{i\pm 1}(kr'), \quad (9)$$

where μ is the projectile-target reduced mass and k is the relative wave number.

It is convenient to define the trivially equivalent polarization potential as [3]

$$U_i^{bu}(r) = \frac{1}{F_i(kr)} \int V_i(r, r') F_i(kr') dr'. \quad (10)$$

Using eqs. (8) and (9) in eq. (10), we get

$$U_i^{bu}(r) = -i \frac{4\mu(\alpha Z_P Z_T \epsilon^2)^2}{363 \hbar^2 (2l+1)} \times \left\{ (l+1) |S_{i+1}^{opt}| \frac{F_{i+1}(kr)}{F_i(kr)} I_{l, l+1} + l |S_{i-1}^{opt}| \frac{F_{i-1}(kr)}{F_i(kr)} I_{l, l-1} \right\} \frac{1}{r^2}. \quad (11)$$

The Coulomb radial integrals $I_{l, l\pm 1}$ have the value [10]

$$I_{l\pm 1} = \frac{1}{2 \sqrt{l_{\pm}^2 + \eta^2}}, \quad (12)$$

where $\eta = Z_P Z_T \epsilon^2 / \hbar v$ is the usual Sommerfeld parameter. Using the explicit form of the Coulomb radial integrals and assuming that $l \gg 1$ and that $|S_i^{opt}|$ is a slowly varying function of l , we get

$$U_i^{bu}(r) = -i \left[\frac{A^{bu}(l, E)}{r^2} + \frac{B^{bu}(l, E)}{r^3} \right], \quad (13)$$

with

$$A^{bu}(l, E) = \frac{4\pi\mu (Z_T \epsilon)^2 B_{cl}(E1) \uparrow}{9 \hbar^2 \eta} \frac{|S_l^{opt}|}{1 + \bar{l}^2}; \quad B^{bu}(l, E) = \frac{\eta \bar{l}^2}{k} A^{bu}(l, E). \quad (14)$$

Above, we adopted the notation $\bar{l} = l/\eta$, used the relation (eq. (3.13) of ref. [11])

$$\frac{F_{l+1}(kr) + F_{l-1}(kr)}{F_l(kr)} = \frac{2}{\sqrt{1 + \bar{l}^2}} \left(1 + \frac{\bar{l}^2}{\eta kr} \right), \quad (15)$$

and the expression for the cluster-model $B(E1)$ -value

$$B_{cl}(E1) \uparrow = \frac{3\hbar^2 \epsilon^2}{16\pi B\mu_{(11 \text{ Li})}} \left(\frac{Z_{9 \text{ Li}} m_{2n} - Z_{2n} m_{9 \text{ Li}}}{m_{11 \text{ Li}}} \right)^2 = \frac{3 (\alpha Z_P \epsilon)^2}{242 \pi}. \quad (16)$$

III. THE SOFT GIANT DIPOLE MODE POLARIZATION POTENTIAL

In this section we assume that the collision excites the soft giant dipole mode (SM). We show in the following that the resulting polarization potential has the same form as the one calculated in the previous section. We start from the equation (eq. (2) of ref. [6])

$$V(\mathbf{r}, \mathbf{r}') = \langle \mathbf{r}; \phi_0 | v_c Q G_{QQ}^{(+)} Q v_c | \phi_0; \mathbf{r}' \rangle, \quad (17)$$

where the projector Q can be written

$$Q = \sum_{l' JM} \int dr dx |(l' 1) JM r; \phi_1(x) \rangle \langle \phi_1(x); r JM (l' 1) |. \quad (18)$$

The above included states correspond to eigenstates of the total angular momentum, with eigenvalues JM , obtained from the coupling of the orbital angular momentum l' with the intrinsic angular momentum

1 of the SM, and $\phi_1(x)$ stands for the intrinsic SM radial wave function. Using conservation of total angular momentum, the locality of the coupling and the partial waves expansion of $V(\mathbf{r}, \mathbf{r}')$, we get

$$V_i(r, r') = \sum_{l'=l\pm 1} v_{i0,l'l}^i(r) G_l^{(+)}(r, r') v_{i0,l'l}^i(r'), \quad (19)$$

where we have adopted the sudden approximation (replacing the Green's function in the break-up channel by its elastic value) and omitted the subscript c of the Coulomb coupling, for simplicity of notation. Using the expression for the matrix elements of the coupling (eq. (3.19) of ref. [3]), we get

$$v_{i0,l'l}^i(r) \cdot v_{i0,l'l}^i(r') = 4\pi(Z_T\epsilon)^2 B(E1)\uparrow \frac{(2\ell+1)(2\ell'+1)}{3 r^2 r'^2} \begin{pmatrix} l & l' & 1 \\ 0 & 0 & 0 \end{pmatrix}^2 \left\{ \begin{matrix} l & l' & 1 \\ 1 & 0 & l \end{matrix} \right\}^2. \quad (20)$$

Writing the explicit values of the 3-J and 6-J coefficients (see, e.g. ref. [12]), we get

$$V_i(r, r') = \left[\frac{4\pi Z_T^2 \epsilon^2 B(E1)\uparrow}{9} \right] \left\{ \frac{(l+1) G_{i+1}^{(+)}(r, r') + l G_{i-1}^{(+)}(r, r')}{2l+1} \right\} \frac{1}{r^2 r'^2}. \quad (21)$$

Using the Green's function of eq. (9), inserting the above equation in eq. (10) and following the same procedures of the previous section, we get the *SM* polarization potential

$$U_i^{SM}(r) = -i \left[\frac{A^{SM}(l, E)}{r^2} + \frac{B^{SM}(l, E)}{r^3} \right], \quad (22)$$

where the coefficients $A^{SM}(l, E)$ and $B^{SM}(l, E)$ are

$$A^{SM}(l, E) = \frac{4\pi\mu (Z_T\epsilon)^2 B(E1)\uparrow}{9 \hbar^2 \eta} \frac{|S_i^{opr}|}{1 + \bar{l}^2}; \quad B^{SM}(l, E) = \frac{\eta \bar{l}^2}{k} A^{SM}(l, E). \quad (23)$$

By comparing the Coulomb break-up polarization potential (eqs. (13) and (14)) with that due to the excitation of the soft dipole mode (eqs. (22) and (23)) (both treated in the sudden limit), we find that they have the same dependence on r , l and E , and become identical if $B(E1)\uparrow = B_{el}(E1)\uparrow$. Otherwise, the strength of the soft dipole Coulomb polarization potential depends on the model value of $B(E1)\uparrow$. It should be remarked that the width of the soft dipole mode, not taken into account in the above derivation, influences the overall strength. Further discussion of this effect will be made in Section V.

IV. THE EQUIVALENT ANGULAR MOMENTUM-INDEPENDENT POLARIZATION POTENTIAL

The potentials of eqs. (13) and (22) were obtained directly from the two-channel wave function using the Feshbach method. In contrast, the potential of Ref. [1] was derived from the inversion of the semiclassical transition amplitude. These potentials should in principle be different since the former are wave-function equivalent whereas the latter is phase-shift equivalent. We refer the reader to ref. [13] for a careful discussion of this question in relation to the Coulomb quadrupole polarization potential. Similar considerations can be applied in general.

To exhibit in a transparent fashion the above mentioned difference between the potential of ref. [1] and our potentials (eqs. (13) and (22)), we have to remove the l -dependence in the latter. This is easily done within the semiclassical theory. What one has to do is to relate l/η to the corresponding distance of closest approach $r_0(l)$ and then set $r_0(l) \equiv r$.

From the defining equation of $r_0(l)$ for a Rutherford trajectory,

$$\frac{Z_P Z_T \epsilon^2}{r_0(l)} + \frac{\hbar^2 l^2}{2\mu r_0^2(l)} = E, \quad (24)$$

we find

$$\bar{l}^2 = \frac{1 - 2a/r_0(l)}{(a/r_0(l))^2}, \quad (25)$$

where $2a = r_0(0) \equiv Z_P Z_T e^2 / E$ is the distance of closest approach for a head-on collision. Accordingly, from Eq. (25) we have

$$(1 + \bar{l}^2)^{-1} = \frac{(r_0(l) - a)^2}{a^2}. \quad (26)$$

When we use eqs. (25) and (26) in our l -dependent potentials, eqs. (13) and (22), the equivalent l -independent versions are immediately obtained by setting $r_0(l) \equiv r$. Thus we find for, e.g., eq. (22), the following l -independent wave-function equivalent potential (to be consistent with ref. [1], we set $|S_l^{pp'}| = 1$)

$$\bar{U}(r) = -i \frac{U_0}{(r-a) r^2}; \quad U_0 = \frac{4\pi Z_T^2 e^2 B(E1)^\dagger}{9 \hbar v} \quad (27)$$

The above potential has exactly the same strength as that of ref. [1], but it shows an important energy-dependent difference in the r -dependence. Calling $\bar{U}^{pse}(r)$ the phase-shift equivalent polarization potential of ref. [1],

$$\bar{U}^{pse}(r) = -i \frac{U_0}{(r-a)^2 r}, \quad (28)$$

we get the ratio

$$\frac{\bar{U}(r)}{\bar{U}^{pse}(r)} = \frac{r-a}{r}. \quad (29)$$

V. EFFECT OF THE SOFT MODE WIDTH ON THE POLARIZATION POTENTIAL

In deriving eq. (22), we have assumed that the soft mode has infinite lifetime. It is argued, though, that this mode decays into the break-up channel with a width of about 1 MeV [2]. It is therefore essential to take it into account. A simple way of doing this is to treat the soft mode as an exit doorway and, accordingly, the Green's function that appears in eq. (19) becomes

$$G_i^{(+)}(r, r') = \int_0^\infty dk \frac{F_i(kr) F_i(kr')}{\left(E^{(+)} - \varepsilon_1 + i\frac{\Gamma}{2} - \frac{\hbar^2 k^2}{2\mu}\right)}. \quad (30)$$

The Green's function of eq. (30) is well defined and it reduces to that of eq. (9) if one makes the sudden approximation ($\varepsilon_1 = 0$) and takes the limit $\Gamma \rightarrow 0$ (leaving out, for simplicity, $|S_l^{pp'}|$). In the general case of $\Gamma \neq 0$, this Green's function is quite different and we leave its detailed evaluation for a future work. We should mention that the formal apparatus for such an evaluation has been worked out by Schwinger [14].

In the case where Γ is very large, one attains the adiabatic limit usually considered in connection with giant resonance polarizability. From ref. [3], we thus have

$$G_i^{(+)}(r, r') \simeq \frac{-1}{-i\Gamma/2} \int_0^\infty dk F_i(kr) F_i(kr') = -i \frac{\pi}{\Gamma} \delta(r - r'), \quad (31)$$

where the relation $\int_0^\infty dk F_i(kr) F_i(kr') = \frac{\pi}{2} \delta(r - r')$ has been used. With the above form of the resonant Coulomb Green's function, we obtain the local, l -independent, purely absorptive, polarization potential

$$\bar{U}(r) = -i \left(\frac{8\pi^2 (Z_T e)^2 B(E1)^\dagger}{9 \Gamma} \right) \frac{1}{r^4}. \quad (32)$$

The above simple exercise should convince the reader that the the width of the soft dipole mode changes the r -dependence of the polarization potential. In the case of a not so large Γ , the r -dependence should be a combination of terms of the type suggested by eqs. (22) and (32). Thus, for $r \gg a$, we get

$$\bar{U}(r) = -i U_0 \left\{ \frac{1 - C(\Gamma) [B(E1)^\dagger / B_{cl}(E1)^\dagger]}{r^3} + \frac{D(\Gamma) 2\pi \hbar v / \Gamma}{r^4} \right\}, \quad (33)$$

with $C(\Gamma = 0) = 1$, $D(\Gamma = 0) = 0$ and $C(\Gamma = \infty) = 0$, $D(\Gamma = \infty) = 1$. Work to evaluate these functions for arbitrary Γ is in progress.

VI. APPLICATION

To exemplify the use of the results derived in the previous sections, we carry out a detailed study of the $^{11}\text{Li} + ^{208}\text{Pb}$ collision.

A. The Coulomb polarization potential

In our numerical calculations, we use the barrier radius $R_B = 11$ fm [15] and dissociation threshold energy $B = 0.3$ MeV [16].

In fig. 1, we show the l -dependence of the coefficients $A(l, E)$ (fig. 1a) and $B(l, E)$ (fig. 1b), at several center of mass energies. We chose to normalize these functions with respect to proper powers of the barrier radius, in order that they have dimension of energy. In this way, they correspond, respectively, to the contributions from the terms in r^{-2} and r^{-3} , appearing in eq. (13). At 50 MeV and 75 MeV, i.e. above the potential barrier, these functions drop abruptly as the angular momentum goes below the grazing value. This behavior results from the small values of $|S_l^{\text{opt}}|$ due to the strong absorption produced by the optical potential. We observe that at $l = 0$, $B(l, E) = 0$ and the potential behaves as r^{-2} . At the other extreme limit, $l = \infty$, the coefficient $A(l, E)$ approaches zero as l^2 and $B(l, E)$ goes to the constant value

$$B(l \rightarrow \infty, E) = \frac{4\pi(Z_T e)^2 B(E1)}{9\hbar v}. \quad (34)$$

Consequently, the potential behaves as r^{-3} in this limit. This behavior is shared by the l -independent potential of ref. [1] and our l -independent polarization potential.

In fig. (2), we show $U_l(r)$ as a function of r , at $E = 50$ MeV, for several angular momentum values. Except for a few partial waves near the grazing angular momentum, the potential is dominated by the term $B(l, E)/r^3$. For this reason, it changes very little with l , even for very large values ($l \sim 500$). This fact suggests the approximate formula

$$U_l(r) \simeq -i \left(\frac{4\pi\mu Z_T^2 e^2 B(E1)}{9\hbar^2 k} \right) \frac{1}{r^3}. \quad (35)$$

In fig. (3), we compare our potential of eq. (22), for two angular momentum values, with the l -independent potential of eq. (27) and with the phase-shift-equivalent polarization potential of ref. [1] (eq. (28)). The comparison is made at $E_{CM} = 50$ MeV. We see that these potentials are very similar, the main qualitative difference being the longer range of the $l = 50$ potential, arising from the stronger r^{-2} term in eq. (13).

B. Angular distributions

To calculate angular distributions, we must adopt a “background” optical potential. For this purpose, we use a $^9\text{Li} + ^{208}\text{Pb}$ optical potential, $V^b(r)$, arising from the interaction of the target with the projectile core, and add a contribution from the neutron halo (only real part), $V^h(r)$.

Our choice of $V^b(r)$ was based on the study of Robson et al [17], involving collisions of ^{12}C and ^{16}O projectiles on ^{208}Pb targets, over a similar E/A range. These authors used fixed radius and difuseness parameters (the same for both real and imaginary parts) and depths in the range $45 < |U_0(\text{MeV})| < 60$; $20 < |W_0(\text{MeV})| < 30$. They obtained reasonable fits for both projectiles, in a wide energy range above the threshold anomaly.

For the contribution from the neutron halo, we adopted the tail of the folding potential of ref. [1], extrapolated to smaller separations by a Woods-Saxon shape.

Our optical potential can then be written

$$V^{\text{opt}}(r) = \frac{V_o^b}{1 + \exp\left(\frac{r-R^b}{a^b}\right)} + \frac{V_o^h}{1 + \exp\left(\frac{r-R^h}{a^h}\right)}, \quad (36)$$

with

$$\begin{aligned}
V_o^b &= -(50 + 25 i) \text{ MeV} & V_o^h &= -4.28 \text{ MeV} \\
R^b &= 1.256 (208^{1/3} + 9^{1/3}) \text{ fm} & R^h &= 10.6 \text{ fm} \\
a^b &= 0.56 \text{ fm} & a^h &= 2 \text{ fm} .
\end{aligned} \tag{37}$$

The result of our calculation is shown in fig. 4. It is clear that the effect of the Coulomb dipole polarization potential is quite drastic, at the low CM energy considered. Our conclusions are therefore qualitatively similar to those of ref. [1]. We differ from these authors in the details of $V^{\sigma p'}$. This, however, is not relevant for the exploratory study presented here.

We mention in passing that the damping in the angular distribution attributed to the Coulomb dipole polarizability, can be worked out in closed form, using the semiclassical formula

$$\frac{d\sigma}{d\sigma^{\sigma p'}} = \exp \left[-\frac{2}{E} \int_{\rho_0}^{\infty} \frac{\text{Im} \{U_i(\rho/k)\}}{\sqrt{1 - 2\eta/\rho - \hat{l}^2/\rho^2}} d\rho \right], \tag{38}$$

where $\rho \equiv kr$, $\rho_0 \equiv \rho_0/k$ is the classical turning point for a pure Rutherford trajectory, $\hat{l} \equiv l + 1/2$ and $\sigma^{\sigma p'}$ is the elastic cross-section in the absence of the polarization potential. Taking $U_i(r)$ from eq. (22), we obtain by a straightforward evaluation of the integral in eq. (38)

$$\frac{d\sigma}{d\sigma^{\sigma p'}} = \exp \left[-\frac{2R_B^2 k^2 A(l, E)}{\eta E} \right]. \tag{39}$$

where $A(l, E)$ is given in eq. (23).

From the Rutherford relation $l/\eta = \cot(\theta/2)$, we then have (ignoring the factor $|S_i^{\sigma p'}|$ for simplicity),

$$\frac{d\sigma}{d\sigma^{\sigma p'}} = \exp \left[-\frac{16\pi k^2 B(E1)}{Z_p^2 e^2} \sin^2(\theta/2) \right] \tag{40}$$

For the system $^{11}\text{Li} + ^{208}\text{Pb}$ at $E_{CM} = 50$ MeV, and taking for $B(E1) \dagger = 2.25 \text{ fm}^2 e^2$, we find

$$\frac{d\sigma}{d\sigma^{\sigma p'}} = \exp [-35 \sin^2(\theta/2)] \tag{41}$$

This function drops very rapidly as θ is increased, just as Fig. 4 shows.

VII. CONCLUSIONS

In this paper, we have evaluated the dynamic polarization potential in the elastic channel arising from the Coulomb coupling to the “soft” dipole excitation in neutron-rich nuclei. It is found that when the soft mode is treated as the continuum (direct Coulomb break-up) the strength of the polarization potential is proportional to the pure cluster-model value $B_{cl}(E1) \dagger$. In contrast, when the dipole mode is treated as an excited state, then the strength is proportional to the $B(E1) \dagger$ value associated with the particular model used to generate the state.

The polarization potentials were found to be absorptive and to depend on angular momentum and energy. We also constructed l -independent versions of our wave-function-equivalent polarization potentials, using the semiclassical approximation. These potentials turned out to have a different radial dependence when compared with the phase-shift equivalent potential of ref. [1]. The effect of the potential on the elastic scattering of ^{11}Li from ^{208}Pb at $E_{CM} = 50$ MeV was then calculated and shown to lead to very strong damping of the elastic cross-section.

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FIGURES

FIG. 1. The coefficients $A(l, E)$ (a) and $B(l, E)$ (b), as functions of $\bar{l}$, for three CM energies. The system considered is $^{16}\text{O} + ^{208}\text{Pb}$. See text for further details.

FIG. 2. The l -dependent polarization potential at $E_{CM} = 50$ MeV, for several angular momentum values. See text for details.

FIG. 3. The r -dependence of the l -dependent dipole polarization potential for two values of l . Also shown are the equivalent l -independent potential and the phase-shift-equivalent potential of ref. [1]

FIG. 4. The effect of the Coulomb dipole polarizability on the elastic scattering angular distribution of $^{16}\text{O} + ^{208}\text{Pb}$ at $E_{CM} = 50$ MeV. See text for details.

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Fig. 1a

Fig. 1b

Fig. 2

Fig. 3

Fig. 4