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Coupled-Channel Effects on Heavy-Ion Sub-Barrier Fusion Within The Doorway Expansion Method*

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Coupled-channel effects on the heavy-ion sub-barrier fusion cross section σ_F are investigated with the Doorway Expansion Method. The case of two coupled channels is considered in detail. It is found that the two-doorway approximation, which leads to a simple result for σ_F , is in excellent agreement with exact coupled-channel calculations. The contributions from the elastic and inelastic channels to the enhanced fusion cross section are discussed.

I. INTRODUCTION

It is by now well established that one-dimensional penetration models substantially underestimate the sub-barrier heavy-ion fusion cross-section for the intrinsic states of the collision partners and perform a restricted coupled-channel (CC) calculation [2,5]. The success of this method is well documented.

The purpose of the present paper is to supply another form of the CC method that is more appropriate in dealing with problems involving a large number of channels. This method, the Doorway Expansion Method (DEM), has been previously employed with great success to study pion-nucleus scattering at intermediate energy [6] and to treat one-channel optical model potential scattering problems [7]. Very recently, the DEM has been extended to coupled-channel problems and applied to heavy-ion collisions [8]. It was found that the convergence of the method in such strongly absorbing systems is very fast. Indeed, the inclusion of just two doorways was sufficient for a reasonable description of the elastic and inelastic cross sections.

It is certainly important to test the method in the case of sub-barrier fusion, where the cross-section becomes small and CC effects are very important [1,2].

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II. THE DEM IN FUSION REACTIONS

In this section we briefly review the main results of Ref. [8] and extend their results to the fusion cross section.

A. The Doorway Expansion Method

We use a basis of doorways and solve the Lippmann-Schwinger equation for the transition operator T in a subspace spanned by this basis. If one writes the potential U as a product of two factors v and $\bar{v}$, this equation takes a convenient form. It can be written as a function of the rescattering operator W , defined as

$$W = \bar{v} G^{(+)} v, \quad (1)$$

where $G^{(+)}$ is the free particle Green's function. The choice of v and $\bar{v}$ depends on the particular scattering problem considered. In first place, we define the entrance channel doorway and its dual space counterpart

$$|D^{(0)}\rangle = N_0 \bar{v} |k\rangle, \quad |\tilde{D}^{(0)}\rangle = \tilde{N}_0 v^\dagger |k\rangle, \quad (2)$$

where the incident plane waves $|k\rangle$ are the eigenstates of H_0 (the free particle Hamiltonian). The factor N_0 can be chosen arbitrarily while $\tilde{N}_0$ is determined from the normalization condition: $\langle \tilde{D}^{(0)} | D^{(0)} \rangle = 1$. The remaining elements of the orthonormal doorway basis are defined through a recursion relation, as in ref [7]. In terms of the doorway basis, the T -operator can be written as

$$T = \sum_{nm} v |D^{(n)}\rangle \Gamma^{(n,m)} \langle \tilde{D}^{(m)} | \bar{v}, \quad (3)$$

where $\Gamma^{(n,m)} = \langle \tilde{D}^{(n)} | (1 - W)^{-1} | D^{(m)} \rangle$. This basis has the convenient property of making the rescattering matrix $W^{(n,m)}$ tridiagonal. So, e.g., the matrix element $\Gamma^{(0,0)}$ takes the form

$$\begin{aligned} \Gamma^{(0,0)} &\equiv \left(N_0 \tilde{N}_0^* \right) \langle k | T | k \rangle \\ &= \frac{1}{1 - W^{(0,0)} - \frac{W^{(0,1)} W^{(1,0)}}{1 - W^{(1,1)} - \frac{W^{(1,2)} W^{(2,1)}}{1 - W^{(2,2)} \dots}}}. \end{aligned} \quad (4)$$

It is frequently convenient to split the total potential U as a sum of the optical potential V (containing the long range Coulomb interaction) and the channel coupling term ΔV , and then include V in the hamiltonian H_0 . In this way, the Green's function $G^{(+)}$ in Eq. (1) becomes distorted by the optical potential. One now factorizes ΔV , choosing $\bar{v} = 1$ and $v = \Delta V$. The entrance channel doorway states are then defined as in Eq. (2), but with the replacement of the plane waves $|k\rangle$ by the distorted waves $|\chi_k^{(+)}\rangle$ ($|\chi_k^{(-)}\rangle$). In this way, the transition operator can be put in the form

$$T = \bar{T} + \Delta T, \quad (5)$$

where $\bar{T}$ is the optical T -operator and ΔT is associated with ΔV . It is convenient to write the coupling interaction in terms of its components in the intrinsic space as

$$\Delta V = \sum_{\alpha\beta} |\alpha\rangle v_{\alpha\beta} \langle\beta|, \quad (6)$$

where $v_{\alpha\beta} = \langle\alpha|v|\beta\rangle$, with round brackets representing scalar products in the intrinsic space. The matrix-elements of the transition operator ΔT can now be written

$$\Delta T_{(k_\alpha, \alpha \leftarrow k_0, 0)} = \sum_{n\beta} \frac{\Gamma^{(n,0)}}{N_0} \left\langle \chi_{k_\alpha}^{(-)} | v_{\alpha\beta} | D_\beta^{(n)} \right\rangle. \quad (7)$$

Above, k_α is the wave vector in channel- α and $|D_\beta^{(n)}\rangle$ stands for the projection of the doorway $|D^{(n)}\rangle$ onto channel- β .

Carrying out partial-wave expansions and considering the simple case of two channels (denoted $|0\rangle$ and $|1\rangle$) and a scalar coupling (this simplifications will be henceforth assumed), Eq. (7) takes the form

$$\Delta T_l(\alpha \leftarrow 0) = \frac{2}{\pi} e^{i(\sigma_{\alpha,l} + \sigma_{0,l})} \sum_{n\beta} \frac{\Gamma_l^{(n,0)}}{N_0} \langle \chi_l^\dagger(k_\alpha) | v_{\alpha\beta} | d_{\beta,l}^{(n)} \rangle . \quad (8)$$

Above, $\sigma_{\alpha,l}$ stands for the Coulomb phase-shift in channel- α , $|d_{\beta,l}^{(n)}\rangle$ is the l -projection of $|D_\beta^{(n)}\rangle$ and α and β take the values 0 and 1.

For the calculation of fusion cross sections it is convenient to write explicit expressions for the l -components of the S-matrix. The elastic S-matrix elements can be written

$$S_l^{(0)} = \bar{S}_l^{(0)} + \Delta S_l^{(0)} , \quad (9)$$

where

$$\bar{S}_l^{(0)} = 1 - i \left(\frac{2\pi\mu}{\hbar^2 k_0} \right) e^{-2i\sigma_{0,l}} \bar{T}_l \quad (10)$$

and

$$\Delta S_l^{(0)} = -i \left(\frac{4\mu}{\hbar^2 k_0} \right) \sum_n \frac{\Gamma_l^{(n,0)}}{N_0} \langle \chi_l^\dagger(k_0) | v_{01} | d_{1,l}^{(n)} \rangle , \quad (11)$$

where μ is the reduced mass. The inelastic S-matrix elements are

$$S_l^{(1)} = -i \left(\frac{4\mu}{\hbar^2 \sqrt{k_0 k_1}} \right) \sum_n \frac{\Gamma_l^{(n,0)}}{N_0} \langle \chi_l^\dagger(k_1) | v_{10} | d_{0,l}^{(n)} \rangle . \quad (12)$$

B. The Fusion Cross Section

The fusion cross section, σ_F , is given by the difference

$$\sigma_F = \sigma_R - \sigma_{inel} , \quad (13)$$

where the reaction and the inelastic cross sections are

$$\sigma_R = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) (1 - |S_l^{(0)}|^2) , \quad (14)$$

$$\sigma_{inel} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) |S_l^{(1)}|^2 . \quad (15)$$

Using the above equations together with those giving the elastic and inelastic components of the S-matrix (Eqs. (9–12)), one evaluates σ_F .

The calculation of the fusion cross section becomes particularly simple in the two-doorway approximation. In this case, each doorway has non-vanishing component in only a channel. The doorways $|D^{(0)}\rangle$ and $|D^{(1)}\rangle$ are fully contained in the channels 0 and 1, respectively. Consequently, Eqs. (11) and (12) reduces to

$$\Delta S_l^{(0)} = -i \left(\frac{4\mu}{\hbar^2 k_0} \right) \Gamma_l^{(0,0)} \langle \chi_l^\dagger(k_0) | v_{01} g_{1,l} v_{10} | \chi_l(k_0) \rangle \quad (16)$$

$$S_l^{(1)} = -i \left(\frac{4\mu}{\hbar^2 \sqrt{k_0 k_1}} \right) \Gamma_l^{(0,0)} \langle \chi_l^\dagger(k_1) | v_{10} | \chi_l(k_0) \rangle , \quad (17)$$

where

$$\Gamma^{(0,0)} = \frac{1}{1 - (N_1 \tilde{N}_1^*)^{-1}} , \quad (18)$$

with

$$(N_1 \tilde{N}_1^*)^{-1} = \frac{\langle \chi_l^\dagger(k_1) | v_{10} g_{0,l} v_{01} g_{1,l} v_{10} | \chi_l(k_0) \rangle}{\langle \chi_l^\dagger(k_1) | v_{10} | \chi_l(k_0) \rangle} . \quad (19)$$

Above, $g_{\alpha,l}$ are the partial-wave components of the optical Green's function in channel α .

III. NUMERICAL RESULTS AND DISCUSSION

In this section we consider the simple coupled-channel problem studied in Ref. [8] and investigate the effects of the coupling on the fusion cross section. The intrinsic degree of freedom, x , describes monopole vibrations with frequency ω and oscillator length b in the target nucleus. We take into account the channels 0 and 1, corresponding to intrinsic states with 0 and 1 phonon, respectively. The coupling interaction is obtained through the use of an oscillating target radius

$$R_T \rightarrow R_T(x) = R_T [1 + x (b/R_T)] , \quad (20)$$

in the construction of the optical potential. Above, the dimensionless intrinsic coordinate x is normalized with respect to the oscillator length b . With the target vibration, the optical potential radius R become x -dependent, in the form

$$R(x) = R_P + R_T(x) . \quad (21)$$

The x -dependence of the optical potential leads to the coupling between the elastic and the excited channels.

For small oscillations, the total potential U can be approximated as

$$U(r, x) \simeq U(r, 0) + \left. \frac{\partial U}{\partial R_T} \right|_{x=0} b x . \quad (22)$$

We then choose the following separation

$$V(r) = U(r, 0) ; \quad \Delta V(r, x) = \left. \frac{\partial U}{\partial R_T} \right|_{x=0} b x . \quad (23)$$

Above, V is diagonal in channel space and ΔV is the channel-coupling term.

In this schematic study, the choice of monopole vibration channels was based on the simple scalar nature of the coupling, rather than on the phenomenological aspects of the nuclear system. Therefore, the reaction Q -value and the oscillator length b , which gives the measure of the coupling strength, are regarded as arbitrary. The results below are obtained for collisions of ^{16}O projectiles on ^{28}Si targets, with the strong absorption optical potential of Ref. [9]. Under these circumstances, the Coulomb barrier is located at $E_{lab} \simeq 28$ MeV.

In Figure 1, we compare the DEM fusion cross section with that obtained with the standard CC code FRESKO [9], for collision energies in the range $E_{lab} = 20 - 38$ MeV. The calculation was made for $b = 1$ fm and $Q = 0$. These parameters lead to appreciable CC effects in the fusion cross section. The DEM calculations were performed with 6 doorways, which guaranteed very good convergence. The comparison indicates that the DEM fusion cross section is very accurate.

In Figure 2, we illustrate the enhancement of the DEM fusion cross section, comparing it with that predicted by the optical model, σ_F^{opt} . We adopted the same b as in Figure 1 and show cross sections in the same energy range. The calculations were made with (a) $Q = 5$ MeV and (b) $Q = 0$. In both cases, the sub-barrier cross section is appreciably enhanced. For $Q = 0$, the enhancement reaches almost one order of magnitude.

In Figure 3, we study the accuracy of the 2-doorway approximation to the fusion cross section, comparing it to the corresponding full CC result. It is very accurate at low energies, ranging from below the Coulomb barrier up to $E \simeq 30$ MeV. At higher energies, the approximation is poorer, although it remains quite reasonable.

We now study the contributions from the elastic and inelastic channels to the enhanced sub-barrier fusion cross section. For this purpose, we neglect the imaginary part of the off-diagonal channel coupling interaction, which is frequently taken to be real. In this case, the fusion cross section gets separate contributions from each channel, in the form

$$\sigma_F = \sigma_F^{(0)} + \sigma_F^{(1)} \quad (24)$$

with

$$\sigma_F^{(0)} = -\frac{k}{E} \langle \Psi_0^{(+)} | \text{Im}\{U_{00}\} | \Psi_0^{(+)} \rangle \quad (25)$$

and

$$\sigma_F^{(1)} = -\frac{k}{E} \langle \Psi_1^{(+)} | \text{Im}\{U_{11}\} | \Psi_1^{(+)} \rangle . \quad (26)$$

Above, $\Psi_i^{(+)}$ ($i = 0$ or 1) are the exact wave functions and U_{ii} are the diagonal matrix elements of the total potential.

In Figure 4, we show the partial wave components of the optical ($\sigma_{F,\ell}^{opt}$) and the CC ($\sigma_{F,\ell}$) fusion cross sections. The decomposition of the CC cross section according to Eqs. (24–26) leads to the elastic ($\sigma_{F,\ell}^{(0)}$) and inelastic ($\sigma_{F,\ell}^{(1)}$) contributions, which are also shown. The elastic channel contribution is slightly larger than the corresponding optical model cross section. However, most of the enhancement of $\sigma_{F,\ell}$ comes from the inelastic term, originated from channel coupling.

It should be remarked that the calculation of the fusion cross section through Eqs. (24–26) requires a larger number of doorways, since the convergence in the calculation of wave functions is slower. Although the expression of the previous section for σ_F converges with 2 doorways, at least 8 doorways are needed in Eqs. (24–26).

IV. CONCLUSIONS

In this paper, the Doorway Expansion Method has been applied to the study of coupled-channel effects on sub-barrier heavy-ion fusion. It is found that for such strong absorption systems the doorway expansion converges very rapidly. In the case of two coupled channels, discussed in details in the present paper, it suffices to take into account two doorways. This allows the construction of a simple formula which contains the main features of the coupled-channel effects. The effect of the coupling to a resonant state, recently studied within a schematic model [10], can also be treated exactly with the DEM. The extension to a larger number of coupled channels and application to data analysis will be reported in a later paper.

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Figure Captions

- Fig. 1:** Comparison between the CC fusion cross sections obtained with the DEM and with the FRESCO code [9].
- Fig. 2:** Coupled-channel effects in the fusion cross section. For details see the text.
- Fig. 3:** The 2-doorway approximation to the DEM fusion cross section.
- Fig. 4:** Channel contributions to the enhanced fusion cross section.

Fig. 1

Fig. 2 a

Fig. 2b

Fig. 3

Fig. 4