OPTICAL MODEL ANALYSIS OF PARITY-NONCONSERVING NEUTRON SCATTERING AT EPITHERMAL ENERGIES

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ABSTRACT

The parity non-conserving (PNC) resonant scattering of neutrons at epithermal energies is discussed within the optical model. We show that the empirical optical model potential with real PNC interaction does not account for the appreciable resonance-averaged difference between positive and negative helicity total \( p \)-neutron cross-sections \( \langle \Delta \sigma \rangle \), for \( ^{232}\text{Th} \), but reproduces the very small \( \langle \Delta \sigma \rangle \) for \( ^{238}\text{U} \). It is suggested that the \( 2p - 1h \) local doorway, previously proposed to account for the sign correlation in \( ^{232}\text{Th} \), is the main source for the large \( \langle \Delta \sigma \rangle \) seen in this system. It is further proposed that if an optical potential is used to account for \( \langle \Delta \sigma \rangle \), a complex PNC interaction is required in cases involving strong sign correlation.

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In a recent paper [1], we proposed a model that explains the sign correlation seen in the data on parity non-conserving neutron scattering from $^{232}$Th at epithermal energies [2]. Our model emphasizes the role of a local $2p - 1h$ doorway state which is responsible for the weak coupling between the $p$ and $s$ waves. In $^{232}$Th, this doorway state is reasonably well separated from other doorways, whereas in $^{238}$U, where no sign correlation is seen [2], there are several $2p - 1h$ doorways present in the energy range of interest. In this paper we address another question also relevant to the parity non-conserving (PNC) neutron reaction, namely to what extent the averaged total cross-section difference $⟨\Delta \sigma_q⟩$ is accounted for by an optical model which contains an appropriate one-body PNC. The quantity $\Delta \sigma_q$ is just $\sigma_{q}^{(+)} - \sigma_{q}^{(-)}$, where $\sigma_q$ is the cross-section at the $q^{th}$ $p$-resonance and the superscripts $+(-)$ refer to positive (negative) helicity neutrons.

In the following we give arguments that indicates that the same local $2p - 1h$ doorway responsible for the large positive value of the resonance averaged longitudinal asymmetry in $n + ^{232}$Th (the sign correlation effect), also has bearing on the optical model analysis. We should mention here that our local $2p - 1h$ doorway states are statistical in nature, in contrast to the collective $\mathcal{O}^-$ doorway (giant monopole) state considered in Refs. [3] and [4] and to the high-lying single particle “doorway” of Ref. [5]. Both of these alternative doorway-type approaches to the sign correlation problem require an unrealistically large parity violating matrix element about two orders of magnitude larger than the normal ($\sim 1$ eV).

Two optical model calculation related to the TRIPLE [2] data have been reported [6] and [7]. These calculations purported to analyze the resonance-averaged longitudinal analyzing power

$$\langle P_q \rangle_q = \frac{\Delta \sigma_q}{\sigma_q^{(+) + \sigma_q^{(-)}}}$$

It was concluded in both of these references that the one-body PNC part of the optical potential has to be more than two orders of magnitude larger than usual estimates in order to reproduce $\langle P_q \rangle_q$ [6] and [7].

From the optical model point of view, the quantity $\langle P_q \rangle_q$ is rather cumbersome to analyze. A more natural quantity to discuss is the ratio of energy averages

$$P_{opt} = \frac{⟨Δσ⟩_E}{2⟨σ⟩_E} \equiv \frac{Δσ_{opt}}{2σ_{opt}}$$

which will behave differently from (1), where $⟨\cdots⟩_E$ implies energy average. It is a simple matter to show that

$$σ_{opt} = σ_0 + \frac{πΓ}{2D}⟨σ⟩_q ,$$

and therefore

$$\frac{2D}{πΓ} Δσ_{opt} = ⟨Δσ⟩_q + \frac{2D}{πΓ} Δσ_0 ,$$

where $⟨Δσ_q⟩_q$ is the cross-section difference at the peak of the resonance ($q$), averaged over $q$, and $D$ and $Γ$ are the average spacing and width of the resonances. From the data of Ref. [2] we constructed $Δσ_q$, for $n + ^{232}$Th and $n + ^{238}$U, shown respectively in Figs. (1) and (2). The average, $⟨Δσ_q⟩_q$, over the resonances in the energy range $1 < E_0 < 400$ eV is $(85 \pm 12)$ mb for $n + ^{232}$Th, and $(18.2 \pm 18.7)$ mb for $n + ^{238}$U. In performing these averages, involving experimental points with error bars, we followed the procedure of Ref. [8]. The data points were scaled by $1/\sqrt{E}$. It is obvious from this analysis that $⟨Δσ_q⟩_q$ for $n + ^{238}$U is consistent with a zero value, whereas for $n + ^{232}$Th it
is appreciable. To understand this difference in behavior between the two rather similar systems we performed an optical model calculation following the procedure of Refs. [6] and [7].

We use the optical model potential employed in [7], which is appropriate in the actinide mass region. In view of Eq. (4) these optical model cross-section differences must be multiplied by the factor $(2D/\pi \Gamma)$, before comparing with the experimental $(\Delta \sigma_q)_q$ given above. The value of $(2D/\pi \Gamma)$ was found to be $297.1$ for $n+^{232}$Th and $197.6$ for $n+^{235}$U. For $^{232}$Th, we used $\Gamma_1 = \Gamma_\gamma = 24$ meV and $D_1 = 11.2$ eV. For $^{238}$U, we used $\Gamma_1 = \Gamma_\gamma = 23.2$ meV and $D_1 = 7.2$ eV (the notation $\Gamma_1$, $D_1$ means width and spacing for the p-wave ($l = 1$) resonances). With the exception of the $D_1$ value for $^{232}$Th, these numbers come from the book by Mughabghab on nuclear resonance parameters and thermal cross sections [9]. We obtained the value for $D_1$ in $^{232}$Th by averaging over the spacings of $^{232}$Th p-wave resonances below 100 eV. The value of $\epsilon_7$, which appears in the PNC piece of the optical model potential was set equal to unity. The parity conserving part of the optical model potential is that of Ref. [10], which is appropriate to the actinides.

The results for $\frac{2\pi}{\pi} \Delta \sigma_{opt}$ are shown in Figs. (3) and (4) for the two systems under discussion. The values of $\frac{2\pi}{\pi} \Delta \sigma_{opt}$ at $E_n = 1$ eV is 0.4 mb for $n+^{232}$Th and 0.68 mb for $n+^{238}$U at $E_n = 1$ eV. Therefore it is reasonable to conclude that the $n+^{238}$U system exhibits a “normal” behavior since its $(\Delta \sigma_q)_q$ is consistent with the optical value of 0.4 mb: namely PNC transitions whose average is zero and whose resonance background, $\Delta \sigma_0$, is basically the low-energy extrapolated optical model result. On the other hand, the $n+^{232}$Th system is “abnormal” in the sense that $(\Delta \sigma_q)_q = 85$ mb (see above) is more than two orders of magnitude larger than the “scaled” optical model result (scaled in the sense of the factor $2D/\pi \Gamma$). This is certainly related to the fact that $(P_q)_q$ is zero in $n+^{238}$U and relatively large in $n+^{232}$Th.

We further investigated the behavior of the two systems by examining the p-wave and s-wave optical transmission coefficients and compared them with the corresponding experimental values, namely $2\pi \frac{\Gamma_0,n}{D_0}$ and $2\pi \frac{\Gamma_1,n}{D_1}$ where $\Gamma_0,n(\Gamma_1,n)$ is the s-wave (p-wave) neutron width. The values of $\Gamma_0,n$ and $\Gamma_1,n$ for $^{232}$Th and $^{238}$U were taken from Refs. [2] and [9]. Again, the comparison showed the “abnormal” nature of $^{232}$Th when compared to $^{238}$U. The experimental values of $2\pi \frac{\Gamma_0,n}{D_0}$, averaged over the resonances, for $n+^{238}$Th are slightly lower ($\sim 20\%$) than the optical transmission coefficients whereas $2\pi \frac{\Gamma_1,n}{D_1}$ shows conspicuously larger values ($\sim 30\%$) than the optical transmission in the energy range, 100 eV $< E_n < 200$ eV. This behavior is not shared by $n+^{238}$U.

The existence of a local doorway that is required to explain the sign correlation has clearly important implications on the energy averaged cross-section. Within the energy range where the averaged cross-section is calculated we are assuming that there is only one doorway present, as we have emphasized earlier [1]. The contribution to the optical interaction that arises from the doorway is given [11] and [12] by

$$\Delta U_{\text{Doorway}}^{(\pm)} = \frac{V_{DP}(r)[V_{DP}(r') \pm V_{D\sigma}^{W}(r')]}{E - E_D + i\Gamma^i_D/2},$$

(5)

to first order in the weak force. Where $V_{DP}(r)$ is an appropriate form factor representing the doorway coupling to channel $i$. In (5) $\Gamma^i_D$ is the spreading width of the $2p - 1h$ doorway, due to its coupling to the compound nuclear states.

It is clear from Eq. (5) that if treated as an optical potential, $\Delta U_{\text{Doorway}}^{(\pm)}$ will contain a PNC part which is complex

$$\Delta U_{\text{Doorway}}^{\text{PNC}} = \frac{2V_{DP}(r)V_{D\sigma}^{W}(r')}{-E_D + i\Gamma^i_D/2},$$

(6)
In the optical model calculation of [7], whose results for $\langle \Delta \sigma \rangle$ are shown in Figs. (3) and (4), an empirical local energy-dependent optical potential [10], was used to generate distorted waves that are then employed to evaluate the first-order perturbation matrix element

$$ f_{\text{PNC}} \propto \text{Im}(\langle \Psi_s^- | U_{\text{PNC}}^\dagger | \Psi_p^+ \rangle) $$

where $U_{\text{PNC}}$ is taken to be the real one-body PNC interaction suggested long time ago by Michel [13]. It is clear from the discussion above that the reported OM calculation is incomplete, since, to say the least, $U_{\text{PNC}}$ must be complex. Further, the imaginary part to be used in a consistent optical model analysis of the TRIPLE data must arise entirely from the doorway (see Eq. (5)). It is, however, conventional to use an empirical complex potential to represent the parity conserving interaction since there are other doorways and, when used at higher energies the averaged potential will contain the contribution of many terms, of the type given in Eq. (5). Roughly speaking, the parity conserving (PC) imaginary part will then be given by

$$ \text{Im}\Delta U_{\text{Doorway}} = -2\pi V_{Dp}(r) V_{Dp}(r') \Gamma_D^\dagger \rho_D $$

where $\rho_D$ is the $2p - 1h$ density of states. It is usually assumed that when an equivalent local potential is constructed from Eq. (8) and extrapolated to low energies, it can be represented by the empirical imaginary potential. On the other hand the PNC interaction to be used must be complex, on account of the local doorway contribution. We believe that the difference between the optical model $\Delta \sigma$ using the real PNC interaction and the data resides in this fact.

We now use the above argument to further pin-down the nature of the local doorway state. We calculate the contribution of the single local doorway to the energy averaged cross-section. This is straightforward, since $\Delta U_D$ is separable. We find, taking for an "optical potential", a background real piece plus the parity conserving part of (6)

$$ \langle \Delta \sigma \rangle_E = \Delta \sigma_0 + \frac{2\pi}{k^2} \frac{\gamma_{Dp} \gamma_{Dp} \Gamma_D}{(E_D^2 + \Gamma_D^2/4)} $$

where $\Gamma_D$ is the total width of the doorway, $\Gamma_D = \Gamma_D^\dagger + \Gamma_D$. The escape width $\Gamma_D^\dagger$ accounts for the doorway decay to the open channels. We obtain from Eqs. (4) and (9) the following form for $\langle \Delta \sigma_q \rangle_q$

$$ \langle \Delta \sigma_q \rangle_q = \left( \frac{\gamma_{Dp}}{\gamma_{Dp}} \right)^2 \left( \frac{2D}{\pi^2} \right) \left( \frac{1}{k^2} \right) \left( \frac{\gamma_{Dp} \Gamma_D}{(E_D^2 + \Gamma_D^2/4)} \right) $$

Note that the factor $\frac{2D}{\pi^2}$ refers to the fine structure compound resonances, whose value for $n + ^{232}\text{Th}$ was found to be $\sim 300$. From Eq. (10) we find that $\langle \Delta \sigma_q \rangle_q$ is independent of energy. This is so since $\gamma_{Dp}^2 \propto (kR)^3$, and $\gamma_{Dp}^2 \propto k^{-1}$. We choose $E_n = 1 \text{ eV}$ to evaluate the RHS of (10). In Ref. [1], $\gamma_{Dp}^2/\gamma_{Dp}$ was identified with $\langle P_q \rangle_q$, which, at this energy, $\sim 0.08$. Using for $\langle \Delta \sigma_q \rangle_q$ the value cited earlier, namely $85 \text{mb}$, we find the following numerical value for the doorway factor inside the square brackets,

$$ \left[ \frac{2\pi \gamma_{Dp}^2 \Gamma_D}{(E_D^2 + \Gamma_D^2/4)} \right] \approx 1.7 \times 10^{-7} $$

As in [1] we take $E_D \sim \Gamma_D = 30 \text{ KeV}$, which then gives for the partial $p$-wave neutron width of the local $2p - 1h$ doorway, $2\pi \gamma_{Dp}^2 \approx 6.3 \text{ meV}$, several orders of magnitude larger than the average.
width of the compound $p$-wave neutron resonances, quite consistent with our doorway picture. At this point it is important to remind the reader of an interesting relation involving the doorway strength function and that of the compound nucleus [12]. This relation reads

$$2\pi \frac{\Gamma_D}{D_D} = 2\pi \left\langle \frac{\Gamma_q}{D_q} \right\rangle_q.$$  \hspace{1cm} (12)

For $p$-resonances in Th, $\left\langle \frac{\Gamma_q}{D_q} \right\rangle_q$, in the energy range of interest, is $2.1 \times 10^{-3}$. In Ref[1] we have calculated $D_{DP} \equiv 1/\rho_{DP}$ which came out to be about 30 KeV. Therefore we get the following value for the average escape width of the $p$-wave $2p - 1h$ doorway, $\Gamma_{DP}^+ \sim 63$ eV. Within our model, the value of the total width of the doorway is assumed to be $\Gamma = 30$ KeV. Thus, most of this width is spreading.

To summarize, the local $p$-wave doorway state argued in Ref. [1] to be responsible for the sign correlation in the $n+^{232}$Th longitudinal asymmetry, is further investigated in the present paper through an analysis of the energy average cross-section difference $\langle \Delta \sigma \rangle$. The $2p - 1h$ doorway state is found to have a $p$-wave width of about 6 meV, several order of magnitude larger than the compound nucleus $p$-wave neutron widths. However, the doorway state in question is completely damped ($\Gamma_D \sim \Gamma_D^+$). Finally if an optical model potential is used to account for $\langle \Delta \sigma_q \rangle_q$, the PNC piece of this potential must be necessarily complex due to the presence of the doorway. We conjecture that the absence of the sign correlation and the optical model anomaly in $^{238}$U and other nuclei stems from the possibility of having two or more doorways, in the low energy interval of interest, as we have emphasized in [1].

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REFERENCES


FIGURE CAPTIONS

Figure 1. The experimental $\Delta \sigma_{p1/2}$ extracted from Frankle et al. [1], for $n+^{232}$Th.

Figure 2. Same as Fig. 1 for $n+^{238}$U (Zhu et al. [1]).

Figure 3. The scaled optical model cross-section difference $\frac{am^2}{4\pi} \Delta \sigma_{opt}$ for $n+^{232}$Th. See text and Ref. [7] for details.

Figure 4. Same as Fig. 3 for $n+^{238}$U.