HEAVY QUARK PHOTOPRODUCTION IN THE SEMIHARD APPROACH AT HERA AND BEYOND

V.A. Saleev
Samara State University, Samara 443011, Russia

N.P. Zotov
Nuclear Physics Institute, Moscow State University, Moscow 119899, Russia

Abstract

Processes of heavy quark photoproduction at HERA energies and beyond are investigated using the semihard (\( k_\perp \) factorization) approach. Virtuality and longitudinal polarization of gluons in the photon - gluon subprocess as well as the saturation effects in the gluon distribution function at small \( x \) have been taken into account. The total cross sections, rapidity and \( p_\perp \) distributions of the charm and beauty quark photoproduction have been calculated. The results are compared with ZEUS experimental data for charm photoproduction cross section.
1 Introduction

The heavy quark electro- and photoproduction has become an increasingly important subject of study [1, 2, 3]. In particular it originates from come into operation of HERA ep-collider energy of which is high enough to achieve the small $x$ physics region. Indeed first experimental results obtained at HERA collider energy of which is high enough to achieve the small $x$ physics region. Indeed recent experimental results obtained at HERA energies resolved photon interactions, jets in photoproduction produced in 'hard parton collisions, significant fraction of events with a large rapidity gap. The measurement of the total charm photoproduction cross section by ZEUS collaboration [5] is a very important stage of researches.

The heavy quark production at HERA is a very interesting and important subject of study. As it is dominated by photon-gluon fusion subprocess (Fig. 1) one can study the gluon distribution functions $G(x, Q^2)$ in small $x$ region (roughly at $x > 10^{-4}$). Secondly this last issue is important for physics at future colliders (such as LHC): many processes at these colliders will be determined by small $x$ gluon distributions.

At HERA energies and beyond heavy quark production processes are of the so-called semihard type [6]. In such processes by definition a hard scattering scale $Q$ (or heavy quark mass $M$) is large as compared to the $\Lambda_{QCD}$ parameter, but $Q$ is much less than the total center of mass energies: $\Lambda_{QCD} \ll Q \ll \sqrt{s}$. Last condition implies that the processes occur in small $x$ region: $x \approx M^2/s \ll 1$. In such a case the perturbative QCD expansion has large coefficients $O(\ln^n(s/M^2)) \sim O(\ln^n(1/x))$ besides the usual renormalization group ones which are $O(\ln^n(Q^2/\Lambda^2))$. So, in perturbative QCD the heavy quark photoproduction cross section (as result of photon-gluon fusion process) has the form [7]: $\sigma_{\gamma g} = \sigma_{\gamma g}^0 + \alpha_s \sigma_{\gamma g}^1 + ...$, where $\sigma_{\gamma g}^1$ was calculated by Ellis and Nason [8]. The photon - gluon fusion cross section in low order decreases vs. $s$ at $s \to \infty$: $\sigma_{\gamma g}^0 \sim M^2/s \ln(s/Q^2)$. But in the same limit $\sigma_{\gamma g}^1 \to Const$, because heavy quark photoproduction cross section is dominated by the contribution of the gluon exchange in the $t$ - channel. These results in breakdown of standard perturbative QCD expansion and the problem of summing up all contributions of the order $(\alpha_s \ln(Q^2/\Lambda^2))^n, (\alpha_s \ln(1/x))^n$ and $(\alpha_s \ln(Q^2/\Lambda^2) \ln(1/x))^n$ in perturbative QCD appears in calculation of $\sigma_{\gamma g}$.

It is known that summing up the terms of the order $(\alpha_s \ln(Q^2/M^2))^n$ in leading logarithm approximation (LLA) of perturbative QCD leads to the linear DGLAP evolution equation for deep inelastic structure function [9]. Resummation of the large contributions of the order of $(\alpha_s \ln(1/x))^n$ leads to the BFKL evolution equation [10] and its solution gives the drastical increase of the gluon distribution: $xG(x, Q^2) \sim x^{-\omega_0}, \omega_0 = (4N_c \ln 2/\pi)\alpha_s(Q_0^2)$. The sharp growth of the parton density as $x \to 0$ makes the parton-parton interactions very important, which in turn makes the so-called GLR evolution equation essentially nonlinear [6].

The growth of the parton density at $x \to 0$ and interactions between par-
tons induce substantial screening (shadowing) corrections which restore the unitarity constrains for deep inelastic structure functions (in particular for a gluon distribution) in small \(x\) region [11]. These facts break the assumption of the standard parton model (SPM) about \(x\) and transverse momentum factorization for a parton distribution functions. We should deal with the transverse momentum factorization (\(k_t\) factorization) theory [12, 13] or with the so-called semihard approach [11] in case of accounting for shadowing corrections at small \(x\) besides taking into account the virtuality and the longitudinal polarizations of initial gluons.

In semihard approach [6, 11] screening corrections stop the growth of the gluon distribution at \(x \to 0\). This effect was interpreted as saturation of the parton (gluon) density: gluon distribution function \(xG(x, Q^2)\) becomes proportional to \(Q^2R^2\) at \(Q^2 \leq q_0^2(x)\) and \(\sigma \sim \frac{1}{x} xG(x, Q^2) \sim R^2\). The parameters \(R\) and \(q_0^2(x)\) can be considered as new phenomenological parameters: the \(R\) is related to the size of hadron or of black spots in a hadron and the parameter \(q_0^2\) is a typical transverse momentum of partons in the parton cascade of a hadron, which leads to natural infrared cutoff in semihard processes. The parameter \(q_0^2(x)\) increases with \(s\) [11, 14]. So because of the high parton density in small \(x\) region a standard methods of perturbative QCD can not be used [6, 11, 14, 15]. In this point the semihard approach [6, 11] differs from the works [12, 13, 16] where after choice of structure function at starting value \(Q_0^2\) the QCD evolution is calculated using certain equations.

In the region where the transverse mass of heavy quark \(M_\perp = \sqrt{M^2 + p_\perp^2} \ll \sqrt{s}\) one need to take into account the dependence of the photon-gluon fusion cross section on the virtuality and polarizations of initial gluon. Thus in semihard approach the matrix elements of this subprocess differ from the ones of SPM (see for example [7, 13]).

As for the SPM calculations of the next-to-leading (NLO) cross section of the photoproduction of heavy quarks the review may be found in papers [2, 17]. The results of [8] have been confirmed by Smith and Van Neerven [18]. The further results for the electro- and photoproduction of heavy quarks are obtained in Refs. [19, 20, 21]. Authors of Ref. [17] note that for beauty quark production the NLO corrections are large and various estimates of corrections lead to theoretical uncertainties of the order of a factor of 2 to 3. As it was estimated in [22] the total cross section for beauty quark production at HERA will be only a few tens of percent large than the one-loop results [8].

For charm quark theoretical uncertainties are even higher. It is known that there is also strong mass dependence of the results of calculations for charm quark production. Since the mass of the charm quark is small for perturbative QCD calculations the resumme procedure [12] is less reliable for charm quark production [22].

In Refs. [23, 24] we used the semihard approach to calculate the total and differential cross sections of the heavy quarkonium, \(J/\Psi\) and \(\Upsilon\), photoproduc-
tion. We obtained the remarkable difference between the predictions of the semihard approach and the SPM especially for $p_\perp$- and $z$-distributions of the $J/\Psi$ mesons at HERA.

In present paper we investigate the open heavy quark photoproduction processes in the semihard approach, which was used early in Ref. [11] for calculation of heavy quark production rates at hadron colliders and for prediction of $J/\Psi$ and $\Upsilon$ photoproduction cross sections at high energies in Refs. [23, 24].

Some of the results derived in this paper have already been presented in Ref. [25].

2 QCD Cross Section for Heavy Quark Electroproduction

We calculate the total and differential cross sections (the $p_\perp$ and rapidity distributions) of charm and beauty quark photoproduction via the photon-gluon fusion QCD subprocess (Fig.1) in the framework As for the SPM calculations of the next-to-leading (NLO) cross section of the semihard ($k_\perp$ factorization) approach [6, 11]. First of all we take into account the transverse momentum of gluon $q_\perp$, its the virtuality $q^2_\perp = -q_\perp^2$, and the alignment of its polarization vectors along its transverse momentum such as $\epsilon_\mu = q_{2\perp} / |q_{2\perp}|$ [11, 13].

Let us define Sudakov variables of the process $ep \rightarrow QQX$ (Fig.2):

$$
\begin{align*}
p_1 &= \alpha_1 P_1 + \beta_1 P_2 + p_{1\perp}, \\
p_2 &= \alpha_2 P_1 + \beta_2 P_2 + p_{2\perp}, \\
q_1 &= x_1 P_1 + q_{1\perp}, \\
q_2 &= x_2 P_2 + q_{2\perp}
\end{align*}
$$

(1)

where

$$
p_{1\perp} = p_{2\perp} = M^2, \\
q_{1\perp} = q_{2\perp}, \\
q_1^2 = q_2^2, \\
q_2^2 = q_{2\perp}^2,
$$

$p_1$ and $p_2$ are 4-momenta of the heavy quarks, $q_1$ is 4-momentum of the photon, $q_2$ is 4-momentum of the gluon, $p_{1\perp}$, $p_{2\perp}$, $q_{1\perp}$, $q_{2\perp}$ are transverse 4-momenta of these ones. In the center of mass frame of colliding particles we can write $P_1 = (E, 0, 0, E)$, $P_2 = (E, 0, 0, -E)$, where $E = \sqrt{s}/2$, $P_1^2 = P_2^2 = 0$ and $(P_1 P_2) = s/2$. Sudakov variables are expressed as follows:

$$
\begin{align*}
\alpha_1 &= \frac{M_{1\perp}}{\sqrt{s}} \exp(y^*_1), \\
\alpha_2 &= \frac{M_{2\perp}}{\sqrt{s}} \exp(y^*_2), \\
\beta_1 &= \frac{M_{1\perp}}{\sqrt{s}} \exp(-y^*_1), \\
\beta_2 &= \frac{M_{2\perp}}{\sqrt{s}} \exp(-y^*_2)
\end{align*}
$$

(2)

where $M_{1,2\perp} = M^2 + p_{1,2\perp}^2$, $y^*_1, y^*_2$ are rapidities of heavy quarks, M is heavy quark mass.

From conservation laws we can easily obtain the following conditions:

$$
q_{1\perp} + q_{2\perp} = p_{1\perp} + p_{2\perp}, \\
x_1 = \alpha_1 + \alpha_2, \\
x_2 = \beta_1 + \beta_2
$$

(3)
The differential cross section of heavy quark electroproduction has form:

\[
\frac{d\sigma}{d^2p_{1\perp}}(e p \rightarrow Q \bar{Q} X) = \int dy_1^* dy_2^* \frac{d^2q_{1\perp}}{\pi} \frac{d^2q_{2\perp}}{\pi} \frac{|\tilde{M}|^2 \Phi_\gamma(x_1, q_{1\perp}^2) \Phi_\gamma(x_2, q_{2\perp}^2)}{16\pi^2 (x_1 x_2 s)^2} \tag{4}
\]

For photoproduction process it reads:

\[
\frac{d\sigma}{d^2p_{1\perp}}(\gamma p \rightarrow Q \bar{Q} X) = \int dy_1^* \frac{d^2q_{2\perp}}{\pi} \frac{\Phi_p(x_2, q_{2\perp}^2)|\tilde{M}|^2}{16\pi^2 (x_2 s)^2 \alpha_2} \tag{5}
\]

We use generalized gluon structure function of a proton \(\Phi_g(x_2, q_{2\perp}^2)\) which is obtained in semihard approach. When integrated over transverse momentum \(q_{2\perp} = (0, q_{2\perp}, 0)\) of gluon up to some limit \(Q^2\) it becomes the usual structure function giving the gluon momentum fraction distribution at scale \(Q^2\):

\[
\int_0^{Q^2} \Phi_g(x, q_{2\perp}^2) dq_{2\perp}^2 = x G_p(x, Q^2). \tag{6}
\]

In our calculation we use the following phenomenological parametrizations [11]:

\[
\Phi_p(x, q_{2\perp}^2) = C \frac{0.05}{x + 0.05} (1 - x)^3 f_p(x, q_{2\perp}^2), \tag{7}
\]

\[
\Phi_g(x, q_{2\perp}^2) = C_1 (1 - x)^5 f_g(x, q_{2\perp}^2), \tag{8}
\]

where

\[
f_p = 1, \quad q_{2\perp}^2 \leq q_0^2(x), \quad f_p = \left(\frac{q_0^2(x)}{q_{2\perp}^2}\right)^2, \quad q_{2\perp}^2 > q_0^2(x), \tag{9}
\]

and \(q_0^2(x) = Q_0^2 + \Lambda^2 \exp(3.56 \sqrt{\ln(x_0/x)})\), \(Q_0^2 = 2 GeV^2\), \(\Lambda = 56\ MeV\), \(x_0 = 1/3\). The normalization factor \(C \simeq 0.97\ mb\) of the structure function \(\Phi_p(x, q_{2\perp}^2)\) in (7) was obtained in [11] where \(b\bar{b}\)-pair production at Tevatron energy was described. Thus the normalization constant \(C\) includes the so-called soft \(K\)-factor, which takes into account \(\alpha_s\) corrections due to soft gluon radiation. The typical value of \(K\)-factor in hard hadron-hadron interactions is \(K = 2 - 2.5\). As it was noted in Ref. [11] the choice of \(C = 0.97\ mb\) corresponds to the upper limit of heavy quark hadroproduction cross sections. For the normalization factor \(C_1\) in (8) we shall use value of \(C = 0.65\ mb\), which gives better description of existing data on charm quark photoproduction at fixed target energies \(\sqrt{s_{ep}} = 10 - 30\ GeV\).

In (4) \(\Phi(x_1, q_{1\perp}^2)\) is the well known virtual photon spectrum in Weizsacker-Williams approximation [26] before the integration over \(q_{1\perp}^2\):

\[
\Phi(x_1, q_{1\perp}^2) = \frac{\alpha}{2\pi} \left[ \frac{1 + (1 - x_1)^2}{x_1 q_{1\perp}^2} - \frac{2m_c^2 x_1}{q_{1\perp}^4} \right]. \tag{10}
\]
In this paper we consider only heavy quark photoproduction processes.

The effective gluon distributions $xG(x, Q^2)$ obtained from (7), (8) increase as $x^{-\omega_0}$ at not very small $x$ ($0.01 < x < 0.15$), where $\omega_0 = 0.5$ corresponds to the BFKL Pomeron singularity [10]. This rise goes continuously up to $x = x_0$, where $x_0$ being a solution of the equation $q_0^2(x_0) = Q^2$. In the region $x < x_0$ there is the saturation of the gluon distribution: $xG(x, Q^2) \simeq CQ^2$ or $C_1 Q^2$.

The square of matrix element of partonic subprocess $\gamma^* g^* \to Q \bar{Q}$ can be written as follows:

$$|M|^2 = 16\pi^2 c_Q^2 \alpha_s(x_{12s})^2 \left[ \frac{1}{(\hat{u} - M^2)(\hat{t} - M^2)} - \frac{1}{q_{1\perp}^2 q_{2\perp}^2} \left( 1 + \frac{\alpha_2 \beta_1 s}{\hat{t} - M^2} + \frac{\alpha_1 \beta_2 s}{\hat{u} - M^2} \right)^2 \right]$$

For real photon and off-shell gluon it reads:

$$|M|^2 = 16\pi^2 c_Q^2 \alpha_s(x_{12s})^2 \left[ \frac{\alpha_2^2 + \alpha_1^2}{(\hat{t} - M^2)(\hat{u} - M^2)} + \frac{2M^2}{q_{2\perp}^2} \left( \frac{\alpha_1}{\hat{u} - M^2} + \frac{\alpha_2}{\hat{t} - M^2} \right)^2 \right]$$

where $\alpha_2 = 1 - \alpha_1$ and $\hat{s}$, $\hat{t}$, $\hat{u}$ are usual Mandelstam variables of partonic subprocess

$$\hat{s} = (p_1 + p_2)^2 = (q_1 + q_2)^2, \quad \hat{t} = (p_1 - q_1)^2 = (p_2 - q_2)^2, \quad \hat{u} = (p_1 - q_2)^2 = (p_2 - q_1)^2, \quad \hat{s} + \hat{t} + \hat{u} = 2M^2 + q_{1\perp}^2 + q_{2\perp}^2.$$  

### 3 Discussion of the Results

The results of our calculations for the total cross sections of $c$- and $b$-quark photoproduction are shown in Figs. 3 and 4. Solid curves correspond to the semihard approach predictions and dashed curves correspond to the SPM results with the GRV LO parametrization of the gluon distribution [27]. We used in our calculations $m_c = 1.5$ GeV and $m_b = 4.75$ GeV. Fig. 3a shows us that the solid curve for charm quark cross section obtained in semihard approach with parametrization (7) describes new ZEUS data [5] as well as data from earlier fixed target experiments in the range $15 < \sqrt{s} < 30$ GeV. The curve of the parametrization (8) (Fig. 3b) describes better the experimental data at low energies ($6 < \sqrt{s} < 30$ GeV) but gives worse description ZEUS data [5]. There are some differences between the theoretical curves and ZEUS experimental data (Figs. 3). If these ones will remain valid for new data, then its will show to essential resolved photon contribution [5, 28] and a sensitivity of charm photoproduction cross section to the photon structure function [29].

We would like to note that our results for the parametrization (7) coincide with the results from Ref. [28] for pointlike component of the charm photoproduction cross section for MRSD' parton density at $\mu = m_c$ and ones for parametrization (8) correspond to the charm photoproduction cross section in Ref. [28] for MRSA parton density.
At the energy range $\sqrt{s_{\gamma p}} \geq 500$ GeV the saturation effects in the gluon distribution function are important for charm quark photoproduction in semihard approach (Fig. 4): the $c$-quark cross section grows more slowly at these energies (solid curve). The beauty quark photoproduction cross section predicted by semihard approach is larger than the one predicted by SPM at all energies (Fig. 4). Our conclusion for $b$-quark photoproduction is the same as in Ref. [11], where $b$-quark hadroproduction was described using semihard approach.

The $p_{\perp}$ distributions for $c$- and $b$-quark photoproduction in the semihard approach (solid curves) and in the SPM (dashed curves) at the energy $\sqrt{s_{\gamma p}} = 200$ GeV are shown in Fig. 5. The curves are obtained in the semihard approach for charm quark photoproduction show the saturation effects in low $p_{\perp}$ region ($p_{\perp} < 2$ GeV/c). At high $p_{\perp}$ region ($3 < p_{\perp} < 20$ GeV/c) the heavy quark photoproduction $p_{\perp}$ distributions obtained in the semihard approach are higher than the ones of the SPM (with GRV LO parametrization of gluon distribution). This behaviour of $c$-quark $p_{\perp}$ distributions in the $k_{\perp}$ factorization approach results from the off mass shell subprocess cross section as well as the saturation effects of the gluon structure function, because of $m_{c}^{2} \approx q_{0}(x)^{2}$ at HERA energies. In the case of beauty quarks $m_{b}^{2} >> q_{0}(x)^{2}$ and similar effects are absent.

Fig. 6 show the comparison of heavy quark rapidity distribution (in the photon-proton center of mass frame) obtained in the different models: solid curve shows the $y$ distribution semihard approach, dashed curve shows the one in SPM. The effects discussed above are sufficiently large near the kinematic boundaries, i.e. at big value of $|y^{*}|$. We see that the difference between solid and dashed curves can’t be degrade at all $y^{*}$ via change of normalization of both models.

4 Conclusions

We showed that the semihard approach describes experimental data for the charm photoproduction cross section at low and HERA energies, leads to the saturation effects for the total cross section of charm quark photoproduction at $\sqrt{s_{\gamma p}} \geq 500$ GeV as well as to exceeding over the SPM prediction for beauty quark total cross section at the energy range of $\sqrt{s_{\gamma p}} = 100 - 500$ GeV, and predicts a marked difference for rapidity and transverse momentum distributions of charm and beauty quark photoproduction, which can be studied already at HERA $e p$—collider.

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References


**Figure captions**

1. QCD diagrams for open heavy quark photoproduction subprocesses

2. Diagram for heavy quark electroproduction

3. a) The total cross section for open charm quark photoproduction: solid curve - the semihard approach for the parametrization (7), dashed curve - the SPM. The solid dots are the ZEUS measurements [5] and the open dots are earlier measurements from fixed target experiments.

   b) The solid curve - the semihard approach for the parametrization (8). The experimental data as in Fig. 3a

4. The total cross section for open charm and beauty quark photoproduction: curves as in Fig. 3a

5. The $p_\perp$ distribution for charm and beauty quark photoproduction at $\sqrt{s_{\gamma p}} = 200$ GeV: curves as in Fig. 3a

6. The $y^*$ distribution for charm and beauty quark photoproduction: at $\sqrt{s_{\gamma p}} = 200$ GeV: curves as in Fig. 3a
Fig. 1

Fig. 2