BV quantization of a vector-tensor gauge theory with topological coupling

R. Amorim* and J. Barcelos-Neto†
Instituto de Física
Universidade Federal do Rio de Janeiro
RJ 21945-970 - Caixa Postal 68528 - Brasil

Abstract

We use the BV quantization method for a theory with coupled tensor and vector gauge fields through a topological term. We consider in details the reducibility of the tensorial sector as well as the appearance of a mass term in the effective vectorial theory.

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*Electronic mails: ift0100l @ ufrj and amorim @ ifsul.ufrj.br
†Electronic mails: ift03001 @ ufrj and barcelos @ vms1.nce.ufrj.br
1 Introduction

The quantization method due to Batalin and Vilkovisky (BV) [1, 2] has been shown to be a powerful functional procedure to deal with a wide variety of gauge theories. In the BV scheme, covariance, reducibility, openness of gauge algebras and the presence of gauge anomalies are taken into account in a very natural way.

In this work we consider the BV method to quantize the coupled vector-tensor gauge theory present by Allen and collaborators [3] and recently treated by Lahiri [4] from the canonical point of view. It might be opportune to mention that tensor gauge theories have attracted much attention, formerly connected with the theory of strings, but also with cosmic strings, vortices and black holes [5]. Another use of tensorial gauge theories can be seen in connection with the appearance of a mass term for vector gauge fields, in the context of an effective theory [4]. As tensor gauge theories are reducible, the BV scheme is a natural quantization procedure for them.

In the work of ref. [4], it was not directly taken into account the question of reducibility of constraints related to the tensor gauge fields. This is an important point because if one use that all the constraints are independent, the theory has zero tensor degrees of freedom, what is not actually true [6]. This point is naturally considered here and the integration over the tensorial sector can be done without infinities, leading to an effective massive vectorial theory. We mention that in references [4], the presence of this mass term was considered at classical level. Here, we investigate how it comes quantically as a pole in the propagator of the gauge fixed effective vector theory.

Our work is organized as follows. In Sec. 2 we discuss the BV quantization of a theory with topological coupling of vector and tensor gauge fields. The obtainment of the effective mass for the vector gauge field is achieved at Sec. 3. We leave Sec. 4 for some concluding remarks.

2 Vector coupled to tensor gauge fields

The theory we are going to deal is described by the following Lagrangian density [3, 4]

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{12} H_{\rho\sigma\tau} H^{\rho\sigma\tau} + \frac{m}{4} \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu} F_{\rho\lambda}, \]

where \( F_{\mu\nu} \) and \( H_{\rho\sigma\tau} \) are totally antisymmetric tensors written in terms of the potentials \( A_{\mu} \) and \( B_{\mu\nu} \) (also antisymmetric) through the curvature tensors

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \]
\[ H_{\rho\sigma\tau} = \partial_\rho B_{\sigma\tau} + \partial_\sigma B_{\rho\tau} + \partial_\tau B_{\rho\sigma}. \]
In (1), $\epsilon^{\mu\nu\rho}$ is the totally antisymmetric symbol and $m$ is a mass parameter. It is easy to see, by using the (coupled) Euler-Lagrange equations for $A^\mu$ and $B^{\mu\nu}$, as well as the Jacobi identity, that $F_{\mu\nu}$ satisfy a massive Klein-Gordon equation, with mass parameter given by the factor $m$ appearing in (1).

We observe that both $F_{\mu\nu}$ and $H_{\mu\nu\rho}$ are invariant under the gauge transformations

$$\delta A^\mu = \partial^\mu \Lambda, \quad \delta B^{\mu\nu} = \partial^\mu \Lambda^\nu - \partial^\nu \Lambda^\mu,$$

(4)

(5)

where $\Lambda$ and $\Lambda^a$ are (before fixing the gauge) generic functions of spacetime. The reducible character of this theory is seen from the fact that if we choose the gauge parameter $\Lambda^a$ as the gradient of some scalar $\Omega$ we have that $B^{\mu\nu}$ does not change under the gauge transformation (5). Actually, in this situation

$$\delta B^{\mu\nu} = \left( \partial^\mu \partial^\nu - \partial^\nu \partial^\mu \right) \Omega, = 0.$$

(6)

Expressions (4) and (5) can be rewritten as

$$\delta A^\mu(x) = \int d^4y \ R^\mu(x,y) \Lambda(y), \quad \delta B^{\mu\nu}(x) = \int d^4y \ R^{\mu\nu}(x,y) \Lambda^\nu(y),$$

(7)

(8)

where

$$R^\mu(x,y) = \partial^\mu \delta(x-y), \quad R^{\mu\nu}(x,y) = \partial[\mu \delta(x-y) \dot{\epsilon}^\nu].$$

(9)

(10)

are the generators of the gauge symmetries, and

$$Z^\rho(x,y) = \partial^\rho \delta(x-y)$$

is the reducibility operator [1, 2] corresponding to (6) since

$$\int d^4y \ Z^\rho(x,y) \ R^{\mu\nu}(y,z) = 0.$$

(11)

(12)

In expression (10), as well as in some forthcoming ones, the antisymmetrization is performed in a normalized way, i.e. $K_{\mu\nu} = (K^{\mu\nu} - K^{\nu\mu})/2$ for some $K^{\mu\nu}$ tensor.
We are now ready to write down the solution of the classical master equation. From expressions (9 - 11) we notice that the algebra of the gauge generators is closed and Abelian. To fix the notation, it is convenient to distinguish between both sectors (vectorial and tensorial) of the theory. Let us so generically denote by \( c \) and \( d \) the ghosts associated respectively to the vectorial and to the tensorial sectors. In this way, the solution we are looking for reads

\[
S = S_0 + \int d^4x d^4y \left[ i A_\mu^*(x) \partial_\mu \delta(x - y) c(y) + 2 i B_\mu^*(x) \partial_\mu \delta(x - y) \delta_\nu^!(y) + d_\mu^*(x) \partial_\mu \delta(x - y) d(y) \right],
\]

\[
= S_0 + \int d^4x \left[ i A_\mu^* \partial^\mu c + i B_\mu^* \partial^\mu d^\nu + d_\mu^* \partial^\mu d \right],
\]

where \( S_0 \) is the action corresponding to the Lagrangian (1) and \( A_\mu^*, B_\mu^*, c^*, d_\mu^* \) are antifields of the BV formalism.

To fix the gauge freedom of the vectorial sector, we introduce the gauge-fixing fermionic functional

\[
\psi = - \int d^4x \bar{c} \left( \partial^\mu A_\mu - \frac{\alpha}{2} b \right) + \cdots.
\]

and extend the classical action to

\[
\tilde{S} = S + \int d^4x \bar{c}^* b + \cdots
\]

Here, dots are representing the corresponding quantities for the tensorial sector which we are going to discuss soon. After substituting the antifields \( A_\mu^* \) and \( \bar{c}^* \) by the derivatives of \( \psi \) with respect to \( A_\mu \) and \( \bar{c} \) we arrive, for the vectorial sector, at the usual Faddeev-Popov expression for the vacuum functional with covariant Gaussian gauge-fixing.

The fixation of the tensorial counterpart is not so simple. First of all we need to have a covariant gauge fixing with the same reducibility character of the gauge freedom of \( H_{\mu\nu\rho} \). It may be chosen as

\[
\Xi^\nu = \partial_\mu B^{\mu\nu}.
\]

We notice that \( \Xi^\nu \) is divergenceless. To implement (16) we first observe that in (14) and (15) we have used the ghosts \( c, c^*, \bar{c}, \bar{c}^* \) to fix the vectorial gauge invariance of (1). As we have already used the unbarred quantities \( d^\mu, d^\mu^*, \bar{d} \) and \( d^\nu \) in the tensorial sector of (13), we expect to use the additional pairs \( \bar{d}_\mu, \bar{d}^\nu, \bar{d}^\nu \) and \( \bar{d}^\nu \) in a similar fashion. This gives the tensorial sector of the BV action and the corresponding fixing fermion functional.
\[ S = \ldots + \int d^4 x \, \bar{e}_\mu e^\mu, \]  
(17)

\[ \psi = \ldots - \int d^4 x \left[ \bar{d}_\mu \left( \partial_\nu B^{\nu\mu} - \frac{\beta}{2} e^\mu \right) + \bar{d} \, \partial_\mu d^\mu \right], \]  
(18)

where \( e^\mu \) is an auxiliary field that plays a similar role as \( b \). As we can observe, the reducibility of the original theory has not been completely fixed yet. Even though the ghost gauge invariance

\[ d^\mu \longrightarrow d^\mu + \partial^\mu \zeta \]  
(19)

has been fixed in (18), there remains to consider the further invariance

\[ \bar{d}^\mu \longrightarrow \bar{d}^\mu + \partial^\mu \zeta \]  
(20)

do the complete action. This has already been discussed by Henneaux and Teitelboim in ref.[2]. The solution comes by introducing further pairs \((\eta, \eta^*)\) and \((f, f^*)\) in the theory. This is achieved by adding the term \(-\int d^4 x \, \eta \partial^\mu \bar{d}_\mu\) to the fixing fermion \( \psi \) and \( i \int d^4 x \, \eta^* f \) to the action. In these expressions, \( f \) can be considered as the ghost corresponding to symmetry (20) and \( \eta \) can be seen as an auxiliary field, on the same footing as \( b \) and \( b^\mu \).

Putting the considerations above all together, we can write the complete BV action as

\[ \bar{S} = S_0 + \int d^4 x \left( i \, A_\mu^* \partial^\mu e + \bar{e}_\mu b + i \, B^{\mu \nu}_\mu \partial^\mu d^\nu \right. \]

\[ + \, \bar{d}_\mu \partial^\mu \bar{d} + \bar{d}_\mu e^\mu + i \bar{d}_\mu \bar{f} + i \eta^* f \right) \]  
(21)

and the complete gauge fixing fermion functional as

\[ \psi = -\int d^4 x \left[ \bar{e} \left( \partial_\mu A^\mu - \frac{\alpha}{2} b \right) + \bar{d}_\mu \left( \partial_\nu B^{\nu\mu} - \frac{\beta}{2} e^\mu \right) \right. \]

\[ + \, \bar{d} \partial_\mu d^\mu + \eta \partial^\mu \bar{d}_\mu \right]. \]  
(22)

At this stage, it might be elucidative to present a table of parity and ghost numbers for fields and antifields of the theory.
\[ \epsilon [ A^\mu, B^\mu, b, d, \bar{d}, f, \bar{f}^*, e^*, c^*, \bar{c}^*, d^*_\mu, \bar{d}^*_\mu, \eta] = 0, \]
\[ \epsilon [ A^*_\mu, B^*_\mu, b^*, d^*, \bar{d}^*, f, \bar{f}, e^*_\mu, c, \bar{c}, d^\mu, \bar{d}^\mu, \eta^*] = 1, \]  
\[ (23) \]
\[ g h(d^*) = -3, \]
\[ g h(e^*, d^\mu, \bar{d}^\mu, f^*) = -2, \]
\[ g h(A^*_\mu, B^*_\mu, \bar{c}, b^*, \bar{d}^\mu, e^*_\mu, \eta^*, \bar{f}) = -1, \]
\[ g h(A^\mu, B^\mu, \bar{c}^*, b, \bar{d}^\mu, e^\mu, \eta, \bar{f}^*) = 0, \]
\[ g h(e, d^\mu, \bar{d}^\mu, f) = 1, \]
\[ g h(d) = 2. \]  
\[ (24) \]

Making use of the tables above, we observe that action (21) actually has even Grassmannian parity and ghost number zero, as expected.

With the gauge-fixing functions, antifields are obtained as usual

\[ A^*_\mu = \frac{\delta \psi}{\delta A^\mu} = \partial_\mu \bar{c}, \]
\[ B^*_\mu = \frac{\delta \psi}{\delta B^\mu} = \partial_\mu \bar{d}_\nu, \]
\[ \bar{c}^* = \frac{\delta \psi}{\delta \bar{c}} = -\partial_\mu A^\mu + \frac{\alpha}{2} b, \]
\[ d^*_\mu = \frac{\delta \psi}{\delta d^\mu} = \partial_\mu \bar{d}, \]
\[ \bar{d}^* = \frac{\delta \psi}{\delta \bar{d}^\mu} = -\partial_\nu B^\nu + \frac{\beta}{2} e^\mu + \partial_\mu \eta, \]
\[ \bar{d} = \frac{\delta \psi}{\delta \bar{d}} = -\partial_\mu d^\mu, \]
\[ \eta^* = \frac{\delta \psi}{\delta \eta} = -\partial^\mu \bar{d}_\mu. \]
\[ b^* = \frac{\delta \psi}{\delta b} = \frac{\alpha}{2} \bar{c}, \]
\[ e^*_\mu = \frac{\delta \psi}{\delta e^\mu} = \frac{\beta}{2} \bar{d}_\mu. \]  
\[ (25) \]

Now, the combination of (24) and (25) leads to

\[ S = S_0 + \int d^4 x \left[ i \partial_\mu \bar{c} \partial^\mu c + \left( \partial_\mu A^\mu - \frac{\alpha}{2} b \right) b + i \partial_\mu \bar{d}_\nu \partial^\mu d^\nu \right. \]
\[ + \partial_\mu \bar{d} \partial^\mu d + \left( \partial_\mu B^\mu - \frac{\beta}{2} e^\mu - \partial^\mu \eta \right) e^\mu - i \partial_\mu d^\mu \bar{f} + i f \partial^\mu \bar{d}_\mu \right]. \]  
\[ (26) \]
Once the process of construction of the gauge fixed BV action is done and as we
know that the theory is free of anomalies [2], the next step for the BV quantization
is to write the vacuum functional (or its external current dependent generalizations)
\[ Z = \int [d\mu] \exp\{i\mathcal{S}\}, \] (27)
where \( \mathcal{S} \) is given by (26) and \([d\mu]\) represents the Liouville measure for all the fields
appearing there. If we introduce external currents, we are able to generate all the
Green’s functions of the theory in a perturbative scheme, by the usual derivative
expansion in the external sources. An interesting point we are going to discuss in
the next section is how the functional integrations over the antisymmetric tensor
field can be performed in a closed way, leading to an effective massive theory for
the vectorial field.

3 Massive vectorial effective theory

As we have mentioned in the last section, the classical equations of motion for the
fields \( A^\mu \) and \( B^{\mu\nu} \) can be manipulated in order to show that \( F^{\mu\nu} \) satisfies a massive
Klein-Gordon equation. The same kind of feature could be obtained for \( H^{\mu\nu\rho} \),
after eliminating \( F^{\mu\nu} \). We can make two comments: the first one is that these
results are obviously classical. The second one is that are not the fields themselves
that satisfy massive Klein-Gordon equations, but their curvature tensors. To see
how corresponding features appear at quantum level, we are going to obtain an
effective theory, first by functionally integrating over the auxiliary fields \( b, e^\mu \) and
\( \eta \) to introduce Gaussian fixing terms in the action. After that, we get for the \( B^{\mu\nu} \)
depending part of the gauge-fixed effective action

\[
\mathcal{S}_B = \int d^4x \left[ -\frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{2\beta} (\partial_\nu B^{\mu\rho})^2 + \frac{m}{4} \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu} F_{\rho\lambda} \right],
\]
\[
= \int d^4x \left[ \frac{1}{4} B_{\nu\lambda} O_{\alpha\beta} B^{\alpha\beta} + \frac{1}{2} D^{\nu\lambda} B_{\nu\lambda} \right]. \tag{28}
\]

In order to simplify the notation we have defined the operator

\[
O_{\alpha\beta} = \frac{1}{2} \delta_{[\alpha}^{\beta\gamma} \partial_\gamma \partial_\nu + \frac{2}{\beta} \delta^{[\alpha} \partial^{\beta]} \partial_\nu
\]

and the dual tensor

\[
D^{\nu\lambda} = \frac{m}{2} \epsilon^{\mu\nu\rho\lambda} F_{\rho\lambda}. \tag{29}
\]

In expression (29), \( \delta_{[\alpha}^{\beta\gamma} = 6 \delta^{[\alpha} \delta^{\beta]} \delta^{\gamma]} \) values 1 if \( \mu, \nu, \lambda \) are an even permutation
of \( \alpha, \beta, \gamma \).
To perform the integral in the $B$’s we need the inverse of $O^\alpha_\beta$ given by (29), in such a way that

$$O^\alpha_\beta \left( O^{-1} \right)^{\rho^\sigma}_{\mu^\nu} = \delta^\alpha_\rho \delta^\beta_\sigma.$$  \hspace{1cm} (31)

It is just a matter of algebraic calculation to show that

$$(O^{-1})^{\rho^\sigma}_{\mu^\nu} = \frac{1}{2} \left[ \delta^{\rho^\sigma}_{\mu^\nu} - A \left( 1 - \frac{\beta}{2} \right) \delta^{\rho^\sigma}_{[\mu^\nu]} \delta^{\rho^\sigma}_{[\mu^\nu]} \right].$$  \hspace{1cm} (32)

With the aid of (32), we can see that after functionally integrating over $B^{\mu^\nu}$, the part of the effective action which had depended of the tensorial field acquires the form

$$\bar{S}_B = - \frac{1}{4} \int d^4 x \ D^{\mu^\nu} \left( O^{-1} \right)^{\rho^\sigma}_{\mu^\nu} D^{\rho^\sigma},$$

$$= - \frac{1}{4} \int d^4 x \ D^{\mu^\nu} \left[ \frac{1}{2} \delta^{\rho^\sigma}_{\mu^\nu} - \frac{2}{7} (1 - \frac{\beta}{2}) \partial^\nu \partial^\rho \right] D^{\rho^\sigma}. \hspace{1cm} (33)$$

By using the explicit form of $D^{\mu^\nu}$ given by (30) and the Jacobi identity satisfied by $F^{\mu^\nu}$, we observe that the term proportional to $1 - \frac{\beta}{2}$ in (33) vanishes identically. The final result reads

$$\bar{S}_B = - \frac{m^2}{4} \int d^4 x \ F^{\mu^\nu} \frac{1}{2} F^{\mu^\nu},$$

$$= - \frac{m^2}{2} \int d^4 x \left( A^{\mu^\nu} A_{\mu^\nu} + \frac{\partial \cdot A}{2} \right) \partial^\nu A^\nu. \hspace{1cm} (34)$$

At last we have that the complete effective action can be written as

$$\bar{S} = \int d^4 x \left[ \frac{1}{2} A^{\mu} \left( -m^2 \right) A^\mu - \frac{1}{2} \partial^\mu A^\mu \left( 1 - \frac{1}{\alpha} - \frac{m^2}{k^2} \right) \partial^\nu A^\nu \right] + S_{\text{ghost}}, \hspace{1cm} (35)$$

where $S_{\text{ghost}}$ represents the ghost dependent part of the effective action which appears in the vacuum functional (27).

The operator which acts on (the quadratical part of) $A^\mu$ in (34), written in momentum space, reads

$$G_{\mu^\nu} = - \left[ (k^2 + m^2) \eta_{\mu^\nu} - \left( 1 - \frac{1}{\alpha} + \frac{m^2}{k^2} \right) k_{\mu} k_{\nu} \right]. \hspace{1cm} (36)$$

Its inverse gives the propagator for the vectorial field in momentum space. It is just given by
which presents a pole in $k_0 = \not k^2 + m^2$, showing that at quantum level the elimination of the tensorial fields is also traduced by the introduction of a mass term for the vectorial field. Observe however that here it becomes clear that it is the vectorial field itself that effectively acquires a mass.

4 Conclusion

With the aid of the Batalin and Vilkoviski procedure, we have constructed the vacuum functional for a field theoretical model consisting of vector and tensor gauge fields interacting through a topological term. The reducibility of the tensorial sector was taken properly into account. After a covariant gauge-fixing was implemented, we were able to integrate over the tensorial fields, obtaining a massive vectorial effective theory as output. The propagator for the vectorial bosons have been calculating, showing the expected pole at $\not p^2 + m^2$.

As a final comment, we observe that the introduction of non-Abelian gauge symmetries is possible for a corresponding extended model \cite{7}. Its BV formulation, as well as some phenomenological related consequences for this non-Abelian version of the present model are under study and will be presented elsewhere \cite{8}.

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References


