Big Bang Nucleosynthesis in Crisis

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Abstract

We present a new evaluation of the constraint on the number of light neutrino species ($N_\nu$) from big bang nucleosynthesis, which suggests a discrepancy between the predicted primordial light element abundances and those inferred from observations. The best fit for the combined data is $N_\nu = 2.0 \pm 0.3 \ (1\sigma)$ and the upper limit is $N_\nu < 2.5 \ (95\% \ C.L.)$. The data are inconsistent with the Standard Model ($N_\nu = 3$) at the 99.7\% C.L. Relaxed constraints on $N_\nu$ for different $^4$He observational uncertainties and different $^3$He survival factors are explored.
Along with the Hubble expansion and the cosmic microwave background radiation, big bang nucleosynthesis (BBN) provides one of the key quantitative tests of the standard big bang cosmology. The predicted primordial abundances of $^4\text{He}$, $^2\text{D}$, $^3\text{He}$, and $^7\text{Li}$ [1,2], which span almost ten orders of magnitude, have been used to constrain the effective number of light neutrino species ($N_\nu$) $^1$ [3,4,1,5]. The neutrino counting includes anything beyond the Standard Model [such as a right-handed (sterile) neutrino] that contributes to the energy density. This constraint is complementary to neutrino counting from the invisible width of $Z$ decays ($N_\nu^Z$), which is sensitive to a much larger mass range ($\lesssim M_Z/2$, where $M_Z$ is the $Z$ mass), but only to neutrinos fully coupled to the $Z$; the current result is $N_\nu^Z = 2.988\pm0.023$ [6], in agreement with the Standard Model ($N_\nu^Z = 3$).

The primordial $^4\text{He}$ abundance is sensitive to the competition between the early universe expansion rate and the weak interaction rate. The expansion rate depends on the overall density and hence on $N_\nu$, while the weak rate is normalized via the neutron lifetime. Recent improvements in neutron lifetime measurements have significantly reduced the uncertainty in the $^4\text{He}$ prediction and, coupled with increasingly accurate astronomical data on extragalactic $^4\text{He}$, have led to tighter constraints on $N_\nu$; at 95% C.L. $N_\nu < 4$ in 1989 [4], < 3.3 in 1991 [1], and < 3.04 in 1994 [5]. However, a constraint as strong as $N_\nu < 3.04$ hints that the standard theory with $N_\nu = 3$ may not provide a good fit to the observations.

In this Letter we present new BBN limits on $N_\nu$ and the baryon-to-photon ratio ($\eta$) from simultaneous fits to the primordial $^4\text{He}$, $^2\text{D}$, $^3\text{He}$ and $^7\text{Li}$ abundances [hereafter we use the notation $Y_p$ ($^4\text{He}$ mass fraction), $y_{2p} = D/\text{H}$, $y_{3p} = ^3\text{He}/\text{H}$, and $y_{7p} = ^7\text{Li}/\text{H}$, fractions by number, respectively] inferred from the astrophysical observations. In particular, we incorporate new constraints on $y_{2p}$ [7], which are based on a generic chemical evolution model [8] and which significantly improve the prior constraints [9,1]. Our likelihood analysis

$^1$Neglecting the baryon contribution, the total energy density $\rho_{\text{tot}}$ depends on $N_\nu$ as $\rho_{\text{tot}} = \rho_{\gamma} + \rho_{e} + N_\nu\rho_{\nu}$, where $\rho_{\gamma}$, $\rho_{e}$, and $\rho_{\nu}$ are the energy density of photons, electrons and positrons, and massless neutrinos (one species), respectively.
systematically incorporates the theoretical and observational uncertainties. The theoretical uncertainties and their correlations are estimated by the Monte Carlo method [10,5,11,12]. Non-Gaussian uncertainties in the observations, such as the adopted systematic error in the value of $Y_p$, the upper and lower limits for $D$, and the model-dependent $^3$He survival parameter ($g_3$), are treated in a statistically well-defined way.

We adopt a primordial helium abundance estimated from low metallicity HII regions [13]:

$$Y_p = 0.232 \pm 0.003 \text{ (stat)} \pm 0.005 \text{ (syst)}, \quad (1)$$

assuming a Gaussian distribution for the $1\sigma$ statistical uncertainty and a flat (top hat) distribution with a half width of 0.005 for the systematic uncertainty [12].

New D constraints were obtained in Refs. [7,8], using D and $^3$He observations in the solar wind and meteorites [14] and a generic chemical evolution model:

$$y_{2p} = (1.5 - 5.5) \times 10^{-5} \quad (2)$$
$$y_{3p} \leq 1.7 \times 10^{-5}. \quad (3)$$

Although these constraints ($1\sigma$) are independent of any specific model for primordial nucleosynthesis, standard BBN or otherwise, they do depend on the adopted $^3$He survival factor $g_3$. We account for the uncertainty in $g_3$ with a flat (top hat) distribution in the range 0.25–0.50 estimated from Refs. [9,15,1]. When the observational bounds in Eqs. 2 and 3 are convolved with the BBN predictions (which are a function of $\eta$ with $N_e$ fixed at 3), even tighter constraints on D and $^3$He may be inferred [7]: $y_{2p} = (2.3^{+1.7}_{-0.8}) \times 10^{-5}$ and $y_{3p} = (1.0 \pm 0.2) \times 10^{-5}$ at $1\sigma$. The resulting $2\sigma$ upper bound to $y_{2p}$ is roughly a factor of 2 lower than the corresponding bound in Ref. [1] and this has the effect of raising the lower bound on the allowed range of $\eta$. Our central value for $y_{2p}$ is an order of magnitude smaller than the abundance inferred from a possible D detection in absorption against a high redshift QSO [16,17], but consistent with that reported for a different QSO absorption system [18].
We estimate the primordial $^7$Li abundance from the metal-poor stars in our Galaxy’s halo:

$$y_{7p} = (3.0 \pm 1.0) \times 10^{-10} \quad (1\sigma).$$

(4)

This estimate is consistent with other recent determinations [19], but we adopt a larger uncertainty to allow for possible stellar depletion of $^7$Li, permitting $1 \leq 10^{10}y_{7p} \leq 5$ at 2$\sigma$.

For standard BBN ($N_\nu = 3$), the theoretical predictions with the uncertainties (1$\sigma$) determined by the Monte Carlo technique are displayed as a function of $\eta$ in Fig. 1. Also shown in Fig. 1 are the constraints obtained by our likelihood analysis of the predictions and observations. The result is disturbing: the constraints on $\eta$ from the observed $^4$He and D–$^3$He abundances appear to be mutually inconsistent.

To explore this more carefully, all four elements are fit simultaneously, yielding the likelihood function for $N_\nu$ shown in Fig. 2 (where the likelihood is maximized with respect to $\eta$ for each $N_\nu$). The BBN predictions for the D, $^3$He, and $^7$Li abundances are sensitive to the baryon-to-photon ratio $\eta$, but only weakly dependent on $N_\nu$. The BBN prediction for $^4$He is very weakly dependent on $\eta$ and is approximately proportional to $(N_\nu - 3)$. In our likelihood analysis we have computed the Monte Carlo predictions for all of the element abundances for $1.5 \leq N_\nu \leq 4$ and $10^{-10} \leq \eta \leq 10^{-9}$. The $N_\nu$ and $\eta$ dependences of the uncertainties, the $\eta$ dependence of the correlations among the uncertainties [20,5,12], and the correlations between $\eta$ and the $y_{2p}$ and $y_{3p}$ values have been also included in the likelihood function.

Fig. 2 shows that the Standard Model ($N_\nu = 3$) yields an extremely poor fit. The best fit is for $N_\nu = 2.0 \pm 0.3$, and the upper-limit from the joint likelihood (Fig. 2) is

$$N_\nu < 2.5 \quad (95\% \text{ C.L.)}$$

(5)

The ratio of the likelihood of $N_\nu = 3$ to the best fit $N_\nu = 2.0$ is 0.003. This value provides an estimate of the goodness-of-fit of the standard ($N_\nu = 3$) theory. Possible inconsistencies

$^2$There is no standard procedure to estimate the goodness-of-fit when non-Gaussian uncertainties
between predictions and observations have been noted before [21]. Our results exacerbate this discrepancy to a 3 standard deviation effect, mainly due to our new D constraint.

The result of our simultaneous fit in the $\eta - N_\nu$ plane is shown in Fig. 3. The constraint on the baryon-photon ratio is $\eta = (5.1^{+0.9}_{-0.7}) \times 10^{-10}$ (1$\sigma$).

In setting limits when the likelihood function extends beyond the physical parameter space, it is usually a reasonable (and conservative) prescription to renormalize the probability density distribution within the physical part of parameter space. This implies that one should renormalize the likelihood function for $N_\nu \geq 3$ in constraining any (nonstandard) particle contribution in addition to three massless neutrinos in the Standard Model. We have examined the $N_\nu$ limit in this fashion; the 95% C.L. limit for $N_\nu$ extends to 3.25 (for $\eta = 4.6 \times 10^{-10}$). However, we do not advocate this interpretation since the fit for $N_\nu = 3$ is so poor that this additional constraint for $N_\nu > 3$ is meaningless.

The combined data ($D$, $^3$He, $^4$He, and $^7$Li) with the adopted uncertainties are inconsistent with standard BBN ($N_\nu = 3$). But what if some of the uncertainties have been underestimated? In particular, the systematic uncertainty in the $^4$He observational data may be larger than the estimate in Ref. [13] by a factor of 3 or more [22,11]. To quantify the size of the required systematic uncertainty, we predict the $^4$He abundance with $\eta$ determined by the combined $D-^3$He and $^7$Li constraints. This BBN prediction for $^4$He is $0.248 \pm 0.003$ (1$\sigma$), where the error includes the uncertainties from the D-$^3$He and $^7$Li constraints and from the BBN theory calculation. A comparison to the adopted observed value [Eq. (1)] reveals a difference of order 0.016.

In Fig. 4 we show the $\eta - N_\nu$ constraints when the systematic uncertainty ($\Delta Y$) is allowed to be a free parameter (the observed central value is shifted by $\Delta Y$). To be consistent with $N_\nu = 3$, $\Delta Y$ has to be significantly larger than the adopted systematic error (Eq. 1). When

are involved in a likelihood analysis. In addition to using the ratio of the likelihoods for $N_\nu = 2$ and 3, we have also estimated the goodness-of-fit with the standard $\chi^2$ method by approximating the errors with Gaussian distributions: the results from the two methods are consistent [12].
\( \Delta Y \) is fit as a free parameter with \( N_e \) fixed to 3, we obtain \( \Delta Y = 0.016 \pm 0.005 \) at 1\( \sigma \) (for more detail, see Ref. [12]). It is important to note that even allowing \( \Delta Y \) to change freely, the \(^7\text{Li}\) and ISM D constraints still bound \( \eta \) from above at \( 7 \times 10^{-10} \) (95\% C.L.); ISM D alone bounds \( \eta \) from above at \( 9 \times 10^{-10} \). The claim in Ref. [22] that \( \eta \) can be as large as \( \sim 14 \times 10^{-10} \) is unjustified.

We have also examined how the \( \eta - N_e \) constraint is relaxed when the \(^3\text{He}\) survival factor, which affects the upper limit on \( y_{2p} \), differs from that adopted \( (g_3 = 0.25 - 0.50) \) (Fig. 5). To relax the \( y_{2p} \) upper limit so as to be consistent with the \( Y \) constraint, a significantly smaller \( g_3 \) is required. When \( g_3 \) is allowed to be a free parameter with \( N_e \) fixed to 3, we obtain \( g_3 = 0.04 \pm 0.03 \) (1\( \sigma \)). That is, the destruction of \(^3\text{He}\) by stars must be significantly larger than is implied by stellar and chemical evolution models [15]. We note that stellar \(^3\text{He}\) production effectively increases \( g_3 \) and therefore exacerbates the present discrepancy between theory and observations.

If we take at face value the primordial abundances (as inferred from observational data filtered for D and \(^3\text{He}\) via chemical evolution models), standard BBN with 3 massless neutrinos is excluded. The best fit between predictions and observations is for \( N_e = 2.0 \pm 0.3 \). One way to alter standard BBN is to change the physics of the neutrino sector. For example, if \( \nu_\tau \) has a mass in the range \( 10 \text{ MeV} \lesssim M_{\nu_\tau} \lesssim 24 \text{ MeV} \) (the upper limit is the recent result from ALEPH [23]), BBN production of \(^4\text{He}\) can be either increased or decreased (relative to the standard case), depending on whether \( \nu_\tau \) is stable or unstable on nucleosynthesis time scales (\( \sim 1 \text{ sec} \)). An effectively stable \( \nu_\tau \) (\( \tau \gtrsim 10 \text{ sec} \)) in this mass range always increases \( Y \) relative to the standard case [24] and would thus make for a worse fit with the data. However, if \( \nu_\tau \) has a lifetime \( \lesssim 10 \text{ sec} \) and decays into \( \nu_\mu + \phi \) (where \( \phi \) is a ‘majoron-like’ scalar), \(^3\text{He}\) it is possible to decrease the predicted \( Y \) relative to the standard case (see figures 3 and 7 of Ref. [26]). Such an unstable \( \nu_\tau \) contributes less than a massless neutrino species at the epoch

\(^3\text{Decays with } \nu_e \text{ in the final state can directly alter the neutron-to-proton ratio and thus affect } Y_p \text{ somewhat differently [25].}\)
of BBN, thereby reducing the yield of $^4\text{He}$. For example, a $\nu_e$ with mass $\sim 20$ MeV which decays with a lifetime of $\sim 0.1$ sec reduces $N_\nu$ by $\sim 0.5 - 1$ ($\bar{Y}$ by $\sim 0.006 - 0.013$), thus helping to resolve the apparent conflict between theory and observation. It is also possible to alter the yield of BBN $^4\text{He}$ by allowing $\nu_e$ to be degenerate [27]. If there are more $\nu_e$ than $\bar{\nu}_e$, $Y$ is reduced relative to the standard (no degeneracy) case as the extra $\nu_e$'s drive the neutron-to-proton ratio to smaller values. A reduction of $\bar{Y}$ of $\sim 0.01$ can be accomplished with a $\nu_e$ chemical potential of $\mu_e/T_e \sim 0.03$, corresponding to a net lepton-to-photon ratio of 0.005 (to be compared to the net baryon asymmetry which is smaller by $\sim 7$ orders of magnitude). Lastly, one can relax the assumption that baryons are homogeneously distributed. Inhomogeneous BBN typically results in higher $Y_p$, and therefore does not resolve the $^4\text{He}$-$^3\text{He}$ discrepancy [28].

In summary, the predictions of standard ($N_\nu = 3$) BBN for the primordial $^4\text{He}$ and D abundances appear to be inconsistent with those inferred from observations. To reduce the $^4\text{He}$ prediction to the level consistent with the D constraint, $N_\nu$ is required to be as small as $2.0 \pm 0.3$. This opens a possibility for nonstandard nucleosynthesis scenarios, such as those with massive tau neutrinos, neutrino degeneracy, and new decaying particles. Alternatively, standard BBN is allowed if the inferred primordial $^4\text{He}$ mass fraction has been underestimated by $\Delta Y = 0.016 \pm 0.005$ or if the $^3\text{He}$ survival fraction is as small as $g_3 = 0.04 \pm 0.03$.

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FIG. 1. BBN predictions (solid lines) for $Y_p$, $y_{2p}$, and $y_{7p}$ with the theoretical uncertainties (1σ) estimated by the Monte Carlo method (dashed lines). Also shown are the regions constrained by the observations at 68% and 95% C.L. (shaded regions and dotted lines, respectively).
FIG. 2. The likelihood function for $N_\nu$ when the observations for $Y_p$, $y_2p$, $y_3p$, and $y_7p$ are fit simultaneously. For each $N_\nu$ the likelihood function is maximized for $\eta$. The upper limit is $N_\nu < 2.5$ (95\% C.L.) The fit for the Standard Model ($N_\nu = 3$) is excluded at 99.7\% C.L.
FIG. 3. The combined fit of the observations to $N_\nu$ and $\eta_{10}$ ($\equiv \eta \, 10^{10}$).
FIG. 4. The combined fit of the observations when the systematic uncertainty in the $^4\text{He}$ observation ($\Delta Y_{\text{sys}}$) is fixed to 0, 0.005, 0.010, and 0.015.
FIG. 5. The combined fit of the observations when the $^3$He survival factor ($g_3$) is fixed to 0.10, 0.25, and 0.50.