Effects of Gogny type interactions on the nuclear flow.

F. HADDAD, F. SEBILLE, M. FARINE, P. SCHUCK, 
V. de la MOTA, and B. JOUAULT

Rapport Interne SUBATECH - 95 - 07
EFFECTS OF GOGNY TYPE INTERACTIONS ON
THE NUCLEAR FLOW

F. Haddad¹, F. Sébille², M. Farine³, P. Schuck⁴, V. de la Mota² and
B. Jouault²

1) Cyclotron Institute, Texas A&M University.
   College Station, Texas 77843. USA

2) SUBATECH, Université de Nantes-IN2P3, EMN.
   F-44072 Nantes Cedex 03. France.

3) Ecole Navale, Lanvéoc Poulmic. 29240 Brest Marine. France

4) CNRS-IN2P3 Université Joseph Fourier, Institut des Sciences Nucléaires,
   F-38026 Grenoble Cedex. France

Abstract

A flow analysis on symmetric and asymmetric reactions from 100 to 400
Mev/n is performed in the framework of the semi-classical Landau-Vlasov
approach. In this energy range our results present two different trends. At
lower energies it is governed by the momentum dependence of the nuclear
optical potential, whereas at higher energies its density dependence plays
a crucial role leading to a rather pronounced sensitivity to the incompress-
ibility modulus. The non-locality of the nuclear interaction is relevant for
asymmetric colliding systems. With an incompressibility modulus in the
vicinity of 200MeV, an excellent quantitative description of the flow be-
havior with incident energy and impact parameter or the system mass is
provided.
1 Introduction

Heavy ion reactions at intermediate energies give rise to a large variety of phenomena in which nuclear matter appears under extreme conditions of temperature and/or density. These very excited nuclear systems created in the laboratory would, in principle, be convenient as sources of information on the properties of hot and compressed nuclear matter. One of these properties is the incompressibility of nuclear matter experimental value is still subject of controversy. This parameter (extrapolated to neutron rich matter) is thought to have a crucial importance for the occurrence of supernova explosions. There is a large variety of sources from where informations can be extracted (see for instance [1] and references quoted therein). Nonetheless its determination still remains a great challenge for nuclear physics.

The complexity of such reactions, involving strong dynamical effects, requires detailed theoretical analyses of experimental observables. A large amount of data concerning a great variety of nuclear systems in a broad energy range is available from 4π detectors, with fine granularity, giving access to an event-by-event analysis of nuclear reactions. Several observables concerning the nuclear compressibility have been suggested such as isoscalar monopole resonance or subthreshold particle production. On the other hand, special care from both theoretical and experimental points of view has been devoted to the flow observable, which gives a quantitative estimation of the collective momentum transfer occurring in a reaction, in a wide range of beam energies. Many arguments are pleaded in favor of the flow observable as a convenient tool to get information on nuclear matter properties: for instance it is mostly determined in the overlapping region where conditions close to the nuclear matter are reached and it survives the expansion stage of the reaction.

A direct connection with usual equilibrium concepts is certainly not easy as
the collective motion arises from a dynamical process. Presently, one of the best ways to disentangle the effects (and to figure out their relative strengths) of the gross features of the mean nuclear potential is probably the use of calculations with transport models. Many such models are on the market and have demonstrated their ability to model heavy-ion reactions. They have in particular demonstrated the existence of a complicated interplay between collisions and the mean field in flow creation at different stages of the reaction and the insufficiency of either cascade or hydrodynamical models.

We already performed such an analysis in nuclear collisions at beam energies below 100 MeV/n [2] using the zero-range Zamick interactions, the Gogny D1-G1 finite-range interaction as well as Gogny like interactions with varying effective mass and incompressibility. We evidenced the crucial role played by the non-local component of the nuclear force and by dynamical and out-of-equilibrium effects in the creation of collective momentum. A very weak dependence on nuclear matter incompressibility was noted at these lower energies and it was not possible to deduce from our analysis any new compelling information concerning \( K_\infty \), though certainly results with D1-G1 are compatible with the data.

In this work we want to investigate the interaction of flow and nuclear incompressibility at higher energies using again Gogny type interactions. The finite range Gogny force is one of the best tested effective nuclear forces on the market. Still it has some defects. One which will be relevant here is that the original Gogny force [3] yields an optical potential which has not the right asymptotic behavior at large energies. We will see in this work that repulsion of the non-local nuclear field at large momenta (or energies) is absolutely crucial to account for the flow for bombarding energies at \( E/A > 300 \text{ MeV} \). So heavy ion reactions help to pin down a better phenomenological force. On the other hand, in varying the nuclear incompressibility modulus, we will see that the flow values at \( E/A > 300 \text{MeV} \) depend on it in a marked way permitting an independent determination of this quantity. Our work is very much in line with those of Pan and Danielewicz [4] and of Zhang et al. [5] where, however, less commonly used
effective forces have been employed.

Our framework will be the semiclassical transport model, known as Landau-Vlasov model which will be shortly explained in section 2 (see also Ref.[6]). In section 3 we describe our calculations performed with the Gogny D1-G1 non-local effective interaction and other D1-G1 like interactions. We show that at such high energies the real part of the optical potential has to be adjusted to experimental data (asymptotic value around +30 MeV instead of 0 MeV for D1-G1) in order to increase repulsion. Section 4 gives detailed discussion of the results for symmetric (Nb+Nb, Au+Au) and asymmetric (Ar+Pb) systems using our new interactions. Quantitative comparisons with the calculations of [4, 5] are quite encouraging in spite of some different basic ingredients in the two models. A value of about 200 to 230 MeV for the incompressibility of infinite nuclear matter seems to emerge from our analysis; this is in agreement with the former reference and with also the old 1976 result of Blaizot [7] ¹, but contradicting recent claims of the much higher value of 300 MeV by Sharma et al. [8] (see also [9] for a critical analysis of that value).

# 2 The Model

The Landau-Vlasov model describes the dynamics of nuclear collisions using a phase-space transport equation (which is the lowest member of the BBGKY hierarchy) complemented by a Pauli-blocked Uehling-Uhlenbeck collision term \( I_{\text{coll}}(f) \) [6, 10]. The time evolution of the one-body phase-space distribution function \( f(\vec{r}, \vec{p}, t) \) is then driven by the kinetic equation,

\[
\frac{\partial f}{\partial t} + \{f, H\} = I_{\text{coll}}(f)
\]  

¹see, however, a recent preprint by Blaizot et al. where the old value for \( K_\infty \) is confirmed [7].
where \{ , \} stands for the Poisson bracket and \( H \) is the one-body mean field Hamiltonian. This equation is solved using the projection of the phase space distribution on a moving basis of coherent states (referred to as pseudoparticles) which are gaussians in space and momentum, with frozen widths [6]:

\[
f(\vec{r}, \vec{p}) = \frac{A}{N} \sum_i \omega_i \ g_\chi(\vec{r} - \vec{r}_i) \ g_\phi(\vec{p} - \vec{p}_i)
\]

where the \( \omega_i \) are the projection coefficients determined by the initial conditions, \( \chi \) and \( \phi \) the widths in coordinate and momentum space and

\[
g_\chi(u) = \frac{1}{(2\pi\chi)^{2/3}} \exp\left(-\frac{u^2}{2\chi}\right)
\]

The sampling is done using a Monte-Carlo method; \( N \) is the number of sampled gaussians per nucleon, \( A \) the total number of nucleons and the phase space centers \((\vec{r}_i, \vec{p}_i)\) are randomly drawn according to some density law. The centers of these coherent states are then bound to move according to classical Ehrenfest equations [6]. The solution of the Landau-Vlasov equation gives the time evolution of a swarm of pseudoparticles moving in a mean field, undergoing binary collisions when allowed by the phase space availability. The cross-section of pseudoparticles is scaled on the nucleon-nucleon (N-N) cross-section such that their average mean free path is kept equal to the nucleon one [10]. In this work we use the free isospin and energy dependent N-N cross-section. We do not include any in-medium corrections, which, in spite of intensive theoretical efforts [11, 12], are still not well known. Calculations of Cugnon et al. [11] indicates that in-medium cross-section values do not exceed 0.8 to 1 times the free value.

Our model is able to reproduce basic properties of nuclear matter at zero and finite temperatures for many different interactions with zero or finite range. Estimations of the surface energy from semi-infinite nuclear matter calculations are also correct [13]. The same model was used to study dynamical instabilities arising from fluctuations in the spinodal zone for nuclear matter [14] and characteristic times were extracted for the spinodal decomposition.
3 Effective Interactions

There exist many phenomenological zero-range forces in the literature giving an excellent fit to static and low energy properties of nuclei (in particular of those that are doubly magic) e.g. Skyrme-Orsay [15], Skyrme-Koehler [16]. In this work we are mainly concerned with finite range phenomenological interactions because we believe that they are more theoretically sound for extrapolation to high energies. We thus introduce the non-local feature of the mean-field potential by using D1-G1, the well-known and succesfull (at low energy) Gogny force [3]. In order to evaluate the sensitivity of the collective transverse momentum to the nuclear equation of state, we build a family of Gogny-like interactions, labeled D1-Gx, which differ from D1-G1 by their incompressibility $K_{\infty}$ and effective mass $m^{*/m}$. In the phase-space representation the mean field potential reads (excluding contributions from the spin orbit term),

$$V_q(\vec{r}, \vec{p}) = \left[ \frac{3}{4}(\gamma + 2)\rho^{\gamma+1}(\vec{r}) - \frac{3}{4}\gamma\rho^{\gamma-1}(\vec{r}) (\rho_n^2(\vec{r}) + \rho_p^2(\vec{r})) - \frac{3}{2}\rho^\gamma(\vec{r})\rho_q(\vec{r}) \right] l_3$$

$$+ \sum_{i=1}^{2} \int d\vec{r}' [(W_i + \frac{B_i}{2})\rho(\vec{r}') - (H_i + \frac{M_i}{2})\rho_q(\vec{r}')] exp - \frac{(\vec{r} - \vec{r}')^2}{\eta_i}$$

$$- \frac{2(\sqrt{\pi}\eta_i)^3}{(2\pi)^3} \sum_{i=1}^{2} \int d\vec{k}' \left[ [(W_i + B_i) f_s(\vec{r}, \vec{k}') - (H_i + M_i) f_0(\vec{r}, \vec{k}')] exp - \frac{(\vec{k} - \vec{k}')^2}{4} \right]$$

where $\rho_n(\vec{r})$ and $\rho_p(\vec{r})$ are respectively the neutron and proton density, $\rho(\vec{r})$ the total density and $q$ the isospin degree of freedom.

Our first interaction, labeled D1-G3, has the same infinite nuclear matter properties as D1-G1 except for its incompressibility fixed to 360 MeV. It will be sufficient to illustrate the influence of $K_{\infty}$. On the other hand we know that D1-G1 is not reliable at energies higher than 200 MeV. Experimental data [17] show that the nucleon-nucleus optical potential becomes positive at around 200
MeV for nuclear matter at saturation while D1-G1 optical potential is zero in this region. We have therefore built two new interactions labeled D1-G7 and D1-G8; both have an effective mass of $m^*/m=0.56$ and, respectively, incompressibility of 228 (like D1-G1) and 300 MeV. Moreover they are constrained to have a realistic

Table 1: parameters of the different momentum interactions

<table>
<thead>
<tr>
<th>forces</th>
<th>$i$</th>
<th>$\eta_i$</th>
<th>$W_i$</th>
<th>$B_i$</th>
<th>$H_i$</th>
<th>$M_i$</th>
<th>$T_3$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1-G1</td>
<td>1</td>
<td>0.7</td>
<td>-402.4</td>
<td>-100.</td>
<td>-496.2</td>
<td>-23.56</td>
<td>1350</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.2</td>
<td>-21.3</td>
<td>-11.77</td>
<td>37.27</td>
<td>-68.81</td>
<td>1350</td>
<td>1/3</td>
</tr>
<tr>
<td>D1-G3</td>
<td>1</td>
<td>0.7</td>
<td>-402.4</td>
<td>-100.</td>
<td>-496.2</td>
<td>-23.56</td>
<td>1896</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.2</td>
<td>30.96</td>
<td>35.4</td>
<td>34.56</td>
<td>-63.41</td>
<td>1896</td>
<td>1.24</td>
</tr>
<tr>
<td>D1-G7</td>
<td>1</td>
<td>0.7</td>
<td>-402.4</td>
<td>-70.</td>
<td>-496.2</td>
<td>-23.56</td>
<td>916.723</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.2</td>
<td>61.087</td>
<td>-83.963</td>
<td>95.463</td>
<td>-165.197</td>
<td>916.723</td>
<td>0.35</td>
</tr>
<tr>
<td>D1-G8</td>
<td>1</td>
<td>0.7</td>
<td>-402.4</td>
<td>10.</td>
<td>-496.2</td>
<td>-23.56</td>
<td>1166.97</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.2</td>
<td>79.84</td>
<td>-96.53</td>
<td>66.77</td>
<td>-127.82</td>
<td>1166.97</td>
<td>1.2</td>
</tr>
</tbody>
</table>

value for the optical potential at high energies i.e. an asymptotic value of 30 MeV for energies higher than 200 MeV. Figure 1 shows that D1-G7 (G8) and D1-G1 (G3) are almost identical for low energies and in good agreement with experimental data; contrarily above 150 MeV, D1-G7 (G8) stays close to the data while D1-G1 (G3) is definitely too low.

Table 2: main characteristics of the non-local interactions

<table>
<thead>
<tr>
<th>forces</th>
<th>$m^*/m$</th>
<th>$K_{\infty}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1-G1</td>
<td>0.67</td>
<td>228 MeV</td>
</tr>
<tr>
<td>D1-G3</td>
<td>0.67</td>
<td>360 MeV</td>
</tr>
<tr>
<td>D1-G7</td>
<td>0.56</td>
<td>228 MeV</td>
</tr>
<tr>
<td>D1-G8</td>
<td>0.56</td>
<td>300 MeV</td>
</tr>
</tbody>
</table>
We also checked the density dependence of the D1-G7 optical potential. Figure 2 displays a comparison with the microscopic calculations of Wiringa [18] which takes into account two and three body interactions. The agreement over a range of densities and energies is rather striking in view of the fundamentally different nature of the two calculations.

We also did calculations with the Zamick-like interactions [19], a simplified version of the zero range Skyrme force, in order to have comparisons with previous works. Such interactions, although much less realistic than Gogny-like interactions, drastically reduce the calculation time. The potential is simply,

\[ V(\rho) = a \frac{\rho}{\rho_0} + b \left( \frac{\rho}{\rho_0} \right)^{\sigma+1} \]

(4)

where \( \rho_0 \) is the saturation density of nuclear matter and the free parameters \( a \) and \( b \) are adjusted to give saturation properties of infinite symmetric nuclear matter. We built two such interactions, one referred to as “soft” with an incompressibility \( K_\infty = 200 \) MeV and one referred to as “stiff” with \( K_\infty = 380 \) MeV. Tables 1 to 3 give the parameters and some infinite nuclear matter properties of the interactions used in this work.

<table>
<thead>
<tr>
<th>Zamick forces</th>
<th>a</th>
<th>b</th>
<th>( \sigma )</th>
<th>( K_\infty )</th>
<th>( m^*/m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>soft</td>
<td>-356.</td>
<td>303.</td>
<td>( \frac{1}{6} )</td>
<td>200 MeV</td>
<td>1</td>
</tr>
<tr>
<td>stiff</td>
<td>-123.</td>
<td>70.</td>
<td>1</td>
<td>380 MeV</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: parameters of the Zamick interactions

With a reliable transport model we are in a good position to examine the influence of the nuclear equation of state on the flow with good interaction on a wide range of energies.
4 Discussion of the Results

As it is well established experimentally, the nuclear matter exhibits clear collective behavior in heavy ion collisions, characterized by the so-called collective observables. They are expected to bear signatures of the properties of the nuclear interaction. Among these collective observables, we will focus our attention on the flow observable, defined to quantify the momentum transfer from the longitudinal direction to the transverse one in the reaction plane. It has been (and will be) extensively studied experimentally at intermediate energies through different experimental set-ups, in particular with $4\pi$ detectors. Since important progress has been made in the knowledge of experimental biases, it deserves more detailed comparisons between experimental and theoretical results, particularly in order to solve ambiguities related to the competing role of the different components contributing to the nuclear interaction. Due to the violence of the collisions, and to finite size effects, the resort to dynamical approaches are unavoidable to obtain quantitative information on nuclear matter properties. In this framework, let us address the comparison of the Landau-Vlasov results with the experimental data obtained with $4\pi$ detector arrays above 100MeV/n. In the following we will define the flow parameter according to Ref [20]:

$$F = \left( \frac{d < P_x > / m}{dY} \right)_{Y_{cm}}$$

(5)

where $< P_x >$ is the mean value of the transverse momentum in the reaction plane and $Y$ and $Y_{cm}$ are particle and center of mass rapidities, respectively. It provides informations about the collective momentum transfers which take place in the reaction plane at mid-rapidity. Let us note that in this definition the projectile rapidity is not normalized with respect to the center of mass rapidity, in consequence this must be taken into account for comparisons at different energies.

We begin our analysis with a symmetric system, Nb+Nb, since it has been studied first, and for which experimental data are available [21]. Figure 3 shows the flow as a function of the incident laboratory energy. In order to select the same class of events as in experiments, the calculations have been made at b=4fm.
Let us recall that the experimental data display a rather smooth behavior with particle multiplicity bins, and they do not exhibit a strong variation with the impact parameter. The theoretical results are filtered according to the Plastic Ball/Wall device. The filter provided by the experimentalists has been implemented, and its effects on the results have been found rather small. The open (full) symbols correspond to soft (stiff) Gogny interactions (see Table 2).

In figure 3-a, the results obtained with the D1-G1 and D1-G3 forces are reported with squares. The two forces stay close together up to 250 MeV/n but they separate from one other at higher energies. Those corresponding to the D1-G7 and D1-G8 interactions are shown in figure 3-b. For sake of comparison, the flow values for the stiff local force \(K_\infty=380\) MeV have been represented in figure 3-b by crosses. It can be observed that they follow closely the experimental values, as well as those associated to the D1-G7 parametrization. Up to 200 MeV/n all parametrizations of the Gogny force give quite similar results and they are in agreement with the available data [21]. This indicates that in this energy range the flow observable cannot provide quantitative informations on \(K_\infty\) [2].

On the other hand, the flow is clearly ruled by the behavior of the optical potential at saturation density. As it is well known, the collective transverse motion in a reaction is essentially created by the following mechanisms: the compression related to the density built up, the nucleon-nucleon collisions, and the momentum dependence of the mean field, i.e. when a non local potential is used. This latter effect comes from the fact that the attraction of the potential decreases when the relative velocity of the interacting nucleons is high. Since the relative momentum is maximum in the entrance channel, the velocity dependence acts mainly at the beginning of the reaction before a significant enhancement of the density is established. This early repulsive behavior leads the system to reach lower maximum densities for momentum dependent forces than for local ones. According to Fig.1, below 200 MeV/n as long as the momentum dependence of the different mean-fields is similar, and since in this energy region the slope of the optical potential is the steepest, the rather weak density variations attained
with non-local forces (see [2]) does not allow an important sensitivity of the flow to the $K_\infty$ value. This argument explains why all EOS signatures found out with local forces are washed out when dealing with non-local interactions at incident energies lower than 200 MeV/n. In consequence, a proper description of the dynamics appears to be essential to well understand the construction of collective observables.

When the energy increases, a much clearer influence of the incompressibility modulus on the flow values is recovered. The results corresponding to the soft D1-G7 ($K_\infty = 228$ MeV) force stay close to the data whereas those for the stiff D1-G8 ($K_\infty = 300$ MeV) interaction attain higher values. Let us note that the difference between the flow parameters obtained with the soft and with the stiff forces is, in this case, lower than in Ref [4]. This is due to the fact that our $K_\infty$ values differ in 72 MeV instead of 166 MeV as in Ref [4]. On the other hand, the difference in $K_\infty$ between D1-G1 and D1-G3 is 132 MeV, and we see a more pronounced separation in the corresponding flow values in Fig. 3.a. The optical potential of the two forces is however not realistic (see Fig.1). The reason of the change in the behavior of the flow with respect to the EOS above 200 Mev/n can be seen in Fig. 1. At those energies, the real part of the optical potential reaches a plateau at positive energy and therefore the repulsive character does not change any more with momentum. The sensitivity to the density, and consequently to $K_\infty$, is then recovered, since we know from static considerations [2] that the density-dependent part of the potential constitutes the main contribution to $K_\infty$.

At this point, it is interesting to note from Fig.3.b that the difference in the calculated flow for the two $K_\infty$ values seems to decrease for incident energies higher than 300 MeV/n (in this case b was kept constant for all energies). This aspect can be understood from Fig.4, which also illustrates the behaviour of the flow with the mass of the colliding nuclei. Indeed, in Fig.4 we reported the flow as a function of the impact parameter for a heavier system, the Au+Au reaction at 400 MeV/n, with the D1-G7 and the D1-G8 Gogny forces. We observe that even though the mass of the system is very different to the preceding case (Nb+Nb),
the soft D1-G7 force also describes correctly the experimental \cite{21} flow values in all the impact parameter range. For the hard D1-G8 force, even if in more peripheral collisions the agreement with the data \cite{21} is good, an overestimation of the order of 20\% is observed at small $b$. It appears from Fig.4 that for increasing $K_{\infty}$, at higher energies, instead of a real decrease of the maximum flow value, there is a shift in its location towards smaller impact parameters, where density effects are expected to be stronger. This explains then the findings of Fig.3.b, where the impact parameter has been kept constant.

It is encouraging to pin down complementary informations on the nuclear interaction through the flow analysis at different ranges of incident energy. What we learn from the comparisons between the data and the simulations in Figs.3 is that if the optical potential does not stick to the experimental trends, as for example with the D1-G3 or D1-G1 parametrizations, it is then necessary to introduce a strong increase of the incompressibility modulus in order to get flow values compatible with the experimental data. This strong increase is in contradiction with most of the theoretical as well as the experimental results already obtained in other contexts \cite{22}. We also note that with a rather good representation of the expected behavior of the optical potential, as those displayed by D1-G7 and D1-G8 parametrizations, a sufficiently clear and important difference appears between soft and stiff EOS, which can hardly justify the use of higher incompressibility modulii in the interpretation of the experimental data. This holds true, even though the accuracy of both the experimental and theoretical results does not allow to extract a precise value of $K_{\infty}$. Nevertheless, the stability of the theoretical results as well as the steady convergence of different approaches \cite{6,10} related to various numerical treatments and effective nuclear effective interactions show that a detailed investigation of this topic is of particular interest.

To illustrate this statement, a confrontation between the results of two different models, Boltzmann-Uehling-Uhlenbeck (BUU) and Landau-Vlasov (LV), is reported in Figs. 5. In Fig. 5.a we reproduce the BUU flow yields of Ref. \cite{4} as a function of the proton multiplicity in the Nb-Nb reaction at 400 MeV/n. One
of the implemented forces in this reference was a momentum independent interaction of the Skyrme type (The Zamick force [19]). Diamonds and open circles represent, respectively, the flow values obtained with soft (K_{\infty}=199 \text{ MeV}) and hard (K_{\infty}=371 \text{ MeV}) Zamick forces, in the framework of the BUU approach [4]. Full squares and full circles represent the flow values calculated with the soft (K_{\infty}=200\text{MeV}) and hard (K_{\infty}=380\text{MeV}) versions of the Zamick force with our L-V model. In Fig. 5.b the mean transverse momentum in the reaction plane is displayed as a function of the normalized rapidity Y/Y_{proj} for the same reaction at an impact parameter of 5 Fm. We have plotted the results of our L-V calculations performed with the hard Zamick force for the emitted particles (stars) and for the overall distribution (open circles). In the same figure we represented BUU results concerning the emitted particles (squares) [23] calculated with the same hard EOS as in Fig. 5.a (K_{\infty}=371 \text{ MeV}). The agreement between the two kind of calculations clearly appears, not only from flow plots, but also from the detailed <P_x> versus Y/Y_{proj} scheme, in all the rapidity range.

Despite of the fact that local forces fail to reproduce the real part of the optical potential, surprisingly, they reproduce correctly the experimental trends in symmetric reactions like the Nb+Nb, over the whole energy range here considered when a high incompressibility is introduced. Even if such an observation does not suffice to justify this high value, we still should try to remove this ambiguity. According to [4] a convenient way to do this should be provided by asymmetric systems. We have addressed this question in the case of the Ar+Pb system for a beam energy of 400 MeV per nucleon. This reaction has been performed several years ago using the Diogene detector [24, 25]. This 4\pi detector has large experimental biases and theoretical results must be filtered. The filter used here can be decomposed into two parts: i) a geometrical cut avoids the detection of a particle outside the \Theta_{lab} region between 20 and 132 degrees. ii) an energy cut, which only allows the detection of particles having transverse momenta such that:
\[ P_\perp \geq 0.36 + 0.72 \times Y \quad \text{if} \quad Y \leq 0, \]
\[ P_\perp \geq 0.36 - 0.80 \times Y \quad \text{if} \quad Y > 0, \]

Fig. 6 shows the calculated flow for the stiff Zamick force as well as for the D1-G7 and D1-G8 Gogny forces as a function of the impact parameter. In order to compare with Refs. [4, 5], a linear fitting for rapidities roughly between −0.2 and +0.2 has been done, and the slope in Eq. 5 has been now calculated at \( Y_0 \), the rapidity of intercept \( (< P_x > (Y_0) = 0) \), instead of at \( Y_{cm} \). The non-local forces give similar results which do not depend noticeably on their incompressibility modulii. This finding is in agreement with Ref. [4] and is a particularity of very asymmetric systems.

Also the results corresponding to both Gogny interactions are in good agreement with the data for the whole impact parameter range, even though they slightly overestimate the flow in the vicinity of the maximum. As a matter of fact smearing over some impact parameters as required from experiments should slightly decrease the corrected results in this region. We should like to emphasize here that the momentum dependent part of the mean field is calculated from a full convolution of the force with the momentum distribution, that is:

\[ \int d^3p' \ V(\vec{r} - \vec{p}') \ f(\vec{r}, \vec{p}') \]  

(6)

and we did not replace in \( V \) the momentum \( \vec{p}' \) by its average value \( < \vec{p}' > \) as this was often done in the past [26]. Nevertheless we investigated the effect of this approximation (forces D1-G7* and D1-G8* in Fig.6) and found that this considerably enhances the flow (full symbols), a finding in agreement with the work of Zhang et al. [5]. Also shown are in Fig.6 the results with the stiff Zamick force (crosses). They undershoot the experimental results in the same way as in the two preceeding works on the subject [4, 5]. Let us stress here the fact that our calculations are remarkably stable with respect to the sampling of the nuclear phase space. We have fixed the number of gaussians per nucleon in 120, inorder to minimize numerical fluctuations without exceeding reasonable computing times.
In this way, these fluctuations remain negligible compared with the deviation of our theoretical results from experimental data, whatever the force is.

5 Conclusions

A theoretical analysis of collective transverse momentum appearing in heavy-ion collisions has been addressed in the framework of a full dynamical description. This is imposed by out-of-equilibrium as well as finite-size effects which must be explicitly taken into account in order to get a proper description of the collective mechanisms. In this framework, a special attention has been carried on the flow observable since many progress has been made in the understanding of the experimental biases during the last decade, permitting more reliable comparisons between experimental data and theoretical results.

The main objective remaining the extraction of quantitative as well as qualitative informations on the nuclear matter properties, a first interesting feature is provided by the results related to asymmetric systems. As a matter of fact, it is observed in this case that the non-local character of the nuclear interaction must be definitely taken into account to describe the flow values. Otherwise, it should be necessary to increase so highly the incompressibility modulus that this will lead to contradictions with respect to already observed experimental trends. This fact being established, the theoretical simulations permit to point out a transition in the dynamical build-up mechanisms of the flow observable, in the energy range here considered. For low energies, the dynamics of the entrance phase of the reaction governs the flow behavior which is then completely ruled by the momentum dependence of the optical potential, with practically no sensitivity to the density dependence. This trend is completely inverted at the higher energies, where the optical potential variation with the momentum becomes flatter. In this case the sensitivity of the flow values to the density dependence and, consequently, to the incompressibility modulus is retrieved. This observations are very encouraging since it demonstrates the ability of this observable to provide
rather complex and complementary informations on the nuclear interaction. It will be certainly valuable to pursue experimental and theoretical investigations to obtain more precise informations, despite we can already ascertain together with the findings of other approaches [4, 5] that the experimental data strongly suggest an incompressibility value around 210-220 MeV.

Two of us (Prof. P. Schuck and F. Haddad) wish to thank the NSCL staff for its warmly hospitality and especially Prof. P. Danielewicz for fruitful discussions. This work is partly supported by the United State Department of Energy under Grant No DE-FG05-86ER40256 and by the Robert A. Welch Foundation.

References


P. Schuck, R.W.; Hasse, J. Jaenicke, C. Gégoire, B. Remaud, F. Sébille, and E. Suraud; Prog. in Part. and Nucl. Phys. 22 (1989) 181

   J.P. Blaizot, J.F. Berger, J. Dechargé and M. Girod; Saclay Preprint T95/021


Figure Captions

Figure 1. Momentum dependence of the real part of the optical potential at saturation density with Gogny D1-type forces. Data correspond to Ref.[17].

Figure 2. The real part of the optical potential at densities $\rho=0.2, 0.3, 0.4$ Fm$^{-3}$. Comparison between the D1-G7 interaction and the microscopic UV14+TNI interaction of Wiringa [18].

Figure 3. Flow parameter as a function of the incident energy for the Nb+Nb reaction at $b=4$fm. Stars represent data from Ref.[21]. Full(open) symbols correspond to stiff(soft) Gogny forces and crosses to the stiff Zamick force.

Figure 4. Flow parameter as a function of the impact parameter for the Au+Au reaction at 400MeV/n, calculated with the Gogny forces D1 G7 and D1 G8. Experimental data is from Ref.[21].

Figure 5.a) Flow parameter as a function of proton multiplicity in Nb+Nb at 400 MeV/n. Stars correspond to data [21]. The full(open) symbols correspond to LV (BUU [4]) calculations.

Figure 5.b) Mean transverse momentum in the reaction plane as a function of the normalized rapidity for the same reaction as in (a) with Zamick forces. Squares represent BUU calculations [4] with $K_\infty=371$MeV for the emitted particles, open circles (stars) correspond to LV calculations with $K_\infty=380$MeV for all (emitted) particles.

Figure 6. Flow parameter as a function of the impact parameter for the Ar+Pb reaction at 400MeV/n with different forces. For details see text. The experimental data [21] are represented with stars.
Fig 1:

Fig 2:
Fig 3
Fig 5.a
Fig 6