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Double Giant Resonances

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Abstract

We investigate the non-linear response of a quantum anharmonic oscillator as a model for the excitation of giant resonances in heavy ion collisions. We show that the introduction of small anharmonicities and non-linearities can double the predicted cross section for the excitation of the two-phonon states. These findings suggest that such ingredients must be included in future more complete calculations in order to reduce the huge discrepancy between the previous theoretical predictions and the experimental cross section of double giant resonance states.

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1 Introduction

Soon after their discovery in 1947 nuclear giant resonances (GR) have been interpreted as the first quantum of a collective vibration. In particular, the giant dipole resonance has been understood as the vibration of the proton fluid against the neutron one[1, 2]. Since then, giant resonances have been extensively studied both theoretically and experimentally. The first experimental indication of a possible multiple-excitation of giant resonances (i.e. of the second vibration quantum) dates back to 1977 when some bumps have been observed in the cross-section of heavy ion inelastic scattering [3]. Further evidence has been accumulated in heavy ion inelastic scattering at intermediate energy ($E/A \approx 40 - 100$ MeV) [4, 5]. At the end of the 80's double phonon excitations have been observed in double charge-exchange reactions [6, 7, 15]. The availability of high energy ($E/A \approx 1$ GeV) heavy ion beams has recently made possible to observe quite large cross sections for the Coulomb excitation of such double GR modes[5, 8, 9].

Most theoretical calculations [5] aimed at the interpretation of such processes involve a harmonic picture for the collective vibrations in each of the two colliding nuclei. The excitation of the one- and two-phonon states is described as due to the action of the mean nuclear and/or Coulomb fields of the other nucleus, assumed to be linear in the phonon creation and annihilation operators. This implies that the only elementary process considered is the excitation or deexcitation of one vibration quantum. A common feature of such calculations is that the resulting cross sections of the two phonon states are 2 to 4 times smaller than the experimental ones[5]. In the most advanced calculation of ref. [10] investigating the effects of the spreading width of the one-phonon states, the probability of double excitations is simply expressed in terms of the product of two single-phonon excitation amplitudes. Therefore, from this point of view, the calculation of ref. [10] reduces essentially to a standard linear response in a harmonic approximation [5, 11, 12]. However, from the observed discrepancy between experimental and theoretical cross-section of double excitations, it was concluded that the phonon picture of giant resonances might be inadequate [10].

The purpose of this paper is to show that the observed discrepancy might be a consequence of the anharmonic nature of the considered vibrations and of the non-linear components of the external field. In particular, we will focus on the Coulomb excitation of giant resonances at relativistic energies. We will conclude that even small anharmonicities and non-linearities which do not modify significantly our understanding of the structure and excitation of single giant resonances, might strongly affect the excitation probability of the two-phonon states.

It has been argued that in a 3D calculation the presence of spin and parity quantum numbers may strongly reduce the possibility of introducing couplings between multiphonon states, but one has to take into account the large number of quasi-degenerate collective states with different quantum numbers present in the nucleus from the monopole (GMR), the dipole (GDR), the quadrupole (GQR) to the hexadecapole vibrations. In fact, anharmonicities of giant resonances have already been investigated in the framework of boson mapping methods [13, 14, 15]. The effect of the inclusion of anharmonicities is twofold: to shift the eigenenergies and to change

the eigenstates of the internal hamiltonian into a superposition of states with different numbers of phonons. The typical values of the residual two-phonon interaction have been found to be of the order of one-half to one MeV. The matrix elements mixing one- and two-phonon states are of the same order of magnitude. Many other estimations of the anharmonicities of giant resonances can be found in the literature [5]. They all consistently yield to the same order of magnitude. Experimentally the observations are consistent with a small anharmonicity in the multiphonon energies at maximum of the order of 10%.

Another important ingredient to be considered is that the external field is in principle non-linear. In particular, in an already excited nucleus an external one-body field may induce transitions between particle (or hole) states and therefore induce direct transitions between two excited states. These kinds of non-linearities have already been considered in the past mostly within classical descriptions of heavy ion reactions such as for example in the Copenhagen model of ref. [11] in which one considers the feedback of the deformations onto the coupling potential. However, the consequences of both anharmonicities and non-linearities have never been fully investigated at the quantum mechanical level and in particular their consequences on the transition amplitudes towards specific levels are unknown. Since the anharmonicities modify energies, wave functions and therefore transition matrix elements, they will change the selection rules thus allowing transitions that are forbidden in the harmonic picture. For example they can make possible the direct excitation of two-phonon states in a one step process. The non-linearities are providing new routes for the excitation of multiphonon states [5]. Moreover, they generate a coupling between eigenstates which introduces some dynamical modification of their energy. We will see in this paper that all these factors may contribute to strongly modify the predicted excitation probability reducing the gap between experiment and theory.

2 Microscopic description

For simplicity we will only consider the excitation of one of the two nuclei (say the target), the generalization to the case in which the other one can also be excited being straightforward. The total hamiltonian of the system will be the sum of the internal hamiltonian H_T of the target (which will be denoted by T in the following) and the mean field $W(t)$ of the projectile (denoted by P)

$$H = H_T + W(t) = \sum_i \epsilon_i a_i^\dagger a_i + \frac{1}{4} \sum V_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k + \sum_{ij} W_{ij}(t) a_i^\dagger a_j, \quad (1)$$

where V is a two-body residual interaction and the ϵ_i 's are the Hartree-Fock single particle (s.p.) energies. The external field $W(t)$ depends on time through the relative distance.

The collective states of nuclei are usually described in the framework of the Random Phase Approximation (RPA). The latter can be obtained assuming that the fermion pair operators behave as bosons. To this end one introduces boson degrees

of freedom through the mapping:

$$a_p^\dagger a_h \longrightarrow B_{ph}^\dagger + \dots \quad (2)$$

where the operator B fulfills canonical commutation relations for bosons. We denote by p (particle) the unoccupied s.p. states and by h (hole) the occupied ones. The lowest order (at most quadratic in the B and $B^\dagger$ operators) in such a boson expansion of H_T leads to a boson hamiltonian which can be diagonalized by introducing a Bogoliubov transformation defining new operators (see also ref. [14, 5] for a more general discussion)

$$O_\nu^\dagger \equiv \sum_{ph} \left(X_{ph}^{(\nu)} B_{ph}^\dagger - Y_{ph}^{(\nu)} B_{ph} \right) , \quad (3)$$

where the amplitudes X and Y satisfy the usual RPA equations[15].

The boson hamiltonian takes then the diagonal form of an ensemble of decoupled harmonic oscillators

$$H^{(RPA)} = E_0^{(RPA)} + \sum_{\nu} E_{\nu}^{(RPA)} O_{\nu}^\dagger O_{\nu} \quad (4)$$

where $E_0^{(RPA)}$ is the RPA ground state energy and $E_{\nu}^{(RPA)}$ are the excitation energies of the phonon states ν .

When we substitute the original internal hamiltonian H_T by $H^{(RPA)}$, not only the boson mapping of H_T has been truncated at the lowest order but the pppp, hhhh, ppqh and phhh terms of the residual interaction V have been neglected. This residual interaction gets contributions, for example, from the boson mapping of two-particle (pp) or two-hole (hh) operators

$$a_p^\dagger a_{p'} \longrightarrow \sum_h B_{ph}^\dagger B_{p'h} \quad (5)$$

$$a_h a_{h'}^\dagger \longrightarrow \sum_p B_{ph}^\dagger B_{p'h'} \quad (6)$$

and from the higher orders of the boson mapping of particle-hole pair operators eq.(2) which simulate Pauli blocking effects. The inclusion of the residual interaction among phonons leads to a boson hamiltonian containing products of three, four and more $O^\dagger$ and/or O operators. As a consequence, the spectrum will be no more harmonic and some mixing among different RPA phonons will be present. Thus, the eigenstates of the boson image of H_T will be

$$\begin{aligned} |\phi_\alpha\rangle &= A_\alpha |0\rangle + \sum_{\nu} B_\alpha^\nu O_\nu^\dagger |0\rangle + \sum_{\nu_1 \leq \nu_2} \frac{C_\alpha^{\nu_1 \nu_2}}{\sqrt{1 + \delta_{\nu_1 \nu_2}}} O_{\nu_1}^\dagger O_{\nu_2}^\dagger |0\rangle + \dots \\ &= A_\alpha |0\rangle + \sum_{\nu} B_\alpha^\nu |\nu\rangle + \sum_{\nu_1 \leq \nu_2} C_\alpha^{\nu_1 \nu_2} |\nu_1 \nu_2\rangle + \dots \end{aligned} \quad (7)$$

(for simplicity of notation we do not introduce explicitly the angular momentum coupling).

During the collision, the target nucleus T will perform transitions due to the action of the mean field W of the projectile nucleus P. When expressed in terms of RPA bosons the latter contains at least linear and quadratic terms in the phonon creation and annihilation operators

$$W = W_{00} + \sum_{\nu} W_{\nu}^{10} O_{\nu}^{\dagger} + h.c. + \sum_{\nu_1 \leq \nu_2} W_{\nu_1 \nu_2}^{11} O_{\nu_1}^{\dagger} O_{\nu_2} + \sum_{\nu_1 \leq \nu_2} W_{\nu_1 \nu_2}^{20} O_{\nu_1}^{\dagger} O_{\nu_2}^{\dagger} + h.c. + \dots \quad (8)$$

Using the boson mapping (2) truncated at the second order and the definition of the RPA bosons eq.(3) it is easy to show that

$$W_{\nu}^{10} = \sum_{ph} (W_{ph} X_{ph}^{\nu*} + W_{hp} Y_{ph}^{\nu*}) \quad (9)$$

is the standard linear response expression, whereas

$$W_{\nu\nu'}^{11} = \sum_{php'h'} (W_{pp'} \delta_{hh'} - W_{hh'} \delta_{pp'}) (X_{ph}^{\nu*} X_{p'h'}^{\nu'} + Y_{ph}^{\nu*} Y_{p'h'}^{\nu'}) \quad (10)$$

$$W_{\nu\nu'}^{20} = \sum_{php'h'} (W_{pp'} \delta_{hh'} - W_{h'h} \delta_{pp'}) X_{ph}^{\nu*} Y_{p'h'}^{\nu'} \quad (11)$$

provide new excitation routes. It is obvious that adding more terms to the mapping (2) would result in higher order non-linear contributions to W . At the considered order, we see that the W^{11} term describes transitions among one-phonon states, in particular through transitions between two particle or two hole states. As shown in ref.[16, 17, 18] the latter terms can lead to an increase of the population of the two-phonon states. The second non-linear term W^{20} introduces a direct connection between the ground state and the two-phonon states. This connection is made possible through the pp and hh components of the residual interaction and through the presence of RPA correlations (in particular of the 2p2h type) in the ground state.

As an example of these new excitation routes, let us consider the Coulomb excitation of the $|GDR \otimes GQR\rangle$ state coupled to $J=1$ corresponding to the excitation of one dipole GDR and of one quadrupole GQR. This state can be populated first via the usual two step process involving the linear terms in W which is, however, strongly suppressed because in addition to the dipole excitation it involves a quadrupole transition. It can also be populated directly through the W^{20} term. In addition to these possibilities one should also consider another process consisting first in the excitation of the GDR followed by a transition to the GQR due to the W^{11} terms in the excitation operator and finally in a new dipole excitation to reach the $|GDR \otimes GQR\rangle$ state. Thus we expect a depopulation of the GDR and an enhanced population of the $|GDR \otimes GQR\rangle$ state. Moreover, the presence of anharmonicities will introduce some mixing of states with different number of phonons (see eq. 7), opening different routes to access the considered region. In particular, the $|GDR \otimes GQR\rangle$ state can be populated via its mixing with the pure GDR state. This direct transition being E1 will not be suppressed in the Coulomb excitation.

3 A Simple Model

In order to investigate the relative consequences of the anharmonic terms in the hamiltonian describing the target nucleus and of the non-linear terms present in the external field describing the action of the projectile, we will study in the present paper a simplified model. More complete calculations are in progress and will be published in a forthcoming paper [19]. Let us then consider a single harmonic oscillator of frequency ω and of mass m ($\hbar = 1$) whose hamiltonian is

$$H^{(RPA)} = \frac{1}{2m}P^2 + \frac{1}{2}m\omega^2 X^2 = E_0^{(RPA)} + \omega O^\dagger O \quad (12)$$

Since the model we are considering is intended to mimick the excitation of the GDR, the mass of the harmonic oscillator has been taken equal to the reduced mass $NZ/(N+Z)$ of neutrons and protons in the considered nucleus. The frequency ω has been fixed to reproduce the GDR energy. Let us now add anharmonic terms

$$\Delta V = \frac{1}{3}\alpha X^3 + \frac{1}{4}\beta X^4 \quad (13)$$

Such terms simulate the cubic and quartic terms present in the boson image of the hamiltonian H_T of the target nucleus. The strength of these anharmonic terms can be fixed by imposing that some of their matrix elements have values of the order found in microscopic calculations [13]. In the following application we have imposed that the expectation value of the normal ordered $\frac{1}{4}\beta X^4$ operator in the two-phonon state as well as the matrix element of $\frac{1}{3}\alpha X^3$ between the one- and the two-phonon states are of the order of 1 MeV. It should be noticed that this value corresponds to a rather small anharmonicity. The sign of these two components has also some importance. As far as the eigenenergies of the target internal hamiltonian are concerned, the sign of α is irrelevant. It can only be defined with reference to the sign of the external field. The sign of β has a direct consequence on the excitation spectrum of the target nucleus. Indeed if β is positive the potential is stiffer than the harmonic one and the corresponding eigenfrequencies have higher values. Conversely, when β is negative the potential is softer and the energies are reduced. The anharmonicities are expected to produce a reduction of the collectivity of the RPA states. Therefore, for isovector resonances (as the GDR) they should lower the energy of the double resonance. Since the microscopic estimations of the matrix elements of the residual interaction between two-GDR phonon states have been found to be negative[13], we have chosen a negative β . It should be noticed that in the latter case the total hamiltonian is not bound from below making the ground state of the system ill-defined. This can be remedied either adding a small positive term with a higher X power or by performing the calculation in a restricted subspace. We have checked that both procedures do not affect the results we are going to present.

Diagonalizing the internal hamiltonian, $H_T = H^{(RPA)} + \Delta V$, we construct the eigenstates $|\phi_\alpha\rangle$ and the excitation energies E_α . This diagonalization is performed in the basis of the harmonic oscillator states $|n\rangle$, so defining the coefficients C_α^n of the expansion of $|\phi_\alpha\rangle$ on the $|n\rangle$ basis. It should be noticed that the introduction

of ΔV also modifies the excitation energy E_1 of the first phonon state. In order to avoid the trivial consequences of this modification which would have nothing to do with anharmonic effects, the excitation energy E_1 has been kept equal to that of the considered GR by a suitable overall renormalization of the hamiltonian.

In a relativistic reaction between heavy ions the strong transverse electric field

$$E_{\perp}(t) = -\frac{Z_P e \gamma b}{(b^2 + (\gamma v t)^2)^{\frac{3}{2}}} \quad (14)$$

dominates. In eq.14 it is assumed that the projectile of charge Z_P is travelling on a straight line trajectory defined by an impact parameter b and a constant velocity v associated with the Lorentz contraction factor γ [20]. Therefore, the excitation of the transverse GDR degrees of freedom in a nucleus of mass A_T , charge Z_T and neutron number N_T , in the linear response approximation, can be simulated in our one-dimensional oscillator model by the linear external field

$$W(t) = \frac{Z_T N_T e}{A_T} E_{\perp}(t) X = F(t)(O^{\dagger} + O) \quad (15)$$

with

$$F(t) = -\frac{Z_T N_T}{A_T} \frac{Z_P e^2 \gamma b}{(b^2 + (\gamma v t)^2)^{\frac{3}{2}}} \sqrt{\frac{1}{2m\omega}} \quad (16)$$

which corresponds to the excitation of the GDR with 100 % of the energy weighted sum rule.

The next step is the inclusion of non-linear terms in this external field. The most general effects of such non-linearities cannot be fully studied in the present oversimplified model because we do not have neither different angular momenta and parities, nor different kinds of phonons as it is the case in a more realistic calculation where the mutual influence of the GDR, GQR and low lying collective states or other resonances plays an important role. It is however instructive to study these non-linearities in order to see whether they can appreciably modify the predicted excitation probabilities. In an RPA based calculation of the various matrix elements of the external field (see eq.8) we have seen that the dipole matrix element $\langle GDR | W^{11} | GQR \rangle$ (eq.10) is about 1/6 of the matrix element $\langle GDR | W^{10} | 0 \rangle$ (eq.9) with the same sign. Therefore we have added to the external field (eq.15) a term in $O^{\dagger}O$ with such strength. From microscopic calculations, the terms in $O^{\dagger}O^{\dagger}$ and OO are expected to be much smaller than the $O^{\dagger}O$ ones, about a factor 6 smaller. So, $\langle 2 | W^{20} | 0 \rangle$ has been fixed to be 1/36 of $\langle GDR | W^{10} | 0 \rangle$ and we have verified that the latter terms have practically no effect in the presented results.

In order to describe the evolution of the system we can express the state $|\Psi(t)\rangle$ as a superposition of the eigenstates $|\phi_{\alpha}\rangle$ of H_T :

$$|\Psi(t)\rangle = \sum_{\alpha} A_{\alpha}(t) e^{-iE_{\alpha}t} |\phi_{\alpha}\rangle \quad (17)$$

The solution of the time dependent Schroedinger equation in presence of the external field $W(t)$ amounts to the solution of a system of coupled differential equations in the

amplitudes $A_\alpha(t)$

$$\dot{A}_\alpha(t) = -i \sum_{\alpha'} e^{i(E_\alpha - E_{\alpha'})t} \langle \phi_\alpha | W(t) | \phi_{\alpha'} \rangle A_{\alpha'}(t) \quad (18)$$

We have solved this set of equations using the Runge-Kutta method. The probability to excite the internal state of the target $|\phi_\alpha\rangle$ will then be equal to

$$P_\alpha = |\langle \phi_\alpha | \Psi(t = \infty) \rangle|^2 = |A_\alpha(t = \infty)|^2 \quad (19)$$

and the integrated cross sections σ_α can be calculated as

$$\sigma_\alpha = 2\pi \int_{b_0}^{\infty} P_\alpha(b) b db \quad (20)$$

where b_0 has to be taken such that contributions from the nuclear part of the external field can be neglected. Following ref. [10], we have chosen $b_0 = r_0(A_P^{1/3} + A_T^{1/3})$ with $r_0 = 1.5$ fm. In order to check the influence of this parameter we have also performed the calculations of the cross sections σ_α with $r_0 = 1.2$ fm.

4 Excitation probability and cross-section

We have applied our simple model to the reaction analysed in ref. [8], $^{136}\text{Xe} + ^{208}\text{Pb}$ at $E/A = 700$ MeV, where the excitation of the projectile ^{136}Xe has been studied. We have fixed the energy E_1 of the GDR in ^{136}Xe equal to the experimental value 15.2 MeV. In order to investigate the respective roles of anharmonicities and nonlinearities, we have considered different situations which are described below.

4.1 Linear excitation of a harmonic oscillator

First of all, let us study the reference calculation of a linear excitation of a harmonic oscillator, i.e. $\alpha = 0 = \beta$, with only the terms W^{10} and W^{01} of the external field different from zero. In this case, the state of the target at time t has the form of a coherent state [5, 11, 12]

$$|\Psi(t)\rangle = e^{-\frac{|I(t)|^2}{2}} \sum_n \frac{I(t)^n}{\sqrt{n!}} e^{-in\omega t} |n\rangle \quad (21)$$

with

$$I(t) = \int_{-\infty}^t W^{10}(t') e^{-i\omega t'} dt' \quad (22)$$

The probability to find the system in the n -phonon state $|n\rangle$ follows the Poisson distribution

$$P_n = e^{-|I(\infty)|^2} \frac{(|I(\infty)|^2)^n}{n!} \quad (23)$$

It should be noticed that, except for the normalization factor $e^{-|I(t)|^2}$, these results are identical to those obtained with a perturbative calculation. In the first line of

table 1 we show the cross section associated with the one- and two-phonon states σ_1 and σ_2 and the ratio σ_2/σ_1 . These results directly compare with the prediction of the sophisticated perturbative calculation of ref. [10]. Indeed, in this reference the cross section of the first phonon state is predicted to be 1480 mb while the one of the two phonon state is around 50 mb. The difference between these numbers and ours can be attributed to two facts: on one hand, in table 1 we have considered the transverse electric field and the associated dipole excitation; on the other hand, we are performing an exact calculation. When we repeat the calculation taking into account the mutual excitation of longitudinal and transverse degrees of freedom, we get in an exact calculation 1352 mb for the total cross section of the first phonon states and 44 mb for the two phonon states; in perturbation theory these numbers become 1446 mb and 49 mb respectively, in good agreement with those of ref.[10]. As discussed in ref.[10], this result illustrates the fact that at such a high incident energy the inclusion of a width has a minor influence on the total cross section. The calculated cross section for the one-phonon excitation is of the right order of magnitude compared with the experimental value for the total cross section observed in the GDR region $\sigma_1 = 1485 \pm 100$ mb [8, 10]. It should however be noticed that some part of this cross section is expected to be due to the excitation of the GQR. The estimation of this contribution presented in [8] is about 460 mb, whereas in [10] this cross section is estimated to be around 170 mb. Concerning the two-phonon cross section σ_2 , it turns out to be a factor 4 smaller than the experimental cross section of 215 ± 50 mb associated with the bump at twice the GDR energy. It should be noticed that a reduction of the cutoff parameter r_0 cannot solve the problem because it leads to an increase of both cross sections, bringing to a too large value for σ_1 (see table 1). In the following we will show that the introduction of small anharmonicities and nonlinearities can produce a strong increase of the two-phonon cross section without a sizable modification of σ_1 . This is an indication that such effects have to be included in more realistic calculations and that they should give a better agreement between theoretical and experimental results.

4.2 Linear excitation of an anharmonic oscillator

We have calculated the response of the oscillator when the harmonicity of its spectrum is broken, still assuming the external field as linear in the phonon operators $O^\dagger$ and O . In this case no coherent state solution exists and we have to solve numerically the set of differential equations (18).

With reference to previous microscopic calculations of anharmonicities [5, 12], we have chosen the α and β parameters of eq.(13) so that the matrix elements $\langle 2|\alpha\hat{X}^3/3|1\rangle$ and $\langle 2|\beta\hat{X}^4/4|2\rangle$ are of the order of 1 MeV. This gives $\alpha = -38.6$ MeV/fm³ and $\beta = -188.18$ MeV/fm⁴. Correspondingly, the value of E_2 is 28.3 MeV, compatible with the observed position of the double GDR peak, 28.3 ± 0.7 MeV [5, 8]. This reassures us that the values we have chosen for the strength of the anharmonic terms are of a reasonable order of magnitude. In the second line of table 1 the results obtained are shown. We see that the inclusion of anharmonicities strongly enhances the cross section associated with the excitation of the two-phonon state. In

particular, comparing these results with those obtained in the reference calculation, we observe that the cross section of the two-phonon state has been multiplied by 1.7 while the one of the first phonon state remains constant. In order to understand at a qualitative level the origin of this variation, let us calculate P_1 and P_2 within perturbation theory. We have

$$A_1 = \int_{-\infty}^{+\infty} \langle \phi_1 | W(t) | \phi_0 \rangle e^{iE_1 t} dt = \langle \phi_1 | (O^\dagger + O) | \phi_0 \rangle \tilde{F}_1(E_1) \quad (24)$$

where we have introduced the Fourier transform of the external field $\tilde{F}_1(\omega)$ (see eq. (15)). Neglecting the small direct transition contribution, we get for A_2

$$\begin{aligned} A_2 &= \langle \phi_2 | (O^\dagger + O) | \phi_1 \rangle \langle \phi_1 | (O^\dagger + O) | \phi_0 \rangle \int_{-\infty}^{+\infty} F(t) e^{i(E_2 - E_1)t} dt \int_{-\infty}^t F(t') e^{iE_1 t'} dt' \\ &= \frac{1}{2} \langle \phi_2 | (O^\dagger + O) | \phi_1 \rangle \langle \phi_1 | (O^\dagger + O) | \phi_0 \rangle \tilde{F}_2(E_2 - E_1, E_1) \end{aligned} \quad (25)$$

where

$$\tilde{F}_2(E_2 - E_1, E_1) = \tilde{F}_1(E_2 - E_1) \tilde{F}_1(E_1) + \frac{i}{\pi} P \int \frac{d\omega}{\omega} \tilde{F}_1(E_2 - E_1 - \omega) \tilde{F}_1(E_1 + \omega). \quad (26)$$

Let us now consider the ratio P_2/P_1^2 which reads

$$\frac{P_2}{P_1^2} = \frac{1}{4} \frac{|\langle \phi_2 | (O^\dagger + O) | \phi_1 \rangle|^2 |\tilde{F}_2(E_2 - E_1, E_1)|^2}{|\langle \phi_1 | (O^\dagger + O) | \phi_0 \rangle|^2 |\tilde{F}_1(E_1)|^4} \quad (27)$$

In the harmonic limit ($E_2 = 2E_1$) the principal value integral of eq.(26) vanishes. Therefore, since $|\phi_2\rangle$ is a pure two-phonon state the ratio (27) is equal to 1/2.

When anharmonicities are taken into account, this ratio will depart from this value. The first factor contains the effects of the anharmonicities present in the wavefunctions, while the second factor contains the effects of the anharmonicities of the energy spectrum. It turns out that both effects are quite important. To show this, in fig. 1, we report the ratio P_2/P_1^2 , as obtained from four different calculations. The first one, corresponding to the full line, is the complete calculation. The second one, dashed line, is a calculation in which the wavefunctions are those of the harmonic hamiltonian while the energies are the anharmonic ones. The third one, dash-dotted line, was obtained by considering harmonic energies while the wavefunctions are those of the anharmonic hamiltonian. For comparison, the results concerning the linear excitation of the harmonic oscillator are also plotted (dotted line). The perturbative calculation of the linear harmonic case gives for P_2/P_1^2 the value of one-half, while the harmonic coupled-channel calculations predict a value of $\exp(|I(\infty)|^2)/2$ (as follows from eq.(23)). Deviations between the two results appear in the small impact parameters region where the correction due to the average number of phonons $|I(\infty)|^2$ is not negligible. On the contrary, for larger impact parameters both results coincide and we can use perturbation theory to interpret the calculations. Looking at the figure, we see

that the anharmonicities in the wavefunctions give rise to a sizeable enhancement of the ratio with respect to the harmonic limit. This effect is essentially a constant factor, as expected from eq.(27), and indeed the observed increase corresponds to the increase of the ratio of the matrix elements $|\langle \phi_2 | (O^\dagger + O) | \phi_1 \rangle|^2 / |\langle \phi_1 | (O^\dagger + O) | \phi_0 \rangle|^2$. A very big effect comes also from the anharmonicities in the energy spectrum especially for large impact parameters. This behaviour can be qualitatively understood by evaluating analytically the first term in eq.(26) and neglecting the principal value integral in the same equation. In fact, in this approximation the ratio P_2/P_1^2 can be written as

$$\frac{P_2}{P_1^2} \simeq \frac{1}{2} \left[\frac{E_2 - E_1}{E_1} \frac{K_1((E_2 - E_1)\tau)}{K_1(E_1\tau)} \right]^2 \quad (28)$$

where K_1 is the modified Bessel function of order 1 and $\tau \approx b/\gamma v$ is the reaction time. This expression, in the large impact parameter limit, reduces to

$$\frac{P_2}{P_1^2} \longrightarrow \frac{E_2 - E_1}{2E_1} e^{[2(2E_1 - E_2)\tau]} \quad , \quad (29)$$

so P_2/P_1^2 is increasing as function of b because $(2E_1 - E_2)$ is greater than zero. From the physical point of view, the observed effect is related to the increase of the reaction time τ with the impact parameter b . Now, if we turn to the cross section, we observe that the effects of the matrix elements lead to $\sigma_2 = 48$ mb while the modifications of the energy spectrum alone give $\sigma_2 = 40$ mb. Therefore both effects equally contribute to the increase of the two-phonon cross section. In ref.[10] the mixing of a huge number of one- and two-phonon components in the wavefunction of the GDR was considered, leading to a good description of its width. However, in the calculation of the two-phonon excitation amplitude the above described effects of the anharmonicities are missing.

4.3 Non-linear excitation of a harmonic oscillator

The next step is the inclusion, in the external field, of the non-linear terms in the creation and annihilation operators of phonons. Within the present model, with only one mode of excitation, the action of the term in $O^\dagger O$ is essentially to dynamically lower the energy of the multiphonon components of the states $|\phi_\alpha\rangle$ during the collision time. Therefore one may expect an increase of the cross section. On table 1 (third row) we can see that the non-linearities in the external field increase only slightly the excitation probability. This can be easily understood by realizing that in this case the total hamiltonian is still at most quadratic in the field operators so that the exact evolution is still given by a coherent state. Therefore, the Poisson distribution is preserved explaining the observed small effect of the non-linearities alone. Obviously, this effect is also present in a more realistic model, where it will add to the effect of the non diagonal terms in $Q_\nu^\dagger Q_\nu$ (see eq.8) which, as we said before, should lead to a selective enhanced population of some two-phonon state like the $|GDR \otimes GQR\rangle$ one and a depopulation of the GDR.

4.4 Non-linear excitation of an anharmonic oscillator

When both the anharmonicities and non-linearities are included (fourth row of table 1), we observe a strong increase of the two phonon cross section and of the ratio σ_2/σ_1 which is now a factor of 2 larger than that in the harmonic limit. With such prediction the disagreement between experiment and theory starts to be strongly reduced. It should be noticed that only the transverse excitation has been considered here. The same calculation for the longitudinal component of the external field alone gives 245 mb and 2.2 mb for the cross section of the one- and the two-phonon states, respectively (or 347 mb and 5 mb for $r_0 = 1.2$ fm). In a 3-dimensional calculation, the transverse and longitudinal components have to be considered together. Therefore one should have in addition to the two-phonons excited by the transverse field (TT) or by the longitudinal one (LL), the possible excitation of one transverse and one longitudinal (LT). Moreover, in order to compare the theoretical results with the experimental data, one should include more internal degrees of freedom (GDR, GQR, low-lying collective states, etc.). However, at this point, a realistic complete calculation is called for and is now under investigation [19].

4.5 Incident energy dependence

It can be interesting to study the dependence of the effects of the non-linearities and anharmonicities as a function of the projectile velocity. In Fig 2 we can observe that the ratio σ_2/σ_1 is strongly increased by the presence of anharmonicities and non-linearities at any incident velocity. This enhancement is larger at lower energy. This can be understood by recalling that the effect of both anharmonicities and non-linear terms is to lower the energy of the two-phonon state and that at low incident energy the Coulomb excitation probability is very sensitive to variations of the transition energy. It might be interesting to notice that the ratio σ_2/σ_1 presents a maximum around a given velocity which therefore provides optimum conditions for the experimental observations of two-phonon states. Depending upon the parametrisation and in particular upon the value of r_0 the most favorable incident energy lies between 100 MeV/A and 500 MeV/A.

5 Conclusions

We have explored the possibility that the large discrepancy between theoretical results and experimental data on the two-phonon excitation cross section can be reduced when anharmonicities in the internal hamiltonian and non-linearities in the external field are taken into account. In order to have a first insight on the problem, we have considered the simplified model of an anharmonic oscillator. The anharmonicities were introduced in agreement with microscopic calculation, by imposing that the residual interaction between phonons is of the order of 1 MeV. In this way, the main properties of the first excited state were kept unchanged while those of the second one were slightly modified ($\sim 10\%$ variation on its energy). From the presented results we can conclude that the inclusion of small anharmonicities and non-linearities

strongly enhance the excitation cross section of the two phonon states by a factor up to 2 without modifying much the population of the one phonon-state. Therefore, taking into account anharmonicities and non-linearities may fill the gap between the experimental results and the theoretical predictions. Realistic calculations are now in progress [19] in the case of multiple excitation of giant resonances in nuclei.

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TABLE CAPTION

Table 1: Cross-section of the Coulomb excitation of one- and two- phonon states, σ_1 and σ_2 , respectively calculated for a parameter $r_0 = 1.5$ fm. For comparison the value obtained with a parameter $r_0 = 1.2$ fm are also shown in parenthesis. The different rows correspond to different calculations. The first one is the reference calculation of a linear harmonic oscillator. In the second row we present the results of the calculations including only anharmonicities while the third one refers to the case in which only non-linear terms in the external field are included. The last row combines the effects of both non-linearities and anharmonicities.

FIGURE CAPTIONS

Figure 1: Ratio P_2/P_1^2 as a function of the impact parameter b , P_n being the excitation probability of the n -phonon state. All the curves correspond to calculations where only the linear excitation is taken into account. The dotted line shows the results of the reference calculation (harmonic), the solid line those of the complete calculation (anharmonicity in both energy spectrum and wave-function). The dashed line represents the results of the calculation in which only the anharmonicities in the energy spectrum are included. Finally, the dot-dashed line refers to the case where only the mixing in the wavefunctions is included. These calculations correspond to a projectile energy of 700 MeV/A.

Figure 2: Ratio of the excitation cross-section of the two- to the one-phonon states, σ_2/σ_1 , as a function of the incident energy for two parametrisation $r_0 = 1.2$ fm (A) and $r_0 = 1.5$ fm (B). The thin lines correspond to the reference calculation, harmonic and linear, while the thick line is associated with the complete calculation, non-linear and anharmonic.

	$\sigma_1(\text{mb})$	$\sigma_2(\text{mb})$	σ_2/σ_1	σ_1/σ_1^{ref}	σ_2/σ_2^{ref}
lin. and har. (ref)	1153 (1698)	33 (93)	0.029	1.	1.
lin. and anhar.	1267 (1835)	55 (139)	0.044	1.1	1.67
non-lin. and har.	1200 (1762)	36 (101)	0.030	1.04	1.09
non-lin. and anhar.	1352 (1938)	67 (166)	0.05	1.17	2.04