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The W -width in the two Higgs doublet model

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ABSTRACT

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The electroweak 1-loop corrections to the decay width of the W boson into fermions $\Gamma(W^+ \rightarrow f_1 \bar{f}_2)$ is discussed in the extended Higgs sector of the general two Higgs doublet model without any constraint. The relative corrections to the decay $W^+ \rightarrow e^+ \nu_e$ which come, due to the small mass of the electron, exclusively from W boson propagator are unobservably small in the whole parameter region with the maximal correction of $-6 \cdot 10^{-3}\%$. Due to the increasing of the Yukawa couplings with fermion masses, the relative corrections to the decays $W^+ \rightarrow \tau^+ \nu_\tau$ and $W^+ \rightarrow c \bar{s}$ which get also contributions from the vertex become larger, but even after having raised the Yukawa couplings with help of $\tan \beta$, the maximal corrections lie under the percent region and thus give also no observable effect. We discuss also the Higgs sector of the minimal supersymmetric standard model for the decays $W^+ \rightarrow \tau^+ \nu_\tau$ and $W^+ \rightarrow c \bar{s}$. Similarly to the cases in the general two Higgs doublet model the corrections in this model give no sizable effect and hence are not significant.

1 Introduction

The experiments at LEP and SLC have thus far confirmed all predictions of the Standard Model (SM) for fermionic final states, but did not show any indication of a Higgs boson with a mass below 60 GeV [1]. Thus the Higgs sector of the SM remains still as an unsatisfactory region where the experimental confirmation is lacking. On the other hand, the experimentally confirmed relation

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1 \quad (1.1)$$

is satisfied not only in the SM with a single Higgs doublet, but also in other versions of the SM with any number of Higgs doublets. Therefore we can extend, in principle, the Higgs sector of the SM to more Higgs doublets without violating the relation.

At the same time, from the view of, among others, the CP violation or the hierarchy problem, the SM needs to be extended. If we try to solve these problems, we are led to adding additional Higgs doublets to the Higgs sector of the SM. The strongest motivation for two Higgs doublet model (2HDM) may come from minimal supersymmetric standard model (MSSM). In this model, in order to give masses to the two types of quarks, two Higgs doublets are needed; one Higgs doublet for the quarks with isospin $I_3 = +\frac{1}{2}$ and the other for those with $I_3 = -\frac{1}{2}$ [2].

The extension of the SM to the 2HDM results in the production of a new particle spectrum in the Higgs sector. After diagonalizing the mass matrix, it has five physical mass eigenstates: a pair of charged Higgs bosons $H^\pm$, two neutral CP even states H^0, h^0 ('scalars') and one neutral CP odd state A^0 ('pseudoscalar') with masses $M_{H^\pm}, M_{H^0}, M_{h^0}, M_{A^0}$. In the procedure of generating the mass eigenstates, two Higgs mixing angles appear additionally as free parameters: $\tan \beta$ (the ratio of vacuum expectation values) and α . In contrast to the SM which has only one free parameter, the neutral Higgs mass M_H , the 2HDM has thus 6 free parameters.

Besides a direct search for these particles, another way to get information about them is through the calculation of radiative corrections; they appear in the loop calculations as virtual states, and so influence physical predictions. The comparison of the calculations with precision measurements gives indirect information on the masses of these particles.

In contrast to the measurements of the parameters of the Z boson where precise experimental data exist, the observables for the W boson are not known so well – but with the construction of LEP 200 this situation will improve. The precision for the mass will be about 0.1 % and for the width in the percent region.

For an exact comparison between the measurements and the theoretical predictions the calculation of the radiative corrections is necessary. These corrections to higher order of the perturbation theory for the decay of the W boson into fermions were already calculated both in the QCD and in the electroweak SM: the QCD corrections are about 4 % [3, 4], while the electroweak corrections are under 1 % and practically not dependent on the Higgs boson [5].

Since the W decay width is not sensitive to the Higgs sector of the SM, it is interesting to investigate the effect of an extension of the Higgs sector from the SM to the 2HDM on the decay width through the radiative corrections. The aim of this work is to study the influence of the additional free parameters appearing through the extension on the W decay width.

The paper is organized as follows:

In the next section we introduce the Higgs sector of the 2HDM briefly and show its particle spectrum. Before the actual loop calculation we first re-examine in section 3 the decay width of the W boson in the lowest order which will show our conventions and notations. Section 4 contains our main work. We give the necessary formulae for the decay width and the 1-loop contributions to it. The results are presented quantitatively and discussed in section 5. Appendices at the end include the Feynman rules used as well as the detailed formulae and expressions of the calculations.

2 The Higgs Sector of the Two Higgs Doublet Model

The Higgs sector of the 2HDM has two complex scalar doublets Φ_1 and Φ_2 which have vacuum expectation values v_1 and v_2 respectively ($i = 1, 2$):

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix}. \quad (2.1)$$

The CP invariant potential with discrete symmetry, $\Phi_1 \rightarrow \Phi_1$ and $\Phi_2 \rightarrow -\Phi_2$ (or $\Phi_1 \rightarrow -\Phi_1$ and $\Phi_2 \rightarrow \Phi_2$) leading to the vanishing of the flavor changing neutral currents has the form

$$\begin{aligned} V(\Phi_1, \Phi_2) = & \sum_{i=1}^2 (-\mu_i^2 |\Phi_i|^2 + \lambda_i^2 |\Phi_i|^4) \\ & + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 \\ & + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \\ & + \frac{1}{2} \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2], \end{aligned} \quad (2.2)$$

where $\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 are real.

We get the mass spectrum of the physical Higgs bosons after diagonalizing the mass matrix. The main features are as follows:

- In the charged sector the mass eigenstates are given by

$$H^\pm = -\phi_1^\pm \sin \beta + \phi_2^\pm \cos \beta \quad \text{with} \quad M_{H^\pm}^2 = \lambda_4(v_1^2 + v_2^2), \quad (2.3)$$

where

$$\tan \beta = \frac{v_2}{v_1}. \quad (2.4)$$

- In the neutral sector real and imaginary parts of the neutral Higgs fields decouple due to CP invariance.

- In the imaginary (CP odd) sector we have a massive field:

$$A^0 = -\chi_1 \sin \beta + \chi_2 \cos \beta \quad \text{with} \quad M_{A^0}^2 = \lambda_5(v_1^2 + v_2^2). \quad (2.5)$$

- The real (CP even) sector contains two physical Higgs scalars which are mixed by the following mass matrix:

$$M = \begin{pmatrix} 4v_1^2(\lambda_1 + \lambda_3) + v_2^2\lambda_5 & (4\lambda_3 + \lambda_5)v_1v_2 \\ (4\lambda_3 + \lambda_5)v_1v_2 & 4v_2^2(\lambda_1 + \lambda_3) + v_1^2\lambda_5 \end{pmatrix}. \quad (2.6)$$

After diagonalization we get the following mass eigenstates:

$$H^0 = \eta_1 \cos \alpha + \eta_2 \sin \alpha \quad (2.7)$$

$$h^0 = -\eta_1 \sin \alpha + \eta_2 \cos \alpha \quad (2.8)$$

with masses

$$M_{H^0, h^0}^2 = \frac{1}{2}[M_{11} + M_{22} \pm \sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2}] \quad (2.9)$$

where the mixing angle α is given by

$$\begin{aligned} \sin 2\alpha &= \frac{2M_{12}}{\sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2}} \\ \cos 2\alpha &= \frac{M_{11} - M_{22}}{\sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2}}. \end{aligned} \quad (2.10)$$

- In the case of a non-unitary gauge, unphysical Higgs fields corresponding to Goldstone bosons appear additionally:

$$\begin{aligned} G^\pm &= \Phi_1^\pm \cos \beta + \Phi_2^\pm \sin \beta \\ G^0 &= -\chi_1 \cos \beta + \chi_2 \sin \beta. \end{aligned} \quad (2.11)$$

3 The Lowest Order Decay Width

We consider the decay of a W^+ boson¹ into two fermions $f_1(u, c, \nu_l)$ and $\bar{f}_2(\bar{d}, \bar{s}, l^+)$ with $l = e, \mu, \tau$ (figure 1):

$$W^+(q) \longrightarrow f_1(p_1) + \bar{f}_2(p_2). \quad (3.1)$$

Figure 1: Feynman diagram for W^+ decay in lowest order

If we work in the rest frame of the W^+ boson, we get for the energies and velocities of the decay products

$$E_{1,2} = \frac{1}{2M_W} [M_W^2 \pm m_1^2 \mp m_2^2] \quad (3.2)$$

$$\beta_{1,2} = \frac{\kappa}{M_W^2 \pm m_1^2 \mp m_2^2} \quad (3.3)$$

where

$$\kappa = \sqrt{(M_W^2 - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2}. \quad (3.4)$$

The invariant amplitude $\mathcal{M}_0$ in lowest order is given by

$$-i\mathcal{M}_0 = \bar{u}(p_1)\Gamma_\mu^{(0)}v(p_2)\varepsilon^\mu(q)V_{f_1 f_2} \quad (3.5)$$

with the vertex

$$\Gamma_\mu^{(0)} = -i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1}{2}(1 - \gamma_5) \quad (3.6)$$

and the W^+ polarization vector $\varepsilon^\mu(q)$. $V_{f_1 f_2}$ denotes the element of the Kobayashi-Maskawa matrix for hadronic decays and is equal to one for leptonic decays. The gauge coupling g is related with the electromagnetic charge e and the mixing angle $\sin \theta_W$ by $g = e/\sin \theta_W$.

The amplitude (3.5) leads to the lowest order decay width

$$\Gamma_0^{(\alpha)} = \frac{\alpha_{em}}{6} \frac{M_W}{2\sin^2 \theta_W} N_c |V_{f_1 f_2}|^2 \frac{\kappa}{M_W^2} \cdot \left[1 - \frac{m_1^2 + m_2^2}{2M_W^2} - \frac{(m_1^2 - m_2^2)^2}{2M_W^4} \right] \quad (3.7)$$

¹The W^- decay proceeds analogously by charge conjugation.

where N_c is the color factor. If we use the tree level relation between the Fermi constant G_μ and the fine structure constant α_{em} [6]

$$G_\mu = \frac{\pi\alpha_{em}}{\sqrt{2}M_W^2} \cdot \frac{1}{\sin^2\theta_W}, \quad (3.8)$$

we can express $\Gamma_0^{(\alpha)}$ in (3.7) with G_μ :

$$\begin{aligned} \Gamma_0^{(G_\mu)} &= \frac{G_\mu}{6\sqrt{2}\pi} M_W^3 N_c |V_{f_1 f_2}|^2 \frac{\kappa}{M_W^2} \\ &\cdot \left[1 - \frac{m_1^2 + m_2^2}{2M_W^2} - \frac{(m_1^2 - m_2^2)^2}{2M_W^4} \right]. \end{aligned} \quad (3.9)$$

Compared with (3.7), this form has the advantage of being not dependent on the mixing angle $\sin\theta_W$.

4 The 1-Loop Decay Width

For the loop calculation we use the on-shell renormalization scheme discussed in [7, 8]. Since the calculation at the 1-loop level in SM was already done and the 2HDM concerns only an extension of the SM in the Higgs sector, we calculate explicitly only the corrections which appear additionally through the extension. If we try to calculate only these “non-standard” (NS) loop contributions, a small complication appears, since in the general case of the mixing in the neutral Higgs sector neither H^0 nor h^0 can be identified with the “standard” Higgs in a natural way. Only in the limiting case $\sin(\alpha - \beta) = 0(1)$, $H^0(h^0)$ behaves like the standard Higgs. In order to obtain the pure NS contributions we must therefore subtract those from the SM Higgs part. Without loss of generality we take $M_H = M_{H^0}$ as the reference mass for the SM Higgs boson.

At the 1-loop level, the vertex in (3.6) has the following structure:

$$\begin{aligned} \Gamma_\mu^{(1)} &= -i\frac{g}{\sqrt{2}} \left\{ \gamma_\mu \mathbf{L} \left[1 + F_L + \delta Z_1^W - \delta Z_2^W + \delta Z_L - \frac{1}{2}\hat{\Pi}_1(m_1^2) - \frac{1}{2}\hat{\Sigma}'_W(M_W^2) \right] \right. \\ &\quad \left. + \gamma_\mu \mathbf{R} F_R + \frac{p_{1\mu}}{M_W} (H_L \mathbf{L} + H_R \mathbf{R}) \right\} \end{aligned} \quad (4.1)$$

with the right- and left-handed projectors $\mathbf{R}, \mathbf{L} = \frac{1}{2}(1 \pm \gamma_5)$. The vertex is composed of:

- the form factors F_L, F_R, H_L and H_R from the vertex corrections.
- the vertex counterterm $\delta Z_1^W - \delta Z_2^W + \delta Z_L$. In our calculation of the extra Higgs contributions $\delta Z_1^W = \delta Z_2^W$ and δZ_L remains as the only counterterm of the vertex.

- the finite wave function renormalization $-\frac{1}{2}\hat{\Pi}_1(m_1^2)$ for the $I_3 = +1/2$ fermion.
- the finite wave function renormalization $-\frac{1}{2}\hat{\Sigma}'_W(M_W^2)$ for the W boson.

For the derivation of the decay width from the vertex in (4.1) we introduce the following Dirac matrix elements:

$$\begin{aligned} M_0 &= \bar{u}(p_1) \not{\epsilon} \mathbf{L} v(p_2) \\ M_1 &= \bar{u}(p_1) \not{\epsilon} \mathbf{R} v(p_2) \\ M_2 &= \bar{u}(p_1) \mathbf{L} v(p_2) (\epsilon \cdot p_1) / M_W \\ M_3 &= \bar{u}(p_1) \mathbf{R} v(p_2) (\epsilon \cdot p_1) / M_W. \end{aligned}$$

After contraction of $M_i^\dagger$ with M_0 and summation over the spin polarizations we obtain ($p_1^2 = m_1^2, p_2^2 = m_2^2$)

$$\begin{aligned} G_0 &= \sum_{pol} |M_0|^2 = 2M_W^2 - m_1^2 - m_2^2 - \frac{(m_1^2 - m_2^2)^2}{M_W^2} \\ G_1 &= \sum_{pol} M_0 M_1^\dagger = 6m_1 m_2 \\ G_2 &= \sum_{pol} M_0 M_2^\dagger = \frac{m_1}{M_W} \left[-\frac{1}{2} M_W^2 + m_1^2 + m_2^2 - \frac{(m_1^2 - m_2^2)^2}{2M_W^2} \right] \\ G_3 &= \sum_{pol} M_0 M_3^\dagger = \frac{m_2}{m_1} G_2. \end{aligned}$$

Using for the lowest order decay width $\Gamma_0^{(\alpha)}$ in (3.7), we have the decay width in first order:

$$\begin{aligned} \Gamma_{NS} &= \Gamma_0^{(\alpha)} \left\{ 1 + 2\text{Re}F_L + 2\delta Z_L - \hat{\Pi}_1(m_1^2) - \hat{\Sigma}'_W(M_W^2) \right. \\ &\quad \left. + \frac{G_1}{G_0} 2\text{Re}F_R + \frac{G_2}{G_0} 2\text{Re}H_L + \frac{G_3}{G_0} 2\text{Re}H_R \right\} \\ &\equiv \Gamma_0^{(\alpha)} \cdot (1 + \delta_{NS}^\alpha). \end{aligned} \tag{4.2}$$

Since G_1, G_2 and G_3 are suppressed strongly due to the small fermion masses, the terms F_R, H_L and H_R are negligible. Therefore, for the vertex corrections only the form factor F_L is important. The relative non-standard correction δ_{NS}^α is composed of three parts

$$\delta_{NS}^\alpha = \delta_w + \delta_v + \delta_f \tag{4.3}$$

where δ_w, δ_v and δ_f denote corrections from the W propagator, vertex and external fermion propagator respectively. We will discuss these terms separately.

If we use, instead of $\Gamma_0^{(\alpha)}$, the quantity $\Gamma_0^{(G_\mu)}$ in (3.9) as the lowest order decay width, we have the decay width in so-called G_μ -scheme:

$$\Gamma_{NS} = \Gamma_0^{(G_\mu)} (1 - \Delta r_{NS} + \delta_{NS}^\alpha) \equiv \Gamma_0^{(G_\mu)} (1 + \delta_{NS}^{G_\mu}). \tag{4.4}$$

Δr_{NS} is the NS part of Δr which is the 1-loop correction to the μ lifetime of (3.8):

$$G_\mu = \frac{\pi \alpha_{em}}{\sqrt{2} M_W^2} \cdot \frac{1}{\sin^2 \theta_W} (1 + \Delta r) \quad (4.5)$$

with

$$\Delta r = \Delta r_{SM} + \Delta r_{NS}. \quad (4.6)$$

4.1 Corrections to W boson propagator

In our renormalization scheme the W propagator gets a finite wave function renormalization $1 - \frac{1}{2} \hat{\Sigma}'_W(M_W^2)$ where $\hat{\Sigma}'_W(M_W^2)$ is expressed by the derivative of the unrenormalized W self energy Σ_W and the counter term given in [7, 9]:

$$\begin{aligned} \hat{\Sigma}'_W(M_W^2) &= \Sigma'_W(M_W^2) + \delta Z_2^W, \\ \delta Z_2^W &= -\Pi^\gamma(0) - 2 \frac{c_W}{s_W} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{c_W^2}{s_W^2} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right). \end{aligned} \quad (4.7)$$

We note that in our case of considering only the NS Higgs contributions $\Sigma^{\gamma Z}(0) = 0$.

As we see in the above formula, in order to get the finite renormalized quantity $\hat{\Sigma}'_W(M_W^2)$, it is required to calculate not only the self energy of the W boson, but also the self energies of the Z boson, photon and γ - Z mixing which appear in the counterterm. The additional Higgs contributions to these self energies are shown in figure 2. In evaluating these diagrams we consider only the transverse components proportional to $g_{\mu\nu}$. The terms $\sim q^\mu q^\nu$ are negligible. The evaluated analytic expressions can be found in appendix C.

Figure 2: Additional Higgs contributions to the self energies of the W boson, Z boson, photon and γ - Z mixing.

4.2 Corrections to the vertex and external fermion propagator

We calculate the vertices in an approximation of ignoring the fermion masses except in Yukawa couplings, since, due to the small mass of the fermions, their contributions to the decay width are strongly suppressed by the ratio m_f/M_W . From the diagrams in figure 3 we get the expressions for the form factors F_L, F_R, H_L and H_R in (4.1) which are given in appendix C. These form factors are finite except F_L whose singularity is removed by the field renormalization constant for the external fermions δZ_L .

Figure 3: Additional Higgs contributions to the vertex: (a) the decay into two quarks (b) the decay into two leptons. The diagrams are valid also for c and s quarks with the substitutions $u \rightarrow c$, $d \rightarrow s$ and l states for e, μ and τ .

Our renormalization scheme deals with the minimum number of field renormalization constants, assigning a single renormalization constant to each symmetry multiplet of fields. Imposing this condition on the fermion of $I_3 = -\frac{1}{2}$ component fixes the renormalization constant¹

$$\delta Z_L = -\Sigma_L^{f_2}(m_2^2) - m_2^2[\Sigma_L^{f_2'}(m_2^2) + \Sigma_R^{f_2'}(m_2^2) + 2\Sigma_S^{f_2'}(m_2^2)] \quad (4.8)$$

where the self energy has been decomposed according to

$$\Sigma^f(p) = \not{p}L\Sigma_L^f(p^2) + \not{p}R\Sigma_R^f(p^2) + m_f\Sigma_S^f(p^2). \quad (4.9)$$

Since we have the renormalization condition only for $I_3 = -\frac{1}{2}$ component, the residue of the renormalized propagator of the $I_3 = +\frac{1}{2}$ fermion is finite, but in general not

¹For the notation used in this section see [9].

equal to 1, yielding a finite wave function renormalization $1 - \frac{1}{2}\hat{\Pi}_1(m_1^2)$ with

$$\hat{\Pi}_1(m_1^2) = \delta Z_L + \Pi_1(m_1^2), \quad (4.10)$$

$$\Pi_1(m_1^2) = \Sigma_L^{f_1}(m_1^2) + m_1^2 [\Sigma_L^{f_1'}(m_1^2) + \Sigma_R^{f_1'}(m_1^2) + 2\Sigma_S^{f_1'}(m_1^2)]. \quad (4.11)$$

The fermion self energies are obtained from the diagrams of figure 4 and their explicit analytic expressions are listed again in appendix C.

Figure 4: Additional Higgs contributions to the fermion self energies: (a) self energies of quarks (b) self energies of leptons. The diagrams are valid also for c and s quarks with the substitutions $u \rightarrow c$, $d \rightarrow s$ and l states for e, μ and τ .

5 Numerical Results and Discussion

Among the 6 free parameters in the Higgs sector, we fixed the mass of our reference Higgs boson M_{H^0} to 100 GeV , since we are interested in the evaluation of the difference with respect to a given SM Higgs mass $M_H = M_{H^0}$. On the other hand, we set $\alpha = \beta$. With this choice of α the diagrams of the SM Higgs boson drop completely out of those of the entire non-standard part and hence we obtain the pure NS contributions which are not dependent on the SM.

For the numerical evaluation of the virtual correction (4.2) we took, according to the on-shell renormalization scheme, the following set of input parameters [10]:

$$\begin{aligned} \alpha_{em} &= 1/137.0359895 & G_\mu &= 1.16639 \cdot 10^{-5} \\ M_Z &= 91.187 \text{ GeV} & m_e &= 0.511 \text{ MeV} \\ m_\tau &= 1.7771 \text{ GeV} & m_c &= 1.3 \text{ GeV} \\ m_s &= 150 \text{ MeV} & m_t &= 175 \text{ GeV} \end{aligned}$$

In the renormalization scheme used here, M_W is required as an input parameter.

Since this mass is not yet known well experimentally, we determined it with help of the very well known Fermi constant G_μ via the relation

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi \alpha_{em}}{\sqrt{2} G_\mu} \cdot \frac{1}{1 - \Delta r_{SM} - \Delta r_{NS}}. \quad (5.1)$$

Since the NS Higgs contributions to Δr in the vertex and box diagrams are completely negligible, we can restrict Δr_{NS} to the vector boson self energies:

$$\Delta r_{NS} = \frac{\Sigma_W(0) - \delta M_W^2}{M_W^2} - \delta Z_2^W \quad (5.2)$$

with δZ_2^W from (4.7). For the detailed discussion of Δr_{SM} we refer to the literature [11].

5.1 Corrections to the decay $W^+ \rightarrow e^+ \nu_e$

The corrections to the decay $W^+ \rightarrow e^+ \nu_e$ consist exclusively of those from the W propagator, since, due to the small mass of the electron, the vertex corrections are completely negligible. The relative corrections through the W propagator δ_W are given by (4.7):

$$\delta_W = -\hat{\Sigma}'_W(M_W^2) = -\Sigma'_W(M_W^2) - \delta Z_2^W. \quad (5.3)$$

These corrections get large contributions from the term δZ_2^W , when the mass difference between the Higgs bosons is large.

This form is valid in α -scheme (4.2) where we use as input parameter, among others, the fine structure constant α_{em} . However, for the calculation of the W decay width it is more suitable to use G_μ as input parameter. In this G_μ -scheme (4.4), if we neglect the vertex corrections, the relative corrections are given exclusively by the W propagator:

$$\delta_{NS}^{G_\mu} \cong -\Delta r_{NS} - \hat{\Sigma}'_W(M_W^2) = -\frac{\Sigma_W(0) - \delta M_W^2}{M_W^2} - \Sigma'_W(M_W^2) \quad (5.4)$$

which comes from (5.2) and (5.3). In this way, the term δZ_2^W responsible for the large corrections drops out and the corrections are therefore small and stable.

We list the evaluated relative corrections both in the α -scheme and in the G_μ -scheme together with the evaluated masses of the W boson in table 1. We chose $\tan \beta = 0.7^1$ and varied 3 Higgs masses. In the table, δ_{NS}^α stands for the relative corrections in the α -scheme and $\delta_{NS}^{G_\mu}$ for those in the G_μ -scheme. On the other hand, the W mass is also an observable for which experimental measurements are existing. The measured mass of the particle is at present $80.22 \pm 0.26 \text{ GeV}$ [10]. In the cases of the deviation of the evaluated W mass from the measured by more than

¹The results are however not dependent on the actual value of $\tan \beta$.

M_{h^0}	M_{A^0}	$M_{H^\pm}$	M_W	δ_{NS}^α	$\delta_{NS}^{G_\mu}$
100	300	300	80.472	$-3.05 \cdot 10^{-2}$	$-4.83 \cdot 10^{-4}$
100	300	600	81.817*	$-8.50 \cdot 10^0$	$-1.92 \cdot 10^{-4}$
100	600	300	80.051	$2.18 \cdot 10^0$	$-3.73 \cdot 10^{-4}$
100	600	600	80.473	$-3.47 \cdot 10^{-2}$	$-1.32 \cdot 10^{-4}$
600	300	300	80.460	$3.80 \cdot 10^{-2}$	$-2.26 \cdot 10^{-4}$
600	300	600	80.471	$-2.26 \cdot 10^{-2}$	$-1.06 \cdot 10^{-4}$
600	600	300	81.276*	$-4.81 \cdot 10^0$	$-1.54 \cdot 10^{-4}$
600	600	600	80.467	$-1.80 \cdot 10^{-4}$	$-7.89 \cdot 10^{-5}$

Table 1: Relative corrections to the decay $W^+ \rightarrow e^+ \nu_e$ in the two Higgs doublet model with $\tan \beta = 0.7$. The statements of the corrections are in [%] and the masses are given in units of GeV .

two standard deviations we marked these with “*”. The parameter sets of them are therefore already excluded by M_W . The table shows that the corrections in the G_μ -scheme are very small in the whole parameter region and give thus no observable effect.

5.2 Corrections to the decays $W^+ \rightarrow \tau^+ \nu_\tau$ and $W^+ \rightarrow c \bar{s}$

With increasing fermion masses the Yukawa couplings also increase. Thus the corrections to the decays $W^+ \rightarrow \tau^+ \nu_\tau$ and $W^+ \rightarrow c \bar{s}$ get, apart from the contributions from the W propagator, also contributions from the vertex. On the other hand, in 2HDM we have the possibility to increase the Yukawa couplings with help of $\tan \beta$. Since the Yukawa couplings on up-type fermions contain typically the term $\frac{m_u}{M_W} \times \frac{v_1}{v_2}$, the situation $v_1 \gg v_2$ ($\tan \beta \approx 0$) leads to the raising of the couplings on the up-type fermions, while the Yukawa couplings on down-type fermions contain the term $\frac{m_d}{M_W} \times \frac{v_2}{v_1}$, which leads to the raising of the couplings for $v_2 \gg v_1$ ($\tan \beta \gg 0$).

We took $0.7 \leq \tan \beta \leq 70$ as the allowed region for $\tan \beta$ and for the actual calculations chose the two limiting values $\tan \beta = 0.7, 70$. The restriction $\tan \beta \geq 0.7$ comes from the $K - \bar{K}$, $B - \bar{B}$ mixing and the decay of the B meson if the charged Higgs bosons are heavy [12]. On the other hand, we can increase the Yukawa couplings on the down-type fermions of any size by increasing $\tan \beta$, but with $\tan \beta = 70$ we already arrive at the maximal limit of the couplings for the applicability of the perturbation theory. For example, for this value of $\tan \beta$ we get from the quadratic coupling of the b quark on the A^0 boson

$$\left(\frac{e}{2 \sin \theta_W} \tan \beta \frac{m_b}{M_W} \right)^2 \cong 200 \frac{e^2}{4\pi}. \quad (5.5)$$

We list the evaluated relative corrections to the decays $W^+ \rightarrow \tau^+ \nu_\tau$ and $W^+ \rightarrow c\bar{s}$ as well as the evaluated masses of the W boson in tables 2 and 3. The tables show that for $\tan \beta = 70$ we get substantially larger corrections, but their increases with help of $\tan \beta$ are not sufficient to raise the effects in percent region. The entire values remain in the region of a few per mil and are therefore not significant.

The experimental uncertainty of the quark masses c ($1.0 \sim 1.6 \text{ GeV}$) and s ($100 \sim 200 \text{ MeV}$) does not play any role for the corrections. Even for $\tan \beta = 70$ the change of the quantities by variation of the masses within the unknown regions is completely negligible.

We calculated also the NS Higgs contributions to the decay width in MSSM where the following relations among the free parameters in the 2HDM exist and only two of them are therefore independent [2]:

$$\begin{aligned} M_{A^0}^2 &= M_{H^\pm}^2 - M_W^2 \\ M_{H^0, h^0}^2 &= \frac{1}{2} \left[M_{A^0}^2 + M_Z^2 \pm \sqrt{(M_{A^0}^2 + M_Z^2)^2 - 4M_Z^2 M_{A^0}^2 \cos^2 2\beta} \right] \\ \cos 2\alpha &= -\cos 2\beta \left(\frac{M_{A^0}^2 - M_Z^2}{M_{H^0}^2 - M_{h^0}^2} \right) \\ \sin 2\alpha &= -\sin 2\beta \left(\frac{M_{H^0}^2 + M_{h^0}^2}{M_{H^0}^2 - M_{h^0}^2} \right). \end{aligned}$$

In the actual calculation we chose M_{A^0} and $\tan \beta$ as free parameters and evaluated the corrections for $20 \text{ GeV} \leq M_{A^0} \leq 1 \text{ TeV}$ and $\tan \beta = 2, 30$. We list the results in tables 4 and 5. As we see in the tables, the maximal corrections in this model are for the decay $W^+ \rightarrow \tau^+ \nu_\tau$ given by $4.23 \cdot 10^{-2}\%$ for $M_{A^0} = 85 \text{ GeV}$, $\tan \beta = 30$ and for the decay $W^+ \rightarrow c\bar{s}$ by $-1.52 \cdot 10^{-2}\%$ for $M_{A^0} = 20 \text{ GeV}$, $\tan \beta = 2$. Radiative corrections to the MSSM Higgs sector were not considered here. They are however also irrelevant in view of the small effects.

From our calculations of the radiative corrections to the W decay width in the NS part of the Higgs sector we conclude that the corrections give no observable effects in the whole parameter region. The additional corrections lie below a percent, similarly to the minimal standard model. In the 2HDM, as in the SM, the decay width of the W boson is thus practically not dependent on the Higgs sector.

I thank W. Hollik for suggesting this work and many useful discussions as well as P. Weisz for reading this paper.

$\tan \beta$	M_{h^0}	M_{A^0}	$M_{H^\pm}$	M_W	δ_{NS}^α	$\delta_{NS}^{G_\mu}$
0.7	100	300	300	80.472	$-3.05 \cdot 10^{-2}$	$-4.65 \cdot 10^{-4}$
	100	300	600	81.817*	$-8.50 \cdot 10^0$	$-1.84 \cdot 10^{-4}$
	100	600	600	80.473	$-3.47 \cdot 10^{-2}$	$-1.22 \cdot 10^{-4}$
	600	300	300	80.460	$3.80 \cdot 10^{-2}$	$-2.05 \cdot 10^{-4}$
	600	300	600	80.471	$-2.26 \cdot 10^{-2}$	$-8.52 \cdot 10^{-5}$
	600	600	600	80.467	$-1.57 \cdot 10^{-4}$	$-5.53 \cdot 10^{-5}$
70	100	300	300	80.472	$1.50 \cdot 10^{-1}$	$1.80 \cdot 10^{-1}$
	100	300	600	81.817*	$-8.42 \cdot 10^0$	$7.90 \cdot 10^{-2}$
	100	600	600	80.473	$6.15 \cdot 10^{-2}$	$9.61 \cdot 10^{-2}$
	600	300	300	80.460	$2.51 \cdot 10^{-1}$	$2.12 \cdot 10^{-1}$
	600	300	600	80.471	$1.89 \cdot 10^{-1}$	$2.12 \cdot 10^{-1}$
	600	600	600	80.467	$2.36 \cdot 10^{-1}$	$2.36 \cdot 10^{-1}$

Table 2: Relative corrections to the decay $W^+ \rightarrow \tau^+ \nu_\tau$ in the two Higgs doublet model. The statements of the corrections are in [%] and the masses are given in units of GeV .

$\tan \beta$	M_{h^0}	M_{A^0}	$M_{H^\pm}$	M_W	δ_{NS}^α	$\delta_{NS}^{G_\mu}$
0.7	100	300	300	80.472	$-3.05 \cdot 10^{-2}$	$-4.97 \cdot 10^{-4}$
	100	300	600	81.817*	$-8.50 \cdot 10^0$	$-2.40 \cdot 10^{-4}$
	100	600	600	80.473	$-3.47 \cdot 10^{-2}$	$-1.70 \cdot 10^{-4}$
	600	300	300	80.460	$3.80 \cdot 10^{-2}$	$-2.33 \cdot 10^{-4}$
	600	300	600	80.471	$-2.26 \cdot 10^{-2}$	$-1.19 \cdot 10^{-4}$
	600	600	600	80.467	$-1.87 \cdot 10^{-4}$	$-8.58 \cdot 10^{-5}$
70	100	300	300	80.472	$-3.09 \cdot 10^{-2}$	$-8.93 \cdot 10^{-4}$
	100	300	600	81.817*	$-8.50 \cdot 10^0$	$-1.49 \cdot 10^{-3}$
	100	600	600	80.473	$-3.60 \cdot 10^{-2}$	$-1.40 \cdot 10^{-3}$
	600	300	300	80.460	$3.75 \cdot 10^{-2}$	$-6.87 \cdot 10^{-4}$
	600	300	600	80.471	$-2.30 \cdot 10^{-2}$	$-5.74 \cdot 10^{-4}$
	600	600	600	80.467	$-7.34 \cdot 10^{-4}$	$-6.33 \cdot 10^{-4}$

Table 3: Relative corrections to the decay $W^+ \rightarrow c\bar{s}$ in the two Higgs doublet model. The statements of the corrections are in [%] and the masses are given in units of GeV .

$\tan \beta$	M_{A^0}	M_W	δ_{NS}^α	$\delta_{NS}^{G_\mu}$
2	20	80.538	$-3.878 \cdot 10^{-1}$	$-1.512 \cdot 10^{-2}$
	300	80.433	$-2.061 \cdot 10^{-1}$	$-4.641 \cdot 10^{-3}$
	600	80.384	$-2.120 \cdot 10^{-1}$	$-4.479 \cdot 10^{-3}$
	1000	80.344	$-2.129 \cdot 10^{-1}$	$-4.466 \cdot 10^{-3}$
30	20	80.539	$-3.563 \cdot 10^{-1}$	$1.322 \cdot 10^{-2}$
	85	80.488	$-5.096 \cdot 10^{-2}$	$4.232 \cdot 10^{-2}$
	600	80.378	$-1.566 \cdot 10^{-1}$	$1.603 \cdot 10^{-2}$
	1000	80.338	$-1.720 \cdot 10^{-1}$	$3.077 \cdot 10^{-3}$

Table 4: Relative corrections to the decay $W^+ \rightarrow \tau^+ \nu_\tau$ in the minimal supersymmetric standard model. The statements of the corrections are in [%] and the masses are given in units of GeV .

$\tan \beta$	M_{A^0}	M_W	δ_{NS}^α	$\delta_{NS}^{G_\mu}$
2	20	80.538	$-3.878 \cdot 10^{-1}$	$-1.520 \cdot 10^{-2}$
	300	80.433	$-2.062 \cdot 10^{-1}$	$-4.770 \cdot 10^{-3}$
	600	80.384	$-2.121 \cdot 10^{-1}$	$-4.544 \cdot 10^{-3}$
	1000	80.344	$-2.129 \cdot 10^{-1}$	$-4.476 \cdot 10^{-3}$
30	20	80.539	$-3.836 \cdot 10^{-1}$	$-1.408 \cdot 10^{-2}$
	300	80.427	$-1.621 \cdot 10^{-1}$	$-1.727 \cdot 10^{-3}$
	600	80.378	$-1.742 \cdot 10^{-1}$	$-1.545 \cdot 10^{-3}$
	1000	80.338	$-1.766 \cdot 10^{-1}$	$-1.550 \cdot 10^{-3}$

Table 5: Relative corrections to the decay $W^+ \rightarrow c\bar{s}$ in the minimal supersymmetric standard model. The statements of the corrections are in [%] and the masses are given in units of GeV .

A Feynman Rules

We list here the Feynman rules in 't Hooft-Feynman gauge used in our calculations. A complete list of the Feynman rules with Higgs bosons in 2HDM can be found in [2].

1. Vertices with one gauge boson and two Higgs bosons

$$\frac{+ig}{2}[\sin(\beta - \alpha), -\cos(\beta - \alpha)](p + p')^\mu \quad (\text{A.1})$$

$$\frac{-ig}{2}[\cos(\beta - \alpha), \sin(\beta - \alpha)](p + p')^\mu \quad (\text{A.2})$$

$$\frac{g}{2}(p + p')^\mu \quad (\text{A.3})$$

$$\frac{g}{2 \cos \theta_W} [-\sin(\beta - \alpha), \cos(\beta - \alpha)] (p + p')^\mu \quad (\text{A.4})$$

$$\frac{g}{2 \cos \theta_W} [\cos(\beta - \alpha), \sin(\beta - \alpha)] (p + p')^\mu \quad (\text{A.5})$$

$$\frac{-ig \cos 2\theta_W}{2 \cos \theta_W} (p + p')^\mu \quad (\text{A.6})$$

$$-ie(p + p')^\mu \quad (\text{A.7})$$

2. Vertices with two gauge bosons and one Higgs boson

$$igM_W[\cos(\beta - \alpha), \sin(\beta - \alpha)]g^{\mu\nu} \quad (\text{A.8})$$

$$\frac{igM_Z}{\cos\theta_W}[\cos(\beta - \alpha), \sin(\beta - \alpha)]g^{\mu\nu} \quad (\text{A.9})$$

$$ie[M_W, -M_Z \sin\theta_W]g^{\mu\nu} \quad (\text{A.10})$$

3. Four-point-interactions with two gauge bosons and two Higgs bosons

$$\frac{ig^2}{2} g^{\mu\nu} \quad (j = 1, 2, 3) \quad (\text{A.11})$$

$$\frac{ig^2}{2 \cos^2 \theta_W} [1, \cos^2 2\theta_W] g^{\mu\nu} \quad (j = 1, 2, 3) \quad (\text{A.12})$$

The indices $j = 1, 2, 3$ correspond to H^0 , h^0 , A^0 respectively.

$$ie \left[2e, g \frac{\cos 2\theta_W}{\cos \theta_W} \right] g^{\mu\nu} \quad (\text{A.13})$$

4. Yukawa couplings

The Yukawa couplings for leptons are obtained by the substitutions $u \rightarrow \nu_l$ and $d \rightarrow l$.

$$\frac{-ig}{2M_W} \left[m_u \frac{\sin \alpha}{\sin \beta}, m_d \frac{\cos \alpha}{\cos \beta} \right] \quad (\text{A.14})$$

$$\frac{-ig}{2M_W} \left[m_u \frac{\cos \alpha}{\sin \beta}, -m_d \frac{\sin \alpha}{\cos \beta} \right] \quad (\text{A.15})$$

$$\frac{-g}{2M_W} \left[m_u \cot \beta, m_d \tan \beta \right] \gamma_5 \quad (\text{A.16})$$

$$\frac{ig}{2\sqrt{2}M_W} [(m_d \tan \beta + m_u \cot \beta) \pm (m_d \tan \beta - m_u \cot \beta) \gamma_5] \quad (\text{A.17})$$

$$\frac{ig}{2\sqrt{2}M_W} [(m_u - m_d) \mp (m_u + m_d) \gamma_5] \quad (\text{A.18})$$

B One-, Two- and Three-Point Integrals

Scalar and tensor integrals appear in the 1-loop calculations of the radiative corrections. A systematic method of dealing with these integrals is to reduce the complicated tensor integrals to the scalar integrals [13].

B.1 One- and two-point integrals

We define the following tensor integrals:

$$\frac{i}{16\pi^2} A(M_1) \equiv \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{1}{D_1} \quad (\text{B.1})$$

$$\frac{i}{16\pi^2} B_{\{0,\mu,\mu\nu\}}(q^2, M_1, M_2) \equiv \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{\{1, k_\mu, k_\mu k_\nu\}}{D_1 D_2} \quad (\text{B.2})$$

with

$$\begin{aligned} D_1 &= k^2 - M_1^2 + i\varepsilon \\ D_2 &= (k+q)^2 - M_2^2 + i\varepsilon \end{aligned}$$

where $D \equiv 4 - \varepsilon$. The tensor integrals B_μ and $B_{\mu\nu}$ can be decomposed according to Lorentz covariance:

$$B_\mu = q_\mu B_1 \quad (\text{B.3})$$

$$B_{\mu\nu} = g_{\mu\nu} B_{22} + q_\mu q_\nu B_{21}. \quad (\text{B.4})$$

The coefficients B_1 , B_{22} and B_{21} can be reduced algebraically to the scalar 1- and 2-point integrals A and B_0 which have the following forms [6]:

$$A(M) = M^2 \left(\Delta - \log \frac{M^2}{\mu^2} + 1 \right) \quad (\text{B.5})$$

$$B_0(q^2, M_1, M_2) = \Delta - \int_0^1 dx \log \frac{x^2 q^2 - x(q^2 + M_1^2 - M_2^2) + M_1^2 - i\varepsilon}{\mu^2} \quad (\text{B.6})$$

with

$$\Delta = \frac{2}{\varepsilon} - \gamma + \log 4\pi. \quad (\text{B.7})$$

γ denotes the Euler constant. In our calculation we need only B_1 and B_{22} :

$$\begin{aligned} B_1(q^2, M_1, M_2) &= \left[A(M_1) - A(M_2) + (M_2^2 - M_1^2 - q^2) B_0(q^2, M_1, M_2) \right] / 2q^2 \\ B_{22}(q^2, M_1, M_2) &= \frac{1}{3} \left\{ \frac{1}{2} A(M_2) + M_1^2 B_0(q^2, M_1, M_2) \right. \\ &\quad \left. + \frac{1}{2} (q^2 + M_1^2 - M_2^2) B_1(q^2, M_1, M_2) + \frac{M_1^2 + M_2^2}{2} - \frac{q^2}{6} \right\}. \end{aligned}$$

B.2 Three-point integrals

The following tensor integrals appear in the vertex corrections:

$$\frac{i}{16\pi^2} C_{\{0,\mu,\mu\nu\}}(q^2, M_1, M_2, M_3) \equiv \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{\{1, k_\mu, k_\mu k_\nu\}}{D_1 D_2 D_3} \quad (\text{B.8})$$

with

$$\begin{aligned} D_1 &= (k - p_1)^2 - M_1^2 + i\varepsilon \\ D_2 &= (k - p_2)^2 - M_2^2 + i\varepsilon \\ D_3 &= k^2 - M_3^2 + i\varepsilon \end{aligned}$$

and $q^2 = (p_2 - p_1)^2$.

The parameters are specified in the following figure:

The parameters M_i ($i = 1, 2, 3$) indicate the internal masses, the masses of the external fermions ($p_1^2 = m_1^2$ and $p_2^2 = m_2^2$) are ignored. We can decompose the tensor integrals according to Lorentz covariance in the following way:

$$C_\mu = C_1^+(p_1 + p_2)_\mu + C_1^-(p_1 - p_2)_\mu \quad (\text{B.9})$$

$$C_{\mu\nu} = C_2^+(p_1 + p_2)_\mu(p_1 + p_2)_\nu + C_2^-(p_1 - p_2)_\mu(p_1 - p_2)_\nu + C_2^{+-}[(p_1 + p_2)_\mu(p_1 - p_2)_\nu + (p_1 - p_2)_\mu(p_1 + p_2)_\nu] + C_2^0 g_{\mu\nu}. \quad (\text{B.10})$$

In diagrams which are symmetric under $p_1 \leftrightarrow p_2$ it is valid $C_1^- = C_2^{+-} = 0$.

The coefficients can be determined from the scalar 3-point integral C_0 and the 2-point functions B_0 and B_1 as follows[14]:

$$\begin{aligned} -q^2 C_1^+(q^2, M_1, M_2, M_3) &= B_0(q^2, M_1, M_2) - \frac{1}{2} [B_0(0, M_3, M_1) + B_0(0, M_3, M_2)] \\ &\quad + \frac{1}{2} (2M_3^2 - M_1^2 - M_2^2) C_0 \\ q^2 C_1^-(q^2, M_1, M_2, M_3) &= \frac{1}{2} [B_0(0, M_3, M_1) - B_0(0, M_3, M_2)] + \frac{1}{2} (M_2^2 - M_1^2) C_0 \\ C_2^0(q^2, M_1, M_2, M_3) &= \frac{1}{2} M_3^2 C_0 + \frac{1}{4} + \frac{1}{4} [(M_1^2 + M_2^2 - 2M_3^2) C_1^+ \\ &\quad + (M_1^2 - M_2^2) C_1^- + B_0(q^2, M_1, M_2)] \\ -q^2 C_2^+(q^2, M_1, M_2, M_3) &= -C_2^0 + \frac{1}{2} (2M_3^2 - M_1^2 - M_2^2) C_1^+ + \frac{1}{2} B_0(q^2, M_1, M_2) \\ &\quad + \frac{1}{4} [B_1(0, M_3, M_1) + B_1(0, M_3, M_2)] \\ q^2 C_2^-(q^2, M_1, M_2, M_3) &= -C_2^0 + \frac{1}{2} (M_2^2 - M_1^2) C_1^- \\ &\quad - \frac{1}{4} [B_1(0, M_3, M_1) + B_1(0, M_3, M_2)] \\ 4q^2 C_2^{+-}(q^2, M_1, M_2, M_3) &= 2(M_2^2 - M_1^2) C_1^+ + B_1(0, M_3, M_2) - B_1(0, M_3, M_1). \end{aligned}$$

For vanishing external fermion masses the C_0 function is given as follows[15]:

$$C_0(q^2, M_1, M_2, M_3) = - \int_0^1 dy \int_0^y dx (ay^2 + bx^2 + cxy + dy + ex + f)^{-1} \quad (\text{B.11})$$

with

$$a = 0, b = -c = q^2, d = M_2^2 - M_3^2, e = M_1^2 - M_2^2, f = M_3^2 - i\varepsilon. \quad (\text{B.12})$$

Expressed by dilogarithms, the function is given by

$$\begin{aligned} C_0(q^2, M_1, M_2, M_3) = & \frac{1}{q^2} \left\{ \sum_{j=1}^2 \left[Li_2\left(\frac{x_1}{x_1 - y_{1j}}\right) - Li_2\left(\frac{x_1 - 1}{x_1 - y_{1j}}\right) \right] \right. \\ & \left. - Li_2\left(\frac{x_2}{x_2 - y_2}\right) + Li_2\left(\frac{x_2 - 1}{x_2 - y_2}\right) \right\} \end{aligned} \quad (\text{B.13})$$

with

$$\begin{aligned} x_1 &= x_2 = -\frac{d}{c} \\ y_{1j} &= \frac{-c - e \pm \sqrt{(c + e)^2 - 4b(a + d + f)}}{2b} \\ y_2 &= \frac{M_3^2}{M_3^2 - M_1^2}. \end{aligned}$$

C Analytic Expressions for the 1-Loop Diagrams

We list the evaluated 1-loop Feynman diagrams for the self energies of the gauge bosons and external fermions as well as for the vertex. The detailed formulas for the 1-, 2- and 3-point integrals are given in appendix B and the Feynman rules used in our calculations can be found in appendix A.

C.1 Gauge boson self energies

From the Feynman diagrams of figure 2 we get the following expressions for the gauge boson self energies.

1. W boson self energy

$$\begin{aligned} \Sigma_{\{H^0, H^+\}}^W(q^2) &= -\frac{1}{16\pi^2} g^2 \sin^2(\beta - \alpha) B_{22}(q^2, M_{H^0}, M_{H^+}) \\ \Sigma_{\{h^0, H^+\}}^W(q^2) &= -\frac{1}{16\pi^2} g^2 \cos^2(\beta - \alpha) B_{22}(q^2, M_{h^0}, M_{H^+}) \\ \Sigma_{\{A^0, H^+\}}^W(q^2) &= -\frac{1}{16\pi^2} g^2 B_{22}(q^2, M_{A^0}, M_{H^+}) \\ \Sigma_{\{H^0, G^+\}}^W(q^2) &= -\frac{1}{16\pi^2} g^2 \cos^2(\beta - \alpha) B_{22}(q^2, M_{H^0}, M_W) \\ \Sigma_{\{h^0, G^+\}}^W(q^2) &= -\frac{1}{16\pi^2} g^2 \sin^2(\beta - \alpha) B_{22}(q^2, M_{h^0}, M_W) \end{aligned}$$

$$\begin{aligned}
\Sigma_{\{W^+, H^0\}}^W(q^2) &= -\frac{1}{16\pi^2} [g M_W \cos(\beta - \alpha)]^2 B_0(q^2, M_W, M_{H^0}) \\
\Sigma_{\{W^+, h^0\}}^W(q^2) &= -\frac{1}{16\pi^2} [g M_W \sin(\beta - \alpha)]^2 B_0(q^2, M_W, M_{h^0}) \\
\Sigma_{\{h^0\}}^W &= \frac{1}{64\pi^2} g^2 A(M_{h^0}) \\
\Sigma_{\{A^0\}}^W &= \frac{1}{64\pi^2} g^2 A(M_{A^0}) \\
\Sigma_{\{H^+\}}^W &= \frac{1}{32\pi^2} g^2 A(M_{H^+})
\end{aligned}$$

2. Z boson self energy

$$\begin{aligned}
\Sigma_{\{H^0, A^0\}}^Z(q^2) &= -\frac{1}{4\pi^2} \left[\frac{g \sin(\beta - \alpha)}{2c_W} \right]^2 B_{22}(q^2, M_{H^0}, M_{A^0}) \\
\Sigma_{\{h^0, A^0\}}^Z(q^2) &= -\frac{1}{4\pi^2} \left[\frac{g \cos(\beta - \alpha)}{2c_W} \right]^2 B_{22}(q^2, M_{h^0}, M_{A^0}) \\
\Sigma_{\{H^+, H^+\}}^Z(q^2) &= -\frac{1}{4\pi^2} \left[\frac{g \cos 2\theta_W}{2c_W} \right]^2 B_{22}(q^2, M_{H^+}, M_{H^+}) \\
\Sigma_{\{H^0, G^0\}}^Z(q^2) &= -\frac{1}{4\pi^2} \left[\frac{g \cos(\beta - \alpha)}{2c_W} \right]^2 B_{22}(q^2, M_{H^0}, M_Z) \\
\Sigma_{\{h^0, G^0\}}^Z(q^2) &= -\frac{1}{4\pi^2} \left[\frac{g \sin(\beta - \alpha)}{2c_W} \right]^2 B_{22}(q^2, M_{h^0}, M_Z) \\
\Sigma_{\{Z, H^0\}}^Z(q^2) &= \frac{1}{16\pi^2} \left[\frac{g M_Z \cos(\beta - \alpha)}{c_W} \right]^2 B_0(q^2, M_Z, M_{H^0}) \\
\Sigma_{\{Z, h^0\}}^Z(q^2) &= \frac{1}{16\pi^2} \left[\frac{g M_Z \sin(\beta - \alpha)}{c_W} \right]^2 B_0(q^2, M_Z, M_{h^0}) \\
\Sigma_{\{h^0\}}^Z &= \frac{1}{64\pi^2} \frac{g^2}{c_W^2} A(M_{h^0}) \\
\Sigma_{\{A^0\}}^Z &= \frac{1}{64\pi^2} \frac{g^2}{c_W^2} A(M_{A^0}) \\
\Sigma_{\{H^+\}}^Z &= \frac{1}{32\pi^2} \frac{g^2 \cos^2 2\theta_W}{c_W^2} A(M_{H^+})
\end{aligned}$$

3. γ -Z mixing

$$\begin{aligned}
\Sigma_{\{H^+, H^+\}}^{\gamma Z}(q^2) &= -\frac{1}{4\pi^2} \frac{eg \cos 2\theta_W}{2c_W} B_{22}(q^2, M_{H^+}, M_{H^+}) \\
\Sigma_{\{H^+\}}^{\gamma Z} &= \frac{1}{16\pi^2} \frac{eg \cos 2\theta_W}{c_W} A(M_{H^+})
\end{aligned}$$

4. Photon self energy

$$\begin{aligned}\Sigma_{\{H^+, H^+\}}^\gamma(q^2) &= -\frac{1}{4\pi^2}e^2 B_{22}(q^2, M_{H^+}, M_{H^+}) \\ \Sigma_{\{H^+\}}^\gamma &= \frac{1}{16\pi^2}2e^2 A(M_{H^+})\end{aligned}$$

C.2 Vertex corrections

From the Feynman diagrams of figure 3 we get the following expressions for the form factors F_L, F_R, H_L and H_R in (4.1).

C.2.1 Vertex for the decay into two quarks

We give here the expressions for u and d quarks, they are however valid for c and s quarks in the same way with the substitutions $m_u \rightarrow m_c, m_d \rightarrow m_s$.

1. Diagrams with two virtual fermions and one virtual Higgs boson which contribute to the form factor F_R :

$$\begin{aligned}\Gamma_{\mu, \{d, u, H^0\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 + \gamma_5}{2} \frac{1}{16\pi^2} \frac{g^2 m_u m_d}{4M_W^2} \frac{\sin \alpha \cos \alpha}{\sin \beta \cos \beta} \\ &\quad \cdot \{M_W^2 [C_2^+(M_W^2, 0, 0, M_{H^0}) - C_2^-(M_W^2, 0, 0, M_{H^0})] \\ &\quad - 4C_0^2(M_W^2, 0, 0, M_{H^0})\} \\ \Gamma_{\mu, \{d, u, h^0\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 + \gamma_5}{2} \left(-\frac{1}{16\pi^2}\right) \frac{g^2 m_u m_d}{4M_W^2} \frac{\sin \alpha \cos \alpha}{\sin \beta \cos \beta} \\ &\quad \cdot \{M_W^2 [C_2^+(M_W^2, 0, 0, M_{h^0}) - C_2^-(M_W^2, 0, 0, M_{h^0})] \\ &\quad - 4C_0^2(M_W^2, 0, 0, M_{h^0})\} \\ \Gamma_{\mu, \{d, u, A^0\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 + \gamma_5}{2} \frac{1}{16\pi^2} \frac{g^2 m_u m_d}{4M_W^2} \\ &\quad \cdot \{M_W^2 [C_2^+(M_W^2, 0, 0, M_{A^0}) - C_2^-(M_W^2, 0, 0, M_{A^0})] \\ &\quad - 4C_0^2(M_W^2, 0, 0, M_{A^0})\}\end{aligned}$$

2. Diagrams with two virtual Higgs bosons and one virtual fermion which contribute to the form factors F_L and F_R :

$$\begin{aligned}\Gamma_{\mu, \{H^0, H^+, d\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 - \gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_d^2 \tan \beta \sin(\beta - \alpha) \cos \alpha}{4M_W^2 \cos \beta} \\ &\quad \cdot C_0^2(M_W^2, M_{H^0}, M_{H^+}, 0)\end{aligned}$$

$$\begin{aligned}
\Gamma_{\mu,\{h^0,H^+,d\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1+\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_d m_u \cot \beta \sin(\beta-\alpha) \cos \alpha}{4M_W^2 \cos \beta} \\
&\quad \cdot C_0^2(M_W^2, M_{H^0}, M_{H^+}, 0) \\
&\quad -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_d^2 \tan \beta \cos(\beta-\alpha) \sin \alpha}{4M_W^2 \cos \beta} \\
&\quad \cdot C_0^2(M_W^2, M_{h^0}, M_{H^+}, 0) \\
&\quad -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1+\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_d m_u \cot \beta \cos(\beta-\alpha) \sin \alpha}{4M_W^2 \cos \beta} \\
&\quad \cdot C_0^2(M_W^2, M_{h^0}, M_{H^+}, 0) \\
\Gamma_{\mu,\{A^0,H^+,d\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_d^2 \tan^2 \beta}{4M_W^2} \\
&\quad \cdot C_0^2(M_W^2, M_{A^0}, M_{H^+}, 0) \\
&\quad -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1+\gamma_5}{2} \left(-\frac{1}{8\pi^2}\right) \frac{g^2 m_d m_u}{4M_W^2} \\
&\quad \cdot C_0^2(M_W^2, M_{A^0}, M_{H^+}, 0) \\
\Gamma_{\mu,\{H^+,H^0,u\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \left(-\frac{1}{8\pi^2}\right) \frac{g^2 m_u^2 \cot \beta \sin(\beta-\alpha) \sin \alpha}{4M_W^2 \sin \beta} \\
&\quad \cdot C_0^2(M_W^2, M_{H^+}, M_{H^0}, 0) \\
&\quad -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1+\gamma_5}{2} \left(-\frac{1}{8\pi^2}\right) \frac{g^2 m_u m_d \tan \beta \sin(\beta-\alpha) \sin \alpha}{4M_W^2 \sin \beta} \\
&\quad \cdot C_0^2(M_W^2, M_{H^+}, M_{H^0}, 0) \\
\Gamma_{\mu,\{H^+,h^0,u\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_u^2 \cot \beta \cos(\beta-\alpha) \cos \alpha}{4M_W^2 \sin \beta} \\
&\quad \cdot C_0^2(M_W^2, M_{H^+}, M_{h^0}, 0) \\
&\quad -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1+\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_u m_d \tan \beta \cos(\beta-\alpha) \cos \alpha}{4M_W^2 \sin \beta} \\
&\quad \cdot C_0^2(M_W^2, M_{H^+}, M_{h^0}, 0) \\
\Gamma_{\mu,\{H^+,A^0,u\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_u^2 \cot^2 \beta}{4M_W^2} \\
&\quad \cdot C_0^2(M_W^2, M_{H^+}, M_{A^0}, 0) \\
&\quad -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1+\gamma_5}{2} \left(-\frac{1}{8\pi^2}\right) \frac{g^2 m_u m_d}{4M_W^2} \\
&\quad \cdot C_0^2(M_W^2, M_{H^+}, M_{A^0}, 0) \\
\Gamma_{\mu,\{H^0,G^+,d\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_d^2 \cos \alpha \cos(\beta-\alpha)}{4M_W^2 \cos \beta} \\
&\quad \cdot C_0^2(M_W^2, M_{H^0}, M_W, 0) \\
&\quad -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1+\gamma_5}{2} \left(-\frac{1}{8\pi^2}\right) \frac{g^2 m_d m_u \cos \alpha \cos(\beta-\alpha)}{4M_W^2 \cos \beta}
\end{aligned}$$

$$\begin{aligned}
& \cdot C_0^2(M_W^2, M_{H^0}, M_W, 0) \\
\Gamma_{\mu, \{h^0, G^+, d\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 - \gamma_5}{2} \left(-\frac{1}{8\pi^2} \right) \frac{g^2 m_d^2 \sin \alpha \sin(\beta - \alpha)}{4M_W^2 \cos \beta} \\
& \cdot C_0^2(M_W^2, M_{h^0}, M_W, 0) \\
& -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 + \gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_d m_u \sin \alpha \sin(\beta - \alpha)}{4M_W^2 \cos \beta} \\
& \cdot C_0^2(M_W^2, M_{h^0}, M_W, 0) \\
\Gamma_{\mu, \{G^+, H^0, u\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 - \gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_u^2 \sin \alpha \cos(\beta - \alpha)}{4M_W^2 \sin \beta} \\
& \cdot C_0^2(M_W^2, M_W, M_{H^0}, 0) \\
& -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 + \gamma_5}{2} \left(-\frac{1}{8\pi^2} \right) \frac{g^2 m_u m_d \sin \alpha \cos(\beta - \alpha)}{4M_W^2 \sin \beta} \\
& \cdot C_0^2(M_W^2, M_W, M_{H^0}, 0) \\
\Gamma_{\mu, \{G^+, h^0, u\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 - \gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_u^2 \cos \alpha \sin(\beta - \alpha)}{4M_W^2 \sin \beta} \\
& \cdot C_0^2(M_W^2, M_W, M_{h^0}, 0) \\
& -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 + \gamma_5}{2} \left(-\frac{1}{8\pi^2} \right) \frac{g^2 m_u m_d \cos \alpha \sin(\beta - \alpha)}{4M_W^2 \sin \beta} \\
& \cdot C_0^2(M_W^2, M_W, M_{h^0}, 0)
\end{aligned}$$

3. Diagrams with each one virtual gauge boson, Higgs boson and fermion which contribute to the form factors H_L and H_R :

$$\begin{aligned}
\Gamma_{\mu, \{H^0, W^+, d\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \frac{p_{1\mu}}{M_W} \frac{1 + \gamma_5}{2} \\
& \cdot \left(-\frac{1}{16\pi^2} \right) M_W \frac{g^2 m_d \cos(\beta - \alpha) \cos \alpha}{\cos \beta} \\
& \cdot [C_1^+(M_W^2, M_{H^0}, M_W, 0) - C_1^-(M_W^2, M_{H^0}, M_W, 0)] \\
\Gamma_{\mu, \{h^0, W^+, d\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \frac{p_{1\mu}}{M_W} \frac{1 + \gamma_5}{2} \\
& \cdot \frac{1}{16\pi^2} M_W \frac{g^2 m_d \sin(\beta - \alpha) \sin \alpha}{\cos \beta} \\
& \cdot [C_1^+(M_W^2, M_{h^0}, M_W, 0) - C_1^-(M_W^2, M_{h^0}, M_W, 0)] \\
\Gamma_{\mu, \{W^+, H^0, u\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \frac{p_{1\mu}}{M_W} \frac{1 - \gamma_5}{2} \\
& \cdot \left(-\frac{1}{16\pi^2} \right) M_W \frac{g^2 m_u \sin \alpha \cos(\beta - \alpha)}{\sin \beta} \\
& \cdot [C_1^+(M_W^2, M_W, M_{H^0}, 0) + C_1^-(M_W^2, M_W, M_{H^0}, 0)]
\end{aligned}$$

$$\begin{aligned}
\Gamma_{\mu,\{W^+,h^0,u\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \frac{p_{1\mu}}{M_W} \frac{1-\gamma_5}{2} \\
&\cdot \left(-\frac{1}{16\pi^2} \right) M_W \frac{g^2 m_u \cos \alpha \sin(\beta - \alpha)}{\sin \beta} \\
&\cdot [C_1^+(M_W^2, M_W, M_{h^0}, 0) - C_1^-(M_W^2, M_W, M_{h^0}, 0)]
\end{aligned}$$

C.2.2 Vertex for the decay into two leptons

l appearing below states for three leptons e , μ and τ .

1. Diagrams with two virtual Higgs bosons and one virtual fermion which contribute to the form factor F_L :

$$\begin{aligned}
\Gamma_{\mu,\{H^0,H^+,l^-\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_l^2 \tan \beta \sin(\beta - \alpha) \cos \alpha}{4M_W^2 \cos \beta} \\
&\cdot C_0^2(M_W^2, M_{H^0}, M_{H^+}, 0) \\
\Gamma_{\mu,\{h^0,H^+,l^-\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_l^2 \tan \beta \cos(\beta - \alpha) \sin \alpha}{4M_W^2 \cos \beta} \\
&\cdot C_0^2(M_W^2, M_{h^0}, M_{H^+}, 0) \\
\Gamma_{\mu,\{A^0,H^+,l^-\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_l^2 \tan^2 \beta}{4M_W^2} \\
&\cdot C_0^2(M_W^2, M_{A^0}, M_{H^+}, 0) \\
\Gamma_{\mu,\{H^0,G^+,l^-\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \frac{1}{8\pi^2} \frac{g^2 m_l^2 \cos \alpha \cos(\beta - \alpha)}{4M_W^2 \cos \beta} \\
&\cdot C_0^2(M_W^2, M_{H^0}, M_W, 0) \\
\Gamma_{\mu,\{h^0,G^+,l^-\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1-\gamma_5}{2} \left(-\frac{1}{8\pi^2} \right) \frac{g^2 m_l^2 \sin \alpha \sin(\beta - \alpha)}{4M_W^2 \cos \beta} \\
&\cdot C_0^2(M_W^2, M_{h^0}, M_W, 0)
\end{aligned}$$

2. Diagrams with each one virtual gauge boson, Higgs boson and fermion which contribute to the form factor H_R :

$$\begin{aligned}
\Gamma_{\mu,\{H^0,W^+,l^-\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \frac{p_{1\mu}}{M_W} \frac{1+\gamma_5}{2} \\
&\cdot \left(-\frac{1}{16\pi^2} \right) M_W \frac{g^2 m_l \cos(\beta - \alpha) \cos \alpha}{\cos \beta} \\
&\cdot [C_1^+(M_W^2, M_{H^0}, M_W, 0) - C_1^-(M_W^2, M_{H^0}, M_W, 0)] \\
\Gamma_{\mu,\{h^0,W^+,l^-\}}^{(1)} &= -i \frac{g}{\sqrt{2}} \frac{p_{1\mu}}{M_W} \frac{1+\gamma_5}{2}
\end{aligned}$$

$$\cdot \frac{1}{16\pi^2} M_W \frac{g^2 m_l \sin(\beta - \alpha) \sin \alpha}{\cos \beta} \cdot [C_1^+(M_W^2, M_{h^0}, M_W, 0)) - C_1^-(M_W^2, M_{h^0}, M_W, 0)]$$

C.3 Self energies of the external fermions

The self energies are from the Feynman diagrams of figure 4. The fermion masses in the B functions are to be understood in the limit $m_f \rightarrow 0$.

C.3.1 Self energies of the quarks

u and d are written representatively for f_1 and f_2 as in C.2.1.

1. Self energy of the down quark

(a) Diagrams with the neutral Higgs bosons

$$\begin{aligned} \Sigma_{\{d, H^0\}}^d(p) &= \frac{1}{16\pi^2} \left(\frac{gm_d \cos \alpha}{2M_W \cos \beta} \right)^2 \left(\not{p} \frac{1 - \gamma_5}{2} + \not{p} \frac{1 + \gamma_5}{2} \right) \\ &\quad \cdot [B_0(p^2, M_{H^0}, m_d) + B_1(p^2, M_{H^0}, m_d)] \\ \Sigma_{\{d, h^0\}}^d(p) &= \frac{1}{16\pi^2} \left(\frac{gm_d \sin \alpha}{2M_W \cos \beta} \right)^2 \left(\not{p} \frac{1 - \gamma_5}{2} + \not{p} \frac{1 + \gamma_5}{2} \right) \\ &\quad \cdot [B_0(p^2, M_{h^0}, m_d) + B_1(p^2, M_{h^0}, m_d)] \\ \Sigma_{\{d, A^0\}}^d(p) &= \frac{1}{16\pi^2} \left(\frac{gm_d \tan \beta}{2M_W} \right)^2 \left(\not{p} \frac{1 - \gamma_5}{2} + \not{p} \frac{1 + \gamma_5}{2} \right) \\ &\quad \cdot [B_0(p^2, M_{A^0}, m_d) + B_1(p^2, M_{A^0}, m_d)] \end{aligned}$$

(b) Diagram with a charged Higgs boson

$$\begin{aligned} \Sigma_{\{u, H^\pm\}}^d(p) &= \frac{1}{16\pi^2} \frac{g^2}{2M_W^2} \left[(m_u \cot \beta)^2 \not{p} \frac{1 - \gamma_5}{2} + (m_d \tan \beta)^2 \not{p} \frac{1 + \gamma_5}{2} \right] \\ &\quad \cdot [B_0(p^2, M_{H^\pm}, m_u) + B_1(p^2, M_{H^\pm}, m_u)] \end{aligned}$$

2. Self energy of the up quark

(a) Diagrams with the neutral Higgs bosons

$$\begin{aligned} \Sigma_{\{u, H^0\}}^u(p) &= \frac{1}{16\pi^2} \left(\frac{gm_u \sin \alpha}{2M_W \sin \beta} \right)^2 \left(\not{p} \frac{1 - \gamma_5}{2} + \not{p} \frac{1 + \gamma_5}{2} \right) \\ &\quad \cdot [B_0(p^2, M_{H^0}, m_u) + B_1(p^2, M_{H^0}, m_u)] \end{aligned}$$

$$\begin{aligned}
\Sigma_{\{u,h^0\}}^u(p) &= \frac{1}{16\pi^2} \left(\frac{gm_u \cos \alpha}{2M_W \sin \beta} \right)^2 \left(\not{p} \frac{1-\gamma_5}{2} + \not{p} \frac{1+\gamma_5}{2} \right) \\
&\quad \cdot [B_0(p^2, M_{h^0}, m_u) + B_1(p^2, M_{h^0}, m_u)] \\
\Sigma_{\{u,A^0\}}^u(p) &= \frac{1}{16\pi^2} \left(\frac{gm_u \cot \beta}{2M_W} \right)^2 \left(\not{p} \frac{1-\gamma_5}{2} + \not{p} \frac{1+\gamma_5}{2} \right) \\
&\quad \cdot [B_0(p^2, M_{A^0}, m_u) + B_1(p^2, M_{A^0}, m_u)]
\end{aligned}$$

(b) Diagram with a charged Higgs boson

$$\begin{aligned}
\Sigma_{\{d,H^+\}}^d(p) &= \frac{1}{16\pi^2} \frac{g^2}{2M_W^2} \left[(m_d \tan \beta)^2 \not{p} \frac{1-\gamma_5}{2} + (m_u \cot \beta)^2 \not{p} \frac{1+\gamma_5}{2} \right] \\
&\quad \cdot [B_0(p^2, M_{H^+}, m_d) + B_1(p^2, M_{H^+}, m_d)]
\end{aligned}$$

C.3.2 Self energies of the leptons

l states for e , μ and τ as in C.2.2.

1. Self energy of the charged lepton

(a) Diagrams with the neutral Higgs bosons

$$\begin{aligned}
\Sigma_{\{l^-,H^0\}}^l(p) &= \frac{1}{16\pi^2} \left(\frac{gm_l \cos \alpha}{2M_W \cos \beta} \right)^2 \left(\not{p} \frac{1-\gamma_5}{2} + \not{p} \frac{1+\gamma_5}{2} \right) \\
&\quad \cdot [B_0(p^2, M_{H^0}, m_l) + B_1(p^2, M_{H^0}, m_l)] \\
\Sigma_{\{l^-,h^0\}}^l(p) &= \frac{1}{16\pi^2} \left(\frac{gm_l \sin \alpha}{2M_W \cos \beta} \right)^2 \left(\not{p} \frac{1-\gamma_5}{2} + \not{p} \frac{1+\gamma_5}{2} \right) \\
&\quad \cdot [B_0(p^2, M_{h^0}, m_l) + B_1(p^2, M_{h^0}, m_l)] \\
\Sigma_{\{l^-,A^0\}}^l(p) &= \frac{1}{16\pi^2} \left(\frac{gm_l \tan \beta}{2M_W} \right)^2 \left(\not{p} \frac{1-\gamma_5}{2} + \not{p} \frac{1+\gamma_5}{2} \right) \\
&\quad \cdot [B_0(p^2, M_{A^0}, m_l) + B_1(p^2, M_{A^0}, m_l)]
\end{aligned}$$

(b) Diagram with a charged Higgs boson

$$\begin{aligned}
\Sigma_{\{\nu_l,H^-\}}^l(p) &= \frac{1}{16\pi^2} \frac{g^2}{2M_W^2} (m_l \tan \beta)^2 \not{p} \frac{1+\gamma_5}{2} \\
&\quad \cdot [B_0(p^2, M_{H^+}, 0) + B_1(p^2, M_{H^+}, 0)]
\end{aligned}$$

2. Self energy of the neutrino

$$\begin{aligned}
\Sigma_{\{l^-,H^+\}}^{\nu_l}(p) &= \frac{1}{16\pi^2} \frac{g^2}{2M_W^2} (m_l \tan \beta)^2 \not{p} \frac{1-\gamma_5}{2} \\
&\quad \cdot [B_0(p^2, M_{H^+}, m_l) + B_1(p^2, M_{H^+}, m_l)]
\end{aligned}$$

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