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Forward Backward Asymmetry in Top Quark Semileptonic Decay

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ABSTRACT

The semileptonic decay of the top quark $t \rightarrow bW^+ \rightarrow bl^+\nu_l$ is analyzed in the rest system of the W . The forward backward asymmetry of the lepton l^+ with respect to the heavy quark direction is defined and computed. It is argued that this observable will be an ideal tool to study top quark properties at Tevatron and LHC. Higher order QCD corrections are calculated and their structure is elucidated in some detail.

1. Introduction

Recently, evidence has been found for the production of top quarks at the Fermilab Tevatron collider at $\sqrt{s} = 1.8 \text{ TeV}$ ¹. Under the hypothesis that the events are due to top quark production, the top quark mass was determined to be $m_t = 176 \pm 13 \text{ GeV}$ by CDF and $m_t = 199 \pm 20 \text{ GeV}$ by D0. This value was obtained by measuring the invariant mass of the top quark decay products on the basis of the leading standard model processes for top quark production and decay. In combination with the LEP1 analysis there is now conclusive evidence for the existence of a top quark. In the next years CDF and D0 will presumably be able to reduce the error to about $\pm 10 \text{ GeV}$. At the LHC² a really precise measurement of m_t with an expected error of about 1 GeV will be possible.

Once the mass of the top quark is established, the important question arises whether the top quark properties are precisely as predicted by the standard model. In the standard model the couplings of the top quark to the vector bosons are completely fixed by the family repetition structure. For example, the tbW coupling which induces almost all top quark decays should be a pure V-A interaction. In my opinion this coupling should be very carefully tested because it involves two heavy particles, t and W , and it is conceivable that deviations from the standard model show up first in this coupling³. It is at the same level of interest as the study of the ZWW coupling at LEP2. In the leading order in the standard model the tbW coupling is given by

$$\Gamma_\mu(tbW) = \frac{-ig}{\sqrt{2}} V_{bt} \gamma_\mu \frac{1}{2} (1 - \gamma_5)$$

The CKM matrix element V_{bt} will always be set to one in the following because transitions to other flavors than b take place at the per mille level⁴. This can be deduced from the unitarity of the CKM matrix and the known CKM matrix elements. The theoretical accuracy envisaged in this work is of the order $\sim 1\%$ and therefore $V_{bt} = 1$ a good approximation.

The tbW coupling can be tested by studying distributions of top quark decay products from the decay $t \rightarrow bW^+ \rightarrow b f_1 \bar{f}_2$. The cleanest analysis will be possible in the case of the semileptonic decay of the top quark, $f_1 = \nu_e, \bar{f}_2 = l^+$, provided the events can be reconstructed. It has indeed been shown by CDF and D0¹ that in

most of the one-lepton-plus-jet events even the neutrino momentum can be reconstructed, if use is made of the condition $(\nu_e + l^+)^2 = m_W^2$. Therefore, it will be possible to study precisely all sorts of interesting decay distributions, in particular at LHC, where more than 100 000 top quarks per year will be produced. This is the more important because the total width of the top quark cannot be measured very precisely⁵. Some interesting distributions, like the lepton energy distribution, have been already discussed in the literature⁶. Here we suggest another one, an angular distribution, which allows to define an asymmetry. This asymmetry will be at the centre of this article (section 2) and all sorts of corrections to it will be given (section 3).

2. The Asymmetry in Leading Order

Asymmetries have the advantage that they are normalized inclusive quantities and therefore can be measured with much less systematic error than the total width or angular distributions. Forward-backward asymmetries

$$A_{FB} = \frac{F - B}{F + B} \quad (1)$$

play an important role in the LEP1 analysis of Z boson properties⁷. For example, the asymmetry of b quarks⁸ from the process $e^+e^- \xrightarrow{Z} b\bar{b}$ is given by

$$A_{FB}^b = 3 \frac{v_e a_e}{v_e^2 + a_e^2} \frac{v_b a_b}{v_b^2 + a_b^2} + (m_b \text{ corrections}) + (ho \text{ corrections}) \quad (2)$$

where v and a are the corresponding vector and axialvector couplings. Thus it is sensitive to the ratios of couplings a/v and may be used to determine them. The numerator of (2) is determined by the vector-axialvector interference term ($\sim va \cos \theta$) whereas the denominator is given by the total $b\bar{b}$ production rate $F + B$ (even terms in $\cos \theta$). θ is the production angle of the b -quark.

Since they are ratios, forward-backward asymmetries are rather insensitive to many systematic errors and therefore the favourite observables of many experimentalists. They are in a sense totally inclusive quantities because they average the angular distribution in the regions $\cos \theta \in [0.1]$ and $[-1.0]$.

The above features of forward-backward asymmetries in e^+e^- annihilation carry over to the forward-backward asymmetry which may be defined in semileptonic top

quark decay. Consider fig. 1 in which the semileptonic decay of a top quark is shown in the rest system of the W . In this system the 4-momenta of t and b are identical and l and ν are produced back to back. The particle momenta can be parametrized as follows

$$\begin{aligned}
t &= \frac{m_t^2}{2\sqrt{W^2}}(\beta_+, 0, 0, \lambda^{\frac{1}{2}}) \\
b &= \frac{m_t^2}{2\sqrt{W^2}}(\beta_-, 0, 0, \lambda^{\frac{1}{2}}) \\
l &= \frac{\sqrt{W^2}}{2}(1, 0, \sin \theta, \cos \theta) \\
\nu &= \frac{\sqrt{W^2}}{2}(1, 0, -\sin \theta, -\cos \theta)
\end{aligned} \tag{3}$$

where $\beta_{\pm} = 1 \pm \frac{W^2}{m_t^2} - \frac{m_b^2}{m_t^2}$ and $\lambda = 1 + \frac{m_b^4}{m_t^2} + \frac{W^4}{m_t^4} - 2\frac{m_b^2}{m_t^2} - \frac{2W^2}{m_t^2} - 2\frac{m_b^2 W^2}{m_t^2}$. Clearly, there is a loose relation of this process to the e^+e^- process, by crossing.

One may, to a rather good approximation, put the W on-shell $W^2 = m_W^2$. Finite width effects $O(\frac{\Gamma_W}{m_W}) \sim \frac{2}{80}$ can be corrected for afterwards. In the narrow width approximation there is only one free kinematical parameter, the angle θ . The distribution in $\cos \theta$ will be at the heart of this work. In fig. 1 it is made clear that a forward-backward asymmetry $A_{FB} = \frac{F(\cos \theta > 0) - B(\cos \theta < 0)}{F(\cos \theta > 0) + B(\cos \theta < 0)}$ may be defined. In contrast to the total width $F + B^5$, the asymmetry will be precisely measured.

Theoretical analysis of top quark decay is usually carried out in the t -rest system (see⁶), but it can as well be done in the W -rest system provided the event can be reconstructed. This procedure has the further advantage that a decomposition in terms of W polarization contributions can be made. This will later allow us to get additional insight into the structure of the higher order corrections.

In the rest system of the top quark one may consider, for example, the (normalized) lepton energy distribution, $x_l = \frac{2tl}{m_t^2}$, or the distribution in the normalized neutrino energy $x_\nu = \frac{2t\nu}{m_t^2}$ or in the angle between b -quark and leptons etc. At tree-level, all these quantities are equivalent among themselves and to the angle θ defined above. For example, one has $x_l = \frac{1}{2}(\beta_+ - \lambda^{\frac{1}{2}} \cos \theta)$ and $x_\nu = \frac{1}{2}(\beta_+ + \lambda^{\frac{1}{2}} \cos \theta)$ so that indeed only one kinematical degree of freedom exists.

QCD corrections disturb these equivalence relations so that for each observable the QCD corrections are different and must be calculated separately. The QCD

corrections are the topic of the next section. Here we want to study the lowest order distribution $\frac{d\Gamma_0}{d\cos\theta}$. We have

$$d\Gamma_0(t \rightarrow bl^+\nu_l) = \frac{8G_F^2}{m_t} \frac{4(tl)(b\nu)}{(1 - \frac{W^2}{m_t^2})^2 + \frac{\Gamma_W^2}{m_W^2}} \frac{dR_3(t; b, l, \nu)}{(2\pi)^5} \quad (4)$$

where $dR_n(p; p_1, \dots, p_n) = \delta(p - \sum_{i=1}^n p_i) \prod_{i=1}^n \frac{d\vec{p}_i}{2E_i}$ is the phase space.

Let us, for the moment, put $W^2 = m_W^2$ and $m_b = 0$ and include the corrections $0(m_b, \Gamma_W)$ later. The matrix element obtained from a pure $V - A$ decay is

$$4(tl)(b\nu) = \frac{m_t^4}{4} [f_+ + 2f \cos\theta - f_- \cos^2\theta] f_- \quad (5)$$

where $f = \frac{m_W^2}{m_t^2}$ and $f_{\pm} = 1 \pm f$.

The phase space factor in (4) is given by

$$R_3(t; b, l, \nu) = \int_0^{m_t^2} dW^2 R_2(t; b, W) R_2(W; l, \nu) \quad (6)$$

with $R_2(t; b, W) = \frac{\pi}{2} (1 - \frac{W^2}{m_t^2})$ and $R_2(W; l, \nu) = \frac{\pi}{4} \int_{-1}^1 d\cos\theta$.

Using $\int_0^{m_t^2/m_W^2} \frac{dW^2}{(1 - \frac{W^2}{m_t^2})^2 + \frac{\Gamma_W^2}{m_W^2}} \approx \frac{m_W}{\Gamma_W} \pi \delta(1 - \frac{W^2}{m_t^2})$ and $\frac{\Gamma_W}{m_W} = \frac{G_F m_W^2}{6\sqrt{2}\pi}$

we obtain

$$d\Gamma_0(t \rightarrow bl^+\nu_l) = \frac{3G_F m_t^3}{32\sqrt{2}\pi} f_-^2 d\cos\theta [f_+ + 2f \cos\theta - f_- \cos^2\theta] \quad (7)$$

In fig. 2 the distribution in $\cos\theta$ is shown for two values of the top quark mass, $m_t = 160$ GeV and $m_t = 200$ GeV. Also shown is the distribution for a value of $\frac{a}{v}$ different from -1. This will be discussed later.

The forward-backward asymmetry is induced by the term $\sim \cos\theta$ in eq. (7) and given by

$$A_{FB} = \frac{3}{2} \frac{f}{1 + 2f} + 0\left(\frac{\Gamma_W}{m_t}, \frac{m_b}{m_t}, \alpha_s\right) \quad (8)$$

where the corrections will be calculated later. In the denominator of A_{FB} the total width of the top quark decaying to $bl^+\nu_l$ enters:

$$\begin{aligned} \Gamma_t &= \Gamma_t^0 + 0\left(\frac{\Gamma_W}{m_t}, \frac{m_b}{m_t}, \alpha_s\right) \\ \Gamma_t^0 &= \frac{G_F m_t^3}{16\sqrt{2}\pi} 2f_-^2 (1 + 2f) \end{aligned} \quad (9)$$

Notice that a factor f_-^2 cancels between numerator and demoninator of the asymmetry. Note also that for $f \rightarrow \infty$ one formally recovers eq. (2) from eq. (8) with $v_l = v_b = 1, a_l = a_b = -1$. For realistic values of m_t the asymmetry is typically of the order of $\frac{1}{4}$, i.e. relatively large.

Eq.(7) corresponds to an expansion of $\frac{d\Gamma}{d\cos\theta}$ in powers of $\cos\theta$ up to second order. The existence of such an expansion

$$\frac{d\Gamma}{d\cos\theta} = \frac{3}{8}\Gamma_U(1 + \cos^2\theta) + \frac{3}{4}\Gamma_L \sin^2\theta + \frac{3}{4}\Gamma_P \cos\theta \quad (10)$$

is a general feature, independent of the $V - A$ nature of the decay and true to any order in QCD. The various coefficients U, L and P in eq. (10) are related to the following spin polarization combinations of the W :

$$\begin{aligned} U &= (++) + (--), \\ P &= (++) - (--), \\ L &= (00) \end{aligned} \quad (11)$$

This will be discussed in more detail in section 3.

Explicitly, Γ_U, Γ_L and Γ_P are given in lowest order by

$$\Gamma_U^0 = \frac{G_F m_t^3 f_-^2 f}{4\sqrt{2}\pi} \quad (12)$$

$$\Gamma_L^0 = \frac{G_F m_t^3 f_-^2}{8\sqrt{2}\pi} \quad (13)$$

$$\Gamma_P^0 = \frac{G_F m_t^3 f_-^2 f}{4\sqrt{2}\pi} \quad (14)$$

Let us introduce, for a moment, a general V, A interaction at the tbW vertex (keeping the $Wl\nu$ interaction $V - A$): $\gamma_\mu(1 - \gamma_5) \rightarrow \gamma_\mu(v + a\gamma_5)$.

The matrix element for the decay $t \rightarrow bl^+\nu_l$ is then replaced by

$$4(tl)(b\nu) \rightarrow (v + a)^2(t\nu)(bl) + (v - a)^2(tl)(b\nu) - m_b m_t (v^2 - a^2)\nu l \quad (15)$$

The corresponding forward-backward asymmetry is given by

$$\begin{aligned} A_{FB} &= A_{FB}^0 + 0 \left(\frac{\Gamma_W}{m_t}, \frac{m_b}{m_t}, \alpha_s \right) \\ A_{FB}^0 &= \frac{3f}{1 + 2f} \frac{-a/v}{1 + (\frac{a}{v})^2} \end{aligned} \quad (16)$$

and is thus a measure of the ratio of couplings $\frac{g}{v}$. Unfortunately, the standard model value $\frac{g}{v} = -1$ is such that small deviations $\frac{g}{v} = -1 + \delta$ from -1 are not felt by A_{FB} , namely

$$A_{FB} = A_{FB}(\delta = 0) + O(\delta^2) \quad (17)$$

This is an unpleasant feature which is however not true for other small deviations from the standard model, like $\sigma_{\mu\nu}$ -type interactions etc.

Actually, part of the QCD corrections give an example of such an effect. Let us for a moment look at the effects from virtual gluon exchange, diagrams fig. 3. The tbW vertex including virtual gluon exchange is given by⁹

$$\Gamma_\mu(t \rightarrow bW^+) = \frac{-ig}{\sqrt{2}} \left[H \gamma_\mu \frac{1}{2}(1 - \gamma_5) - C_F \frac{\alpha_s}{2\pi} H_- \frac{i\sigma_{\mu\nu} W^v}{2m_t^2} \frac{1}{2}(1 + \gamma_5) \right] \quad (18)$$

$H = 1 + O(\alpha_s)$ is the leading order term rescaled by QCD corrections. The product $\alpha_s \cdot H_-$ plays the role of the small $\sigma_{\mu\nu}$ admixture. H_- is given by⁹

$$H_- = -\frac{\ln f_-}{f} \quad (19)$$

(still working in the limit $m_b = 0, W^2 = m_W^2$).

The corresponding matrix element for $t \rightarrow bl^+\nu_l$ is

$$\left| M_{\text{Born}} + M_{\text{virtual}} \right|^2 = H(tl)(b\nu) - C_F \frac{\alpha_s}{2\pi} H_-(bl)(b\nu). \quad (20)$$

It can now easily be seen that the effect of the $\sigma_{\mu\nu}$ -term is a longitudinal piece $\sim \sin^2 \theta$ in the matrix element and therefore a contribution only to the denominator of the asymmetry, not to the numerator

$$A_{FB} = \frac{3}{2} \frac{f}{1 + 2f + C_F \frac{\alpha_s}{2\pi} \frac{f_- \ln f_-}{f}} \approx A_{FB}^0 \left(1 - C_F \frac{\alpha_s}{2\pi} \frac{f_- \ln f_-}{f(1 + 2f)} \right) \quad (21)$$

where $A_{FB}^0 = \frac{3}{2} \frac{f}{1 + 2f}$. So in contrast to eq. (17) there is really a linear term $O(\alpha_s)$ present in eq. (21).

This will be added to the real gluon emission contribution presented in the next section. It should be noted that A_{FB} gets no contribution from the $O(\alpha_s)$ part of H because this correction cancels between the numerator and denominator of the asymmetry. The infrared and mass singularities of the virtual corrections are solely

in the $O(\alpha_s)$ term of H . This proves (via the Lee Nauenberg theorem) that the real gluon emission diagrams treated in section 3 give a finite correction to A_{FB} . Of course, if instead of A_{FB} one wants to know the corrections to $\frac{d\Gamma}{d\cos\theta}$, infrared and mass singularities will be present and the $O(\alpha_s)$ part of H will contribute. Explicitly

$$\begin{aligned} H &= 1 - C_F \frac{\alpha_s}{2\pi} H_0 \\ H_0 &= 2L_2(f) + \ln f_b (1 - 2\ln f_b - 4\ln f_-) \\ &\quad + \frac{1-2f}{f} \ln f_- + 4 + 4(\ln f_b + 1) \ln \frac{\lambda}{m_t} \end{aligned} \quad (22)$$

Here $f_b = \frac{m_b}{f_-}$ and λ is the gluon mass needed to regularize the infrared singularities. In eq. (22) terms of order m_b have been neglected. The b -quark mass is kept only in the logarithms as a regulator of the mass singularities. Even there it is needed only in intermediate steps because the final result for the QCD corrections to $\frac{d\Gamma}{d\cos\theta}$ (virtual + real) is free of mass and infrared singularities. This will be further discussed in the next section.

For the moment I would like to leave the QCD corrections and return to tree level. I want to take a closer look at the finite width corrections $O(\frac{\Gamma_W}{m_t})$ and b -quark mass corrections to A_{FB} at Born level. We shall see that they are very small ($\leq 1\%$) and conclude from this that terms of order $O(\alpha_s \frac{\Gamma_W}{m_t})$ and $O(\alpha_s \frac{m_b}{m_t})$ are probably negligible. The Born level $O(\Gamma_W, m_b)$ corrections to A_{FB} can be calculated by a Monte-Carlo integration of eq. (4) in the regions $\{\cos\theta > 0\}$ and $\{\cos\theta < 0\}$. The phase space dR_3 is provided, for example, by the program RAMBO written by Ellis, Kleiss and Stirling¹⁰. In table 1 we have given the results for A_{FB} with $m_b = 4.5$ GeV and $\Gamma_W = 2$ GeV for various values of m_t and a/v and compared it to the corresponding values for $m_b = \Gamma_W = 0$. We see that these corrections are typically $\sim 1\%$. For the standard model value $\frac{a}{v} = -1$ and masses $m_t = 160 - 200$ GeV there is a reduction of A_{FB} between -4.0 and -2.2 per mille.

3. Real Gluon Corrections to A_{FB}

The virtual gluon correction to A_{FB} has already been given in eq. (21). It was a simple exercise because it could be treated in the framework of tree level kinematics and did not contain any ultraviolet, infrared or collinear singularity. In this section

we shall explicitly see that the real gluon corrections to A_{FB} (fig. 4) are completely finite, too. The calculation of the diagrams in fig. 4 yields

$$d\Gamma(t \rightarrow bgl^+\nu_l) = \frac{256G_F^2\pi^2}{m_t} \frac{|M(t \rightarrow bgl^+\nu_l)|^2}{(1 - \frac{W^2}{m_W^2}) + \frac{\Gamma_W^2}{m_W^2}} \frac{dR_4(t; b, g, l, \nu)}{(2\pi)^8} \quad (23)$$

with

$$\begin{aligned} |M(t \rightarrow bgl^+\nu_l)|^2 = & -C_F \frac{\alpha_s}{2\pi} \left\{ -\frac{(b\nu)[(lg)(tg) + m_t^2 l(g-t)]}{(tg)^2} \right. \\ & + \frac{(b\nu)[(lg)(tb) - (bl)(tg) + (tl)((bg) - (tg) - 2(tb))]}{(tg)(bg)} \\ & \left. - \frac{(tl)[(\nu g)(bg) - m_b^2 \nu(g+b)]}{(bg)^2} \right\} \end{aligned} \quad (24)$$

This formula is valid for $m_b \neq 0$, $W^2 \neq m_W^2$. However, in the following we shall work with the approximations $m_b = \lambda = 0$ and $W^2 = m_W^2$. This is possible because we shall encounter completely regular expressions in all steps of the calculation of A_{FB} . A priori, the matrix element above has singularities for $tg \rightarrow 0$ and $bg \rightarrow 0$ but these drop out when the contribution to A_{FB} is formed. This will be explicitly shown later.

Real gluon emission in the W rest system implies that the 3-momenta of t - and b -quark are not parallel any more (whereas l and ν are still back to back). Therefore, two forward backward asymmetries of the lepton may be defined, one with respect to the b direction and one with respect to the t direction. In lowest order these two asymmetries, let us call them A_{FB}^{lb} and A_{FB}^{lt} are identical but they get different QCD corrections. In addition to the θ -dependence an azimuthal dependence arises in the matrix element for $t \rightarrow bgl^+\nu_l$. This will be traced to W polarization contributions different from the diagonal ones, Γ_U, Γ_L , and Γ_P , eq. (11), and may be of interest in future studies of top quark decay. To uncover the azimuthal dependence, let us

parametrize the momenta as follows:

$$\begin{aligned}
W &= (m_W, 0, 0, 0) \\
l &= \frac{m_W}{2} (1, \sin \theta \sin \varphi, \sin \theta \cos \varphi, \cos \theta) \\
\nu &= W - l \\
b &= \frac{m_t^2}{2m_W} f_- x_b (1, 0, 0, 1) \\
t &= \frac{m_t^2}{2m_W} f_+ x_t (1, 0, \beta \sin \alpha, \beta \cos \alpha) \\
g &= t - W - b
\end{aligned} \tag{25}$$

Here x_b and x_t are the rescaled quark energies.

$$\begin{aligned}
x_b &= 1 - \frac{2tg}{m_t^2 f_-} \\
x_t &= 1 - \frac{2bg}{m_t^2 f_+}
\end{aligned} \tag{26}$$

The quantities $1 - x_b$ and $1 - x_t$ are the Dalitz variables which will appear in the phase space. By eq. (26) they are related to the propagator poles tg and bg . $f_{\pm} = 1 \pm f$, $f = \frac{m_W^2}{m_t^2}$ are defined as before. $\beta = \sqrt{1 - \frac{4f}{f_+^2 x_t^2}}$ follows from the mass shell condition $t^2 = m_t^2$. The angle α is also completely fixed for given x_b and x_t . This follows from $g^2 = (t - b - W)^2 = 0$:

$$1 - \beta \cos \alpha = 2f \frac{f_+(1 - x_t) + f_- x_b}{f_+ f_- x_t x_b} \tag{27}$$

The angular orientation of the event is defined by the polar angle θ and the azimuthal angle φ . In the parametrization given above θ is the angle between b and l and we are going to discuss A_{FB}^{bl} first. The other asymmetry, A_{FB}^{tl} , is more conveniently discussed in the system where $\vec{t}$ is put along the z -direction. This will be done later.

The phase space can be given in terms of the four variables x_t, x_b, θ and φ :

$$dR_4(t; b, g, l, \nu) = m_t^2 \int_0^{m_t^2} dW \cdot 2 \frac{\pi^2}{32} f_+ f_- \int_0^1 d(1 - x_b) \int_0^{l^*} d(1 - x_t) \int_{-1}^1 d \cos \theta \int_0^{2\pi} d\varphi \tag{28}$$

where the upper limit for $1 - x_t$ is

$$l^* = \frac{f_-^2 x_b (1 - x_b)}{f_+ (1 - f_- (1 - x_b))} \tag{29}$$

The contribution to A_{FB}^{bl} will be obtained by doing the complete integration of the matrix element in the forward and backward regions ($\cos \theta > 0$ resp. $\cos \theta < 0$). Before I start to describe the calculation I want to present the generic form of the QCD effect. Namely, including the real gluon emission, the asymmetry will be of the form

$$A_{FB}^{bl} = \frac{F - B}{F + B} = \frac{F_0(1 - C_F \frac{\alpha_s}{2\pi} F_1) - B_0(1 - C_F \frac{\alpha_s}{2\pi} B_1)}{F_0(1 - C_F \frac{\alpha_s}{2\pi} F_1) + B_0(1 - C_F \frac{\alpha_s}{2\pi} B_1)} \quad (30)$$

where F_0 and B_0 are the lowest order contributions and F_1 and B_1 denote the relative magnitude of the higher order from forward and backward. After some simple manipulations one finds

$$A_{FB}^{bl} = \frac{F_0 - B_0}{F_0 + B_0} \left\{ 1 - C_F \frac{\alpha_s}{2\pi} \frac{2F_0 B_0}{F_0^2 - B_0^2} (F_1 - B_1) \right\} \quad (31)$$

where $\frac{F_0 - B_0}{F_0 + B_0} = \frac{3}{2} \frac{f}{1+2f}$ is the lowest order piece and $\frac{2F_0 B_0}{F_0^2 - B_0^2} = \frac{1+4f+\frac{7}{3}f^2}{3f(1+2f)}$. The task is therefore to calculate the difference $F_1 - B_1$, i.e. to integrate the matrix element (24) over the full phase space region with the condition that the backward piece is weighted negative. This can easily be accomplished because the θ -dependence is always of the form $1, \cos \theta$ or $\cos^2 \theta$. To make this explicit let us express the matrix element eq. (24) in terms of the phase space variables x_b, x_t, θ and φ :

$$\begin{aligned} |M(t \rightarrow bgl^+ \nu_l)|^2 = & -C_F \frac{\alpha_s}{2\pi} \left\{ \frac{3}{8} Z_U (1 + \cos^2 \theta) + \frac{3}{4} Z_L \sin^2 \theta \right. \\ & + \frac{3}{4} Z_P \cos \theta + \frac{3}{\sqrt{2}} Z_N \sin \theta \cos \varphi - \frac{3}{4} Z_T \sin^2 \theta (2 \cos^2 \varphi - 1) \\ & \left. + \frac{3}{\sqrt{2}} Z_F \sin \theta \cos \theta \cos \varphi \right\} \end{aligned} \quad (32)$$

where the coefficients Z depend only on x_t, x_b and the masses. They are given in the appendix. Eq. (32) makes the angular dependence and the decomposition into W polarization contributions¹¹ explicit. Namely, assume that we specify the three independent polarization vectors of the W as $\varepsilon^\pm = \mp(0, 1, \mp i, 0)/\sqrt{2}$ and $\varepsilon^0 = (0, 0, 0, -1)$.

Then there will be nine products of tensor $\varepsilon_\alpha \varepsilon_\beta^*$, namely $(00) := \varepsilon_\alpha^0 \varepsilon_\beta^{0*} = \begin{pmatrix} 0000 \\ 0000 \\ 0000 \\ 0001 \end{pmatrix} \cdot (0+) := \varepsilon_\alpha^0 \varepsilon_\beta^{+*}$, etc. into which both the lepton side and the quark side of the matrixelement

can be expanded. The above terms U, L, P, N, T and F originate from the following orthogonal combinations of tensors:

$$\begin{aligned} U &= (++) + (--) & L &= (00) & P &= (++) - (--) \\ N &= \frac{1}{4i}[(+0) - (0+) + (0-) - (-0)] & T &= \frac{1}{2}[(+-) + (-+)] \\ F &= \frac{1}{4i}[(+0) - (0+) + (-0) - (0-)] \end{aligned} \quad (33)$$

There are three independent tensors which do not arise in the matrixelement eq. (32) namely

$$\begin{aligned} I &= \frac{1}{4}[(+0) + (0+) - (-0) - (0-)] & V &= \frac{1}{2i}[(+-) - (-+)] \\ A &= \frac{1}{4}[(+0) + (0+) + (-0) + (0-)]. \end{aligned} \quad (34)$$

How can one understand that, for instance, $L = (00)$ gives rise to $\frac{3}{4}\sin^2\theta$ on the leptonside? The answer is that $\frac{3}{4}\sin^2\theta$ arises when one multiplies the lepton trace

$$L_{\alpha\beta} = l_\alpha \nu_\beta + \nu_\alpha l_\beta - l\nu g_{\alpha\beta} + i\varepsilon(\alpha\beta l\nu)$$

with $\varepsilon_\alpha^0 \varepsilon_\beta^{0*} = \delta_{\alpha z} \delta_{\beta z}$. Similarly for U, P , etc. The coefficients Z are the corresponding contributions from the quark trace. In the case that we put the top quark along the z -direction (to determine A_{FB}^{tb}) a similar but not identical expansion will hold.

Let us now come back to our asymmetry. The azimuthal integration is easily carried out and shows that N, T and F do not contribute to the forward backward asymmetry at all. The integral over θ must be carried out according to the prescription derived from eq. (31). According to that prescription the following combination of Z_U, Z_L and Z_P has to be formed:

$$Z_{FB} \equiv (1 + 2f)Z_P - 2f(Z_U + Z_L) \quad (35)$$

because the leading order ratio of P to $U + L$ is $2f$ to $1 + 2f$.

The functions $Z_{U,L,P}(x_b, x_t)$ are given in the appendix. They have collinear and infrared singularities for $x_b \rightarrow 1$ and $x_t \rightarrow 1$. However, the combination Z_{FB} is completely finite in these regions. The integration over x_b and x_t is therefore relatively simple and can be done without regularization. Let us write

$$A_{FB}^{bl} = A_{FB}^0 (1 - C_F \frac{\alpha_s}{2\pi} D_{bl}) \quad (36)$$

where

$$A_{FB}^0 = \frac{3f}{2(1+2f)} \quad (37)$$

According to eq. (31) one has

$$\begin{aligned} D_{bl} &= \frac{2F_0 B_0}{F_0^2 - B_0^2} (F_1 - B_1) \\ &= \frac{f_- \ln f_-}{f(1+2f)} + f_+ f_- \int_0^1 d(1-x_b) \int_0^1 d(1-x_t) \frac{3Z_{FB}}{f(1+2f)f_-^2} \end{aligned} \quad (38)$$

D_{bl} gives the relative correction to A_{FB}^{bl} in units of $-C_F \frac{\alpha_s}{2\pi}$. The first term in eq. (38) is the contribution from virtual gluon exchange. Using the expression for Z_{FB} given in the appendix the integration is straightforward and gives

$$D_{bl} = \frac{1}{2f_-^2(1+2f)} \left\{ -2f(3+2f) \ln f - 1 - 2f - 3f^2 + 6f^3 + \frac{2f_-^3 \ln f_-}{f} \right\} \quad (39)$$

For $m_t = 160$ GeV this gives $D_{bl}(f = \frac{1}{4}) = -0.0822$ corresponding to a tiny correction of about 2 per mille. Fig. 5 and table 2 show the magnitude of the correction for other values of m_t . One can see that it changes sign at about $m_t \sim 150$ GeV. Also shown in fig. 5 is the relative magnitude D_{total} of the QCD correction to the top quark semileptonic width defined by*

$$\Gamma_t = \Gamma_t^0 (1 - C_F \frac{\alpha_s}{2\pi} D_{\text{total}}) \quad (40)$$

The semileptonic top width Γ_t is the correction to the denominator of the asymmetry. In the notation of eq. (30): $D_{\text{total}} = \frac{F_0 F_1 + B_0 B_1}{F_0 + B_0}$. Explicitly, it is given by⁹

$$D_{\text{total}} = \frac{2\pi^3}{3} - \frac{5}{2} + 2 \ln f \ln f_- + 4L_2(f) - 2f + \frac{(5+4f) \ln f_- + \frac{2f \ln f}{f_-} - \frac{4f^3(f_- + \ln f)}{f_-^2}}{1+2f} \quad (41)$$

Numerically, D_{total} is much larger than the correction D_{bl} to the asymmetry. For instance, for $m_t = 160$ GeV, one has $D_{\text{total}} = 3.73$ corresponding to a reduction of

*The total width of the top quark is obtained from this by multiplying with $9 + 6 \frac{\alpha_s}{\pi}$ where the term $6 \frac{\alpha_s}{\pi}$ gives the QCD correction to $W^+ \rightarrow q_1 \bar{q}_2$.

Γ_t by about 8.7 percent. From the knowledge of D_{lb} and D_{total} one can derive the relative correction to the numerator of the asymmetry,

$$\frac{F_0 F_1 - B_0 B_1}{F_0 - B_0} = 3.65 \quad \text{for } m_t = 160 \text{ GeV} \quad (42)$$

and also the relative correction F_1 and B_1 from the forward and backward regions. can be obtained, $F_1 = 3.715$ and $B_1 = 3.758$ for $m_t = 160$ GeV. We see that there is a strong cancellation between F_1 and B_1 leading to the small correction to the asymmetry.

The corrections to A_{FB}^{tl} can be obtained in a similar manner. One should just put the top quark along the z direction, $t = \frac{m_t^2}{2m_W^2} f + x_t(1, 0, 0, \beta)$, $b = \frac{m_t^2}{2m_W^2} f - x_b(1, 0, \sin \alpha, \cos \alpha)$. The relations for β , x_t , x_b and $\cos \alpha$ remain intact. The lepton 4 momenta can still be written in the form of eq. (25) although the angles θ and φ have a different meaning. $\theta = \angle(\vec{t}, \vec{l})$ is now the polar angle to determine A_{FB}^{tl} . The expansion eq. (32) is formally the same as before but the coefficients Z_U, Z_L, Z_P etc. are different. Their integration will not be discussed in detail. We just present the result of the calculation, in the form

$$A_{FB}^{tl} = A_{FB}^0 (1 - C_F \frac{\alpha_s}{2\pi} D_{tl}) \quad (43)$$

D_{tl} is given in fig. 5 and table 2 as a function of the top quark mass. It is negative and slightly larger than D_{bl} . For $m_t = 160$ GeV it corresponds to a QCD correction of about +1.5%.

4. Conclusions and Digressions

In this article the orientation of decay products of top quarks has been studied in the W rest system. It has been shown that several differential and also inclusive observables exist which deserve experimental attention. The most promising one seems to be the forward backward asymmetry of the leptons defined in semileptonic decays of the top quark $t \rightarrow bl\nu$. Actually, two asymmetries, one with respect to top and the other one with respect to bottom can be defined. Experimentally, they are easily determined, to the extent that the 4 momenta of l, W and $b(t)$ can be reconstructed. In fact, the angles θ_{bl} resp. θ_{tl} on which the definition of the

asymmetries relies are given in terms of those 4 momenta by

$$\begin{aligned}\cos \theta_{bl} &= 1 - \frac{m_W^2(bl)}{(Wb)(Wl)} \\ \cos \theta_{tl} &= \frac{1}{\beta} \left[1 - \frac{m_W^2(tl)}{(Wt)(Wl)} \right], \quad \beta = \sqrt{1 - \frac{m_t^2 m_W^2}{(Wt)^2}}\end{aligned}\tag{44}$$

Thus the forward backward conditions $\{\cos \theta_{bl} \gtrless 0\}$ resp. $\{\cos \theta_{tl} \gtrless 0\}$ can easily be implemented.

The asymmetries are not only easy to measure but they also show a remarkable stability to all sorts of standard theoretical corrections. We have seen in this article that QCD and finite width corrections are typically of the order of 1% or smaller. We have not considered electroweak corrections but they are to be expected even smaller. Of course, for a real precision analysis all these corrections should be quantitatively known. For QCD and $O(\Gamma_W)$ corrections this has now been achieved. Since first order QCD is so small it is not necessary to consider higher orders to reduce the scale dependence. In all expressions for the asymmetry, α_s may safely be put to $\alpha_s = \alpha_s(m_t)$.

A precision measurement of the asymmetries will allow for a precise test of the $V - A tbW$ coupling. It will not be possible to identify small admixtures of $V + A$ type but anything else like electric or magnetic moment interactions could neatly be discovered. I think it is a serious possibility that the tbW vertex is the first place in which deviations from the standard model show up because it involves two very heavy particles. For a really complete analysis it would be important to have a Monte Carlo program in which the complete QCD matrix elements are included in such a form that a global fit of top quark decay is possible. This is a task for the future when LHC will provide us with high statistic data of decaying top quarks.

In this work we have concentrated on the forward backward asymmetry although the angular analysis eq. (10) and eq. (32) of the top quark decay in the W rest system contains a lot more of information. For example, a calculation of the QCD correction to the longitudinal part Γ_L of the decay width is quite important. In processes involving only one large scale, like $e^+e^- \rightarrow q\bar{q}$ or deep inelastic scattering the longitudinal contribution is usually small. In the case at hand it is very large.

In lowest order the ratio between longitudinal and transverse width is given by

$$\frac{\Gamma_L}{\Gamma_U} = \frac{1}{2} \frac{m_t^2}{m_W^2} (\cong 2 \text{ for } m_t = 160 \text{ GeV!}) \quad (45)$$

It is therefore an interesting question to see how QCD corrections affect this ratio. We want to present the calculation of these corrections in the following for the case of $\theta = \theta_{bl}$. Let us write

$$\begin{aligned} R_L &:= \frac{\Gamma_L}{\Gamma_U} = \frac{\Gamma_L^0 - C_F \frac{\alpha_s}{2\pi} \Gamma_L^1}{\Gamma_U^0 - C_F \frac{\alpha_s}{2\pi} \Gamma_U^1} \\ &= \frac{1}{2f} \left\{ 1 - C_F \frac{\alpha_s}{2\pi} D_R \right\} \end{aligned} \quad (46)$$

with

$$\begin{aligned} D_R &= \frac{2f\Gamma_L^1 - \Gamma_U^1}{\Gamma_U^0} \\ \Gamma_U^0 &= \frac{G_F m_t^3}{16\sqrt{2}\pi} 2f_-^2 \cdot 2f \\ \Gamma_L^0 &= \frac{G_F m_t^3}{16\sqrt{2}\pi} 2f_-^2 \end{aligned} \quad (47)$$

and Γ_L^1 and Γ_U^1 to be calculated. More precisely, for R_L we need only the combination $2f\Gamma_L^1 - \Gamma_U^1$. The contribution to this combination from virtual gluon exchange diagrams is finite and given by

$$D_R(\text{virtual}) = -\frac{f_- \ln f_-}{f} \quad (48)$$

It comes solely from the $\sigma_{\mu\nu}$ -term in eq. (18) and is free of infrared and mass singularities. In this respect the situation for $\frac{\Gamma_L}{\Gamma_U}$ is similar than for $A_{FB} = \frac{3}{4} \frac{\Gamma_P}{\Gamma_U + \Gamma_L}$ in the sense that infrared and mass singularities cancel between numerator and denominator, and the contribution to D_R from real gluon emission is completely finite as well. In fact, one has

$$D_R(\text{real}) = \frac{3f_+}{ff_-} \int_0^1 d(1-x_b) \int_0^{l^*} d(1-x_t) (2fZ_L - Z_U) \quad (49)$$

where the functions $Z_{L,U}(x_b, x_t)$ are given in the appendix and

$$l^* = \frac{f_-^2 x_b (1-x_b)}{f_+ (1-f_-(1-x_b))}. \quad (50)$$

The integration is straightforward. Adding the result of D_R (virtual) we obtain

$$D_R = -\frac{2}{f_-^2} \left\{ -f_-^2 f_- + \left(\frac{f_-}{2} - f_-^2 \right) \ln f_- \right\} - \frac{f_-}{f} \ln f_- \quad (51)$$

Numerically, this gives $D_R = 2.57$ for $m_t = 160\text{GeV}$, corresponding to a correction of about -6% to $\frac{\Gamma_L}{\Gamma_U}$.

Γ_U, Γ_P and Γ_L constitute the full θ -dependence of the partial width after azimuthal angle integration. An equivalent set of observables is

$$\Gamma_U, \Gamma_L, \Gamma_P \leftrightarrow \Gamma_{\text{total}} = \Gamma_U + \Gamma_L, R_L = \frac{\Gamma_L}{\Gamma_U}, A_{FB} = \frac{3}{4} \frac{\Gamma_P}{\Gamma_L + \Gamma_U} \quad (52)$$

Knowing the corrections to $\Gamma_{\text{total}}, R_L$ and A_{FB} (eqs. (36),(39),(41),(46) and (51)) is therefore equivalent to knowing the correction to $\frac{d\Gamma}{d\cos\theta}$. Namely, one has

$$\begin{aligned} \frac{d\Gamma}{d\cos\theta} &= (\Gamma_U + \Gamma_L) \left\{ \frac{3}{8}(1 + \cos^2\theta) \frac{\Gamma_U}{\Gamma_U + \Gamma_L} + \frac{3}{4} \sin^2\theta \frac{\Gamma_L}{\Gamma_U + \Gamma_L} + \frac{\frac{3}{4}\Gamma_P}{\Gamma_U + \Gamma_L} \cos\theta \right\} \\ &= \Gamma_{\text{total}} \left\{ \frac{3}{8}(1 + \cos^2\theta) \frac{1}{1 + R_L} + \frac{3}{4} \sin^2\theta \frac{R_L}{1 + R_L} + A_{FB} \cos\theta \right\} \\ &= \frac{G_F m_t^3}{16\sqrt{2}\pi} 2f_-^2 (1 + 2f)(1 - C_F \frac{\alpha_s}{2\pi} D_{\text{total}}) \left\{ \frac{3}{8}(1 + \cos^2\theta) \frac{2f}{1 + 2f} \left(1 + C_F \frac{\alpha_s}{2\pi} \frac{D_R}{1 + 2f} \right) \right. \\ &\quad \left. + \frac{3}{4} \sin^2\theta \frac{1}{1 + 2f} \left(1 - C_F \frac{\alpha_s}{2\pi} \frac{2f D_R}{1 + 2f} \right) + \frac{3}{2} \frac{f}{1 + 2f} \left(1 - C_F \frac{\alpha_s}{2\pi} D_{bl} \right) \cos\theta \right\} \end{aligned} \quad (53)$$

Conversely, knowing the corrections to $\Gamma_{\text{total}}, R_L$ and R_{FB} one may write down the corrections to Γ_U, Γ_L and Γ_P in the following way

$$\Gamma_{U,L,P} = \Gamma_{U,L,P}^0 (1 - C_F \frac{\alpha_s}{2\pi} D_{U,L,P}) \quad (54)$$

with

$$\begin{aligned} D_U &= D_{\text{total}} - \frac{D_R}{1 + 2f} (\approx 2.02 \text{ for } m_t = 160\text{GeV}) \\ D_L &= D_{\text{total}} + \frac{2f D_R}{1 + 2f} (\approx 4.59 \text{ for } m_t = 160\text{GeV}) \\ D_P &= D_{\text{total}} + D_{bl} (\approx 3.65 \text{ for } m_t = 160\text{GeV}) \end{aligned} \quad (55)$$

These equations once again show to what extent the correction to $\frac{d\Gamma}{d\cos\theta}$ is homogeneous in θ .

Instead of R_L it is sometimes more convenient to use the ratios $\frac{\Gamma_L}{\Gamma_U + \Gamma_L} = \frac{R_L}{1 + R_L}$, $\frac{\Gamma_U}{\Gamma_U + \Gamma_L} = \frac{1}{1 + R_L}$ or $\frac{\Gamma_L - \Gamma_U}{\Gamma_L + \Gamma_U} = \frac{R_L - 1}{R_L + 1}$. For example, $\frac{\Gamma_L - \Gamma_U}{\Gamma_L + \Gamma_U}$ measures the asymmetry between the longitudinal and transverse contributions to the total cross section. The QCD expression for these ratios can easily be obtained from D_R , namely

$$\begin{aligned}\frac{\Gamma_L}{\Gamma_U + \Gamma_L} &= \frac{1}{1 + 2f} \left(1 - C_F \frac{\alpha_s}{2\pi} \frac{2f D_R}{1 + 2f} \right) \\ \frac{\Gamma_U}{\Gamma_U + \Gamma_L} &= \frac{2f}{1 + 2f} \left(1 + C_F \frac{\alpha_s}{2\pi} \frac{D_R}{1 + 2f} \right) \\ \frac{\Gamma_L - \Gamma_U}{\Gamma_U + \Gamma_L} &= \frac{1 - 2f}{1 + 2f} \left(1 - C_F \frac{\alpha_s}{2\pi} \frac{4f D_R}{1 - 4f^2} \right)\end{aligned}\tag{56}$$

In this article we have calculated QCD corrections to two interesting observables in top quark semileptonic decay. There is a related problem in hadronic top quark decays, $t \rightarrow bq_1\bar{q}_2$, namely the angular distribution of the light quarks with respect to the heavy quark beam in the W-rest frame. Since quark and antiquark jets cannot be discriminated it is not possible to determine the forward backward asymmetry, but the ratio $R_L := \frac{\Gamma_L}{\Gamma_U}$ is certainly an interesting quantity whose QCD correction should be calculated. These corrections fall into 2 parts : First, the corrections to the heavy quark side which can be taken from eq. (51), and secondly, the corrections to the light quark side.

Experimentally, R_L will have very high statistics in hadronic top decays and will give very clean information about the amount to which the W is longitudinally polarized.

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Appendix

In this appendix we make the angular decomposition eq. (32) of the matrixelement $M(t \rightarrow bgl\nu)$ explicit. Let us look at the case in which the b -quark defines the z -direction ($\theta = \angle(\vec{b}, \vec{l})$). Instead of $Z_U, Z_L, Z_P, Z_T, Z_F, Z_N$ we shall give the set

$$Z_U + Z_L, 2fZ_L - Z_U, \frac{1+2f}{2f}Z_P - (Z_U + Z_L), Z_T, Z_F, Z_N$$

from which Z_U, Z_L and Z_P can easily be constructed. Among the 6 combinations in this set only $Z_U + Z_L$ contains infrared and collinear singularities. The others are completely finite and can be integrated without regularization as explained in the main text.

$$\frac{4\pi d\Gamma}{d\cos\theta d\varphi} = \frac{3G_F m_t^3}{2\sqrt{2}\pi} f_+ f_- \int_0^1 d(1-x_b) \int_0^U d(1-x_t) \frac{|M(t \rightarrow bgl^+\nu_l)|^2}{m_t^2}$$

with

$$\begin{aligned} \frac{|M|^2}{m_t^2} = & -C_F \frac{\alpha_s}{2\pi} \left\{ \frac{3}{8} Z_U (1 + \cos^2 \theta) + \frac{3}{4} Z_L \sin^2 \theta + \frac{3}{4} Z_P \cos \theta \right. \\ & \left. + \frac{3}{\sqrt{2}} Z_N \sin \theta \cos \varphi - \frac{3}{4} Z_T \sin^2 \theta (2 \cos^2 \varphi - 1) + \frac{3}{\sqrt{2}} Z_F \sin \theta \cos \theta \cos \varphi \right\} \end{aligned}$$

and

$$\begin{aligned} Z_T = & -\frac{f_+ x_t^2 \beta^2 \sin^2 \alpha}{12(1-x_b)(1-x_t)} \\ Z_F = & \frac{\sqrt{2} f_+ x_t^2 \beta^2 \sin \alpha \cos \alpha}{12(1-x_b)(1-x_t)} \\ Z_N = & -\frac{\sqrt{2} x_t \beta \sin \alpha}{12(1-x_b)(1-x_t) f_-} f[f_- + (1-x_t)f_+] \\ Z_U + Z_L = & \frac{1}{12(1-x_b)^2(1-x_t)f_+ f_-} \left\{ (4 + 12f + 8f^2)(1-x_t) \right. \\ & + (-4 - 8f + 4f^2 + 8f^3)(1-x_t)(1-x_b) \\ & + 4(1-f^2)(1-x_t)(1-x_b)^2 + (-2 - 8f - 10f^2 - 4f^3)(1-x_t)^2(1-x_b) \\ & + (-4 + 12f^2 - 8f^3)(1-x_b) + (4 - 12f + 8f^3)(1-x_b)^2 \\ & \left. + (-2 + 6f^2 - 4f^3)(1-x_b)^3 \right\} \\ 2fZ_L - Z_U = & \frac{2f}{3x_b^2 f_-^2 (1-x_b)} \left\{ -f_+(1+2f)(1-x_t) \right. \\ & + (1+2f)(1-f^2)(1-x_t)(1-x_b) + 2f_-(1-f^2)(1-x_b) \\ & \left. - f_-^2(2f+3)(1-x_b)^2 + f_-^2(1-x_b)^3 \right\} \end{aligned}$$

$$\begin{aligned}
\frac{1+2f}{2f}Z_P - Z_U - Z_L = & \frac{1}{3f_-^2x_b(1-x_b)} \{ f_+(1+2f)(1-x_t) \\
& - (1+2f)(1-f^2)(1-x_t)(1-x_b) - f_-(1-2f)(1-x_b) \\
& + f_-^2(1-x_b)^2 \}
\end{aligned}$$

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Table 1: The effect of $O(m_b, \Gamma_W)$ terms on the lowest order result for A_{FB} for various values of m_t and $\frac{a}{v}$.

m_t	$\frac{a}{v}$	$A_{FB}(m_b = \Gamma_W = 0)$	$A_{FB}(m_b = 4.5\text{GeV}, \Gamma_W = 2\text{GeV})$
140	-1	0.296	0.293
160	-1	0.250	0.249
180	-1	0.2124	0.2116
200	-1	0.1818	0.1814
220	-1	0.1569	0.1566
140	-0.4	0.204	0.211
160	-0.4	0.1724	0.1760
180	-0.4	0.1465	0.1486
200	-0.4	0.1254	0.1266
220	-0.4	0.1082	0.1100
m_t	0	0	0
140	+0.2	-0.1140	-0.1190
160	+0.2	-0.0962	-0.0996
180	+0.2	-0.0817	-0.0836
200	+0.2	-0.0699	-0.0708
220	+0.2	-0.0603	-0.0617
140	+0.6	-0.261	-0.266
160	+0.6	-0.221	-0.223
180	+0.6	-0.187	-0.188
200	+0.6	-0.160	-0.162
220	+0.6	-0.138	-0.139
140	+1	-0.296	-0.293
160	+1	-0.250	-0.248
180	+1	-0.212	-0.212
200	+1	-0.182	-0.183
220	+1	-0.157	-0.158

Table 2: Relative magnitude of first order QCD corrections to Γ_{total} ,

A_{FB}^{bl} and A_{FB}^{tl} in units of $-C_F \frac{\alpha_s}{2\pi}$ ($m_W = 80\text{GeV}$):

m_t	D_{total}	D_{bl}	D_{tl}
100	3.09	0.609	-0.176
120	3.53	0.339	-0.368
140	3.65	0.135	-0.524
160	3.73	-0.0822	-0.657
180	3.77	-0.0246	-0.768
200	3.81	-0.385	-0.860
220	3.84	-0.503	-0.937
240	3.87	-0.604	-1.003

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Relative magnitude of
 QCD corrections to Γ_{total} ,
 $A_{\text{FB}}^{\text{bl}}$ and $A_{\text{FB}}^{\text{tl}}$ in units
 of $-C_F \frac{\alpha_s}{2\pi}$

Fig. 6

Relative magnitude of QCD

Corrections to $\frac{d\Gamma}{d \cos \theta_{bl}}$ in units of $-C_F \frac{\alpha_s}{2\pi}$.

Note that for $\cos \theta = -1$ the lowest order vanishes,

$$\left. \frac{d\Gamma}{d \cos \theta} \right|_{\substack{\cos \theta = -1 \\ m_t = 160 \text{ GeV}}} = + \frac{\alpha_s}{2\pi} C_F \frac{G_F m_t^3}{8\sqrt{2} \pi} \quad 0.34$$