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Theoretical definitions of length and charge and second-order electric
fields from steady currents

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Abstract

It is shown in this paper that the well-known definitions of length, the synchronous definition of length (SDL), and the covariant definition of length (CDL), and the corresponding definitions of the macroscopic charge, the synchronous definition of charge (SDC), and the covariant definition of charge (CDC), are inadequate for all physical systems consisting of relatively moving subsystems. Therefore the new definition of length, the alternative synchronous definition of length (ASDL), and of charge, the alternative synchronous definition of charge (ASDC), are introduced, which correctly define the length, volume, and the charge for all physical systems. In ASDL and ASDC the length and the charge for every (moving) subsystem separately are determined simultaneously in the rest frame of the observer. The charge neutrality is consequently redefined in ASDC. The measurable consequence of ASDL and ASDC is the appearance of the second-order electric field outside stationary conductor with steady current. Different experiments which support the ASDL and ASDC are considered.

Another consequence of the ASDL is the appearance of the second-order gravitational fields for physical systems consisting of relatively moving subsystems.

1. Introduction

In this paper different known definitions of length (volume) are examined. The first one is - (i) the usual synchronous definition of length, SDL, in which the length is determined as the spatial distance between two points on the (moving) object as measured by simultaneity in the rest frame of the observer, Sec. 2. The next one is - (ii) the usual covariant definition of length, CDL, in which the length is determined as the spatial distance between two points on the (moving) body as measured by simultaneity in the rest frame of the body, Sec. 3. It is shown in an exact way that the usual SDL does not correctly define the length (volume) for the physical systems consisting of relatively moving subsystems; according to SDL a part of such system contains different numbers of particles, i.e., different matter, in different inertial frames of reference (IFRs). The CDL agrees with SDL in the rest frame of the body, but for the mentioned systems, e.g., a current-carrying conductor (CCC), the rest frame of the ions is not at the same time the rest frame of the electrons. It means that neither CDL is appropriate for the mentioned systems. The new definition - (iii), the alternative synchronous definition of length, ASDL, is introduced in this paper, which correctly defines the length for all systems including CCCs, i.e., the systems consisting of relatively moving subsystems, Sec. 4. Instead of describing, e.g., a wire with current as one system with its length l , we treat it as two separate systems with their separate lengths l_+ and l_- . The lengths of the relatively moving subsystems are defined by ASDL as the spatial distances between two points on each subsystem as measured by simultaneity in the rest frame of the observer separately for each subsystem.

In order to clarify definitions of length and to examine their experimental consequences we employ a simple system, a closed loop with

current. It is explicitly shown using such a CCC that there is the close connection between different definitions of length, volume, and theoretical definitions of charge.

One of the basic laws in electrodynamics (classical and quantum) is the invariance of charge. There is conclusive experimental evidence that the charges of elementary particles and the total charges of bounded physical systems are independent of their motion. We examine here theoretical definitions of the amount of electric charge, the invariance of charge defined by them, the measurable consequences of these definitions, and propose a new general definition, which avoids the difficulties encountered in the known definitions when treating a current-carrying conductor. The well-known definitions of electric charge appearing in the literature are: (e) the usual synchronous definition of charge (SDC), e.g. [1], which is based on the usual SDL, Sec. 2., (f) the usual covariant definition of charge, (CDC), e.g. [2] and [3], which is based on CDL, Sec. 3., and (g) the flux definition of charge (FDC), e.g. [4], which is also based on SDL. (The usual FDC, and the covariant formulation of FDC, will not be considered here, but they will be exposed elsewhere. The invariance of charge defined by the usual FDC is partially discussed, e.g., in [5] and [6].) Recently, (h), an alternative synchronous definition of charge (ASDC) is introduced in [7], but only in an IFR, and for the small piece of matter. ASDC is based on ASDL. In this paper the general form of the ASDC will be presented, Sec. 4., which is valid for the curved space-time too, and for the total charge of an arbitrary, bounded, system. In ASDC the charge neutrality of a CCC is defined in accordance with ASDL, i.e., in a substantially different manner than in the usual SDL and SDC. The measurable consequence of the ASDL and ASDC, and of the new definition of charge neutrality of a CCC, is the

electric field outside **stationary wire with current**, whose first nonvanishing term is $E_{\text{ext.}}^{(2)} \propto v^2/c^2$, (v is the average drift velocity of the electrons, and c is the velocity of light). Different experiments which support the ASDL and the ASDC and their consequence $\vec{E}_{\text{ext.}}^{(2)} \neq 0$ are discussed.

Another consequence of the ASDL is the appearance of the second-order gravitational fields outside physical systems consisting of relatively moving subsystems.

2. The usual synchronous definitions of length and charge

We start the discussion of the different definitions of length and charge with SDL and the SDC, since SDL and SDC are used in every textbook on general physics, and by almost all physicists. The SDL and SDC always (in every reference frame) involve the usual synchronous description. **The length is defined by SDL as the spatial distance between two points on the (moving) object measured by simultaneity in the rest frame of the observer.** Thus the length l , simultaneously determined in some IFR S , between the points A and B on the (moving) object is

$$l^2 = (x_B^i - x_A^i)(x_{Bi} - x_{Ai}), \text{ with } x_B^0 - x_A^0 = 0. \quad (1)$$

In another IFR S' $l'^2 = (x_B'^i - x_A'^i)(x_{Bi}' - x_{Ai}')$, with $x_B'^0 - x_A'^0 = 0$. (The Cartesian spatial coordinates are x^i and time t ($x^0 \equiv ct$). The notation is such that Latin indices run from 1-3, Greek indices run from 0-3, and repeated indices implying summation.) l is taken at some $t=a$ in S , and l' at $t'=b$ in S' ; **t and t' are not related in any way.** Particularly the length l in S between the points A and B on the linear (moving) object, which is along the x -axis, is $l = x_B - x_A$. In S' , which is moving with V relative to S along the common x, x' -axis, $l' = x_B' - x_A'$; the coordinates of the end points $x_{A,B}, x_{A,B}'$ are taken simultaneously in S, S' , respectively. The particles

building that body, i.e., which are on so defined length, are "counted" simultaneously in the rest frame of the observer. If the volume of an extended object is considered then **the boundary of the volume must be determined simultaneously for the given observer.** (In order to avoid different difficulties with "the conventionality of simultaneity", i.e., with "the conventionality of synchronization", we assume that the "standard" or "Einstein" procedure for the clock synchronization is adopted.) If S is the rest frame of the object then its length l is the rest length, while in S' the length l' is contracted relative to l , $l' = l(1 - \vec{B}^2)^{1/2}$, $\vec{B} = \vec{V}/c$.

In the SDC the total charge Q of the physical system, which at the time $t=a$ lies entirely inside a finite region of volume in 3-space, is given in an IFR S with the familiar expression

$$Q = \int_{t=a} \rho dV, \quad (2)$$

where $\rho = \rho_+ + \rho_-$ is the spatial charge density. In another IFR S' $Q' = \int_{t'=b} \rho' dV'$. In a general case of a curved space-time, [1], Sec.10.9.,

$$Q = \int_{t=a} |g|^{1/2} (j^0/c) d^3x, \text{ where } |g| \text{ is the determinant of the metric tensor}$$

$g_{\alpha\beta}$, $j^0 = j_+^0 + j_-^0$ is the time component of the four-current density, which is related to ρ by $j^0 = \rho c (-g_{00})^{-1/2}$. The electric charge δQ of the small piece of matter in an IFR S is $\delta Q = (\rho_+ + \rho_-) dV$, while in S' $\delta Q' = (\rho'_+ + \rho'_-) dV'$. When all charges are treated as discrete point charges the charge Q in SDC is obtained from (2) by "counting" elementary charges whose world lines cross the hypersurface perpendicular to the time axis.

The first important feature of (2) is that - (v) the integral in (2) is $\neq 0$ only in the region of volume in 3-space (i.e. on the hypersurface

$t=a$) in which ρ is $\neq 0$. The second feature of SDC, which is responsible for many errors in the literature, is that - (vi) the SDC is based on SDL and thus involves the usual synchronous description. Consequently in S the integral in (2) is taken over the hypersurface $t=\text{const.}$, while in S' over $t'=\text{const.}$; again t and t' are not related in any way. Then in (2) dV' , (taken at $t'=\text{const.}$ in some improper IFR S'), is the Lorentz contraction (LC) of the proper spatial volume dV , (taken at $t=\text{const.}$ in the proper IFR S), $dV' = dV(1 - \vec{B}^2)^{1/2}$, (only one dimension suffers a Lorentz contraction), but it is not the Lorentz transformation (LT) of dV . In many well known textbooks on electrodynamics, e.g., [1] Sec. 10.9, [2] Secs. 28 and 90, [8] Sec. 11.9, [9] Sec. 18-1, then The Feynman lectures on physics, Vol.2, Sec.13-6, etc., this fact is not properly understood and it is argued that $\delta Q = \rho dV$ in IFRs, (or $\delta Q = \rho \kappa^{1/2} d^3x$ in curvilinear coordinates, where κ is the determinant of the spatial metric tensor κ_{ij} , $dl^2 = \kappa_{ij} dx^i dx^j$), is an invariant, a Lorentz scalar (for IFRs). This invariance of δQ is usually concluded from the invariance of $d^4x = dx^0 dx^1 dx^2 dx^3$, $d^4x' = d^4x$ for IFRs, [2], [8], [9]. However, d^4x' is obtained by the LT of d^4x , which is not the case with dV in (2); in fact δQ , and Q defined by (2), do not transform as tensor quantities on LT. It means that nothing can be said about the invariance of $\delta Q = \rho dV$, (in IFRs), from the invariance of d^4x , contrary to the assertions in the above mentioned literature. Therefore all the proofs of the invariance of the total charge of bounded systems, which are based on the "invariance" of δQ , do not hold.

Similarly in many textbooks, see e.g. [2], [8], the transformation law for the four-current density is derived from the "invariance" of $\delta Q = \rho dV$. But, as it is said, δQ is not in general an invariant. The correct proofs that $(c\rho, \vec{j})$ forms a legitimate four-vector (in IFRs) are given in [10] and in the Appendix 2 of [1].

The third important characteristics of SDC is that - (vii) j^0 , which is the sum $j_+^0 + j_-^0$, is multiplied for both charged subsystems, (they can be, in general, in different states of motion), by only one element of volume in 3-space. Thus, in the case of a wire with current it is, for every observer, the simultaneously determined volume of the "wire", i.e., of the lattice of ions, which enters into the SDL and on account of that into the SDC too.

To illustrate these features we consider a simple example of a rectangular loop **with current** in two IFRs S and S' . The loop is at rest in the IFR S , (the laboratory frame). The S' frame moves with velocity $\vec{V}$ relative to S along the common x, x' -axis in the $+x$ direction. (The generalization to an accelerated frame of reference, (AFR), can be done by means of the generalized Lorentz transformation (GLT), developed in [11], which connects an IFR with AFR. This is given elsewhere, [12]; the ring with current is considered and the origin of the AFR is supposed to move together with an electron along the ring, but the axes of AFR have no ordinary spatial rotation.) Without loss of generality a wire of negligible resistance is considered, and the charge distribution arising from the pinch effect is neglected, and the electron accelerations at the corners of the loop are omitted. For conductors without current, (e.g. for the considered loop before a current is established), and for nonconducting materials, both charged subsystems are at rest relative to each other. Therefore in the SDL there is only one length scale (one volume) common for both subsystems. It holds for every reference frame. Suppose that in S the simultaneously determined element of length dl_0 (on the leg parallel to $\vec{V}$) of the part of a linear wire **without** current, between two points A and B marked on the wire, contains n stationary ions

labeled from l_i to n_i , and n stationary electrons l_e to n_e . The total numbers of the ions, electrons, for the whole closed loop are N_i , N_e , respectively, and the total charge Q_0 is zero, $Q_0=0$. The point A is on the ion l_i and the point B on the ion n_i , and they determine the ends of the length dl_0 of the wire; $dl_0 = x_B(t) - x_A(t) = x_{n_i}(t) - x_{l_i}(t)$, according to SDL, (1). The number density of the ions in S is $n_{+,0} = n/dl_0$, and the same holds for the electrons $n_{-,0} = n_{+,0}$. Then the SDC, (2), gives that in S $\delta Q_0 = (\lambda_{+,0} + \lambda_{-,0})dl_0 = 0$, where $\lambda_{+,0} = en_{+,0} = e(n/dl_0)$, and $\lambda_{-,0} = -e(n/dl_0)$, are the positive, negative, charge per unit length. The ions l_i to n_i and the electrons l_e to n_e constitute that element of length dl_0 . At any t in S dl_0 contains these l_i to n_i ions and l_e to n_e electrons. In Fig. 1. n world-lines of ions and n world-lines of the electrons cross the hypersurface $t=0$ in S between points A and B. Let us consider what will happen with that dl_0 in IFR S'. The method developed in [10], which uses only the relativity of simultaneity, the invariance of an elementary charge e , and/or the invariance of the masses of particles, will be applied. At the time $t=0$, measured by stationary clocks in S, the electron l_e is supposed to coincide with the ion l_i , the event A, and the electron n_e with the ion n_i , the event B. The coordinates of these two simultaneous events in S are set to be $A(x=0, t=0)$ and $B(x=dl_0, t=0)$ respectively. In S' these two events are not simultaneous and their coordinates are $A(x'=0, t'=0)$ and $B(x'=\Gamma dl_0, t'=-\Gamma(V/c^2)dl_0)$, respectively, where $\Gamma=(1-B^2)^{-1/2}$; see Fig.1., where the distance $\overline{AP} = ct'$, and the distance $\overline{AE} = x'$, of the event B. However the SDL is not interested in the same events in different IFRs. Namely according to the SDL the ends of length have to be determined simultaneously in every IFR. It means that in S' one has to find the position of the ion n_i and of the electron n_e at, e.g., $t'=0$, when the ion l_i and the electron l_e are at $x'=0$. These are the points in which the

world-lines of the ion n_i and the electron n_e cross the hypersurface $t'=0$ in S' , i.e., the point B(in S') in Fig.1. Since the ion n_i coincides with the electron n_e at $t' = -\Gamma(V/c^2)dl_0$ at the position $x' = \Gamma dl_0$, it will be at the moment $t'=0$ at the position $x'_{n_i}(t'=0) = \Gamma dl_0 - \Gamma(V/c^2)dl_0 V = dl_0/\Gamma$, (point B(in S') in Fig. 1.). Thus there are $n'_{+,0} = n$ ions on the length $dl'_0 = dl_0/\Gamma$ in S' , and the number density of the ions in S' is enhanced $n'_{+,0} = n/(dl_0/\Gamma) = \Gamma n_{+,0}$. In S' the electrons 1_e to n_e move together with the ions with velocity $v' = -V$ and the position of the electron n_e at $t'=0$ will be $x'_{n_e}(t'=0) = dl_0/\Gamma$, i.e., the same as the position of the ion n_i . The number density of the electrons in S' is also enhanced $n'_{-,0} = n'_{+,0}$, and the number of electrons $n'_{-,0}$ on dl'_0 is again $= n$. Thus we have found that the simultaneously determined element of length dl'_0 between the points A and B(in S') on the "wire", i.e., on the lattice of ions, measured by apparatus stationary in S' , $dl'_0 = x'_{n_i}(t') - x'_{1_i}(t')$, is contracted relative to dl_0 , $dl'_0 = dl_0/\Gamma$. It contains just the same particles 1_i to n_i and 1_e to n_e as dl_0 in S contained, as seen from Fig. 1. This shows that the length defined by SDL for such a **wire without current**, (and also for nonconducting materials), has clear and unambiguous meaning. Notice that the above derivation deals only with the number of particles on dl_0 and that number is found to be unchanged by uniform motion of the length.

Since the SDC is based on SDL the similar conclusions will hold for the charges on dl_0 . Indeed, the number of particles is unchanged, and an elementary charge e is an invariant, which causes that in S' , $\delta Q'_0 = ne - ne = 0$, or it can be written as $\delta Q'_0 = (\lambda'_{+,0} + \lambda'_{-,0})dl'_0$, and it is $=(en'_{+,0} - en'_{-,0})dl'_0 = \delta Q_0 = 0$. Also Q'_0 for the whole loop is $Q'_0 = Q_0 = 0$; **the wire without current is again locally and globally charge neutral.**

From the fact that number of particles on dl_0 is unchanged, and that

the masses of particles are invariants, one concludes that the total mass of the element of length dl_0 is unchanged by its uniform motion. In S that mass on dl_0 is $\delta m_0 = (n_{+,0} m_i + n_{-,0} m_e) dl_0$, and it is $= n m_i + n m_e$. In S' $\delta m'_0 = (n'_{+,0} m_i + n'_{-,0} m_e) dl'_0 = \delta m_0$. To avoid the misunderstanding of this statement we point out that, contrary to the assertions in many textbooks including [1], [4], [8], [9], etc., there is not correctly defined velocity dependent mass. The only properly defined quantity is the rest mass of an elementary particle. Thus like an elementary charge e the mass of an electron m_e is an invariant, and we assume that the same holds for m_i of the ion.

The problem with SDL, and with SDC, arises for a CCC; the two charged subsystems are as a whole in a relative motion, which means that the "object" and its length (volume) are not clearly defined. In fact there are two "objects", and two lengths, (volumes), one for each charged subsystem. Nevertheless the SDL and the SDC again utilize only one length scale, that one of the lattice of ions, which causes that the length of the wire with current is incorrectly defined, and consequently that δQ and δm for a section of a CCC will be changed by motion of that wire. Let us show it. Since a steady current can be set up in the circuit without a net transfer of the electrons to or from the system, the total charge Q of that loop, but with a steady current, must be always zero in every reference frame and for every theoretical definition of the total charge, $Q, Q', \dots = Q_0 = 0$. In SDC, (2), Q in S for the loop with current is $Q = \int_{t=a} (\lambda_+ + \lambda_-) dl_+$, and it must be zero $Q=0$. dl_+ will be denoted as dl , and since the lattice is rigid and at rest $dl=dl_0$; it again contains n ions l_i to n_i between stationary points A and B on the wire at any t in S , $n_+ = n_{+,0}$, and $n_+ = n$, see Fig.2.. Therefore $\delta Q_+ = en$, or it is $= \lambda_+ dl = \delta Q_{+,0}$, where

$\lambda_+ = en_+ = e(n/dl) = \lambda_{+,0}$. The global charge neutrality of the stationary loop with steady current and the fact that SDL and SDC use only one length dl for two "objects", the moving electrons and the stationary ions, imply that the number of moving electrons on dl between points A and B is again equal n . In general these n electrons between A and B are not the electrons 1_e to n_e existing between A and B in that wire without current. However, since all the electrons are equivalent, (by quantum mechanics), and since their number remained unchanged, one can accept that the part of the wire between A and B, (and the whole wire too), is not changed by motion of the electrons; remember that both e and m_e , the mass of the electron, are invariants. Then the number density of the electrons in S is $n_- = n_{-,0}$, and the number of the electrons on dl in S is $n_- = n$, see Fig.2.. The negative charge on dl in S is $\delta Q_- = -en$, or it is $= \lambda_- dl = \delta Q_{-,0}$, where $\lambda_- = -en_- = -e(n/dl) = \lambda_{-,0}$. Thus the stationary CCC is again locally charge neutral $\delta Q = 0$, i.e., $\delta Q_- = -\delta Q_+$, or $\lambda_- = -\lambda_+ (= -\lambda_{+,0})$. The fact that δQ on dl of a part of a CCC is zero is taken as a "natural" assumption in almost all textbooks and papers on electrodynamics. The preceding analysis shows that $\delta Q = 0$ is, in fact, the consequence of a more fundamental assumption about the existence of only one length scale for a CCC in SDL and SDC. Since $\delta Q = 0$, (and $\lambda = 0$), the external electric field for the stationary loop with current is zero, $\vec{E}_{ext.} = 0$; this is known in literature as the Clausius hypothesis, see [13].

The mass of the part dl of the wire with current in S is $\delta m = (n_+ m_i + n_- m_e) dl$, and it is $= n m_i + n m_e = \delta m_0$, i.e., the same as the mass of that part of the wire when there is no current.

By the method from [10] one can determine the length, the charges and the mass for the moving loop in S'. Let us consider the part of the leg

(parallel to $\vec{V}$) in which the electrons in the S frame move with the average drift velocity $\vec{v}$ to the right along the x axis. This part of the CCC whose length is dl in S, (from (2) $dl = x_{n_i}(t) - x_{l_i}(t)$), contains n stationary ions and, according to the above discussion, n moving electrons. Suppose that in S at $t=0$ the ion l_i (point A on the wire), coincides with the electron l_e , and the ion n_i (point B on the wire) coincides with the electron n_e , see Fig. 2. In the same way as for the wire without current one finds that in S' the length dl' is contracted $dl' = dl/\Gamma$. (The length $dl'_+ (= dl')$ in S' is the distance between A and B (in S'), taken at $t'=0$ in Fig. 2., and measured by apparatus stationary in S' , $dl'_+ = x'_{n_i}(t') - x'_{l_i}(t')$.) There are the same n ions l_i to n_i on dl' in S' as were on dl in S, but now they are moving with $-V$. Thus n'_+ on dl' in S' is $= n_+$ on dl in S, and it is $= n$. The number density of the ions in S' is enhanced $n'_+ = n/dl' = \Gamma n_{+,0}$. This result does not depend on the chosen moment of time t' in S' .

The electrons in that leg move in S' with velocity $v' = (v - V)/(1 - vV/c^2)$ (to the right for $v > V$). The position of the electron n_e at $t'=0$ is $x'_{n_e}(t'=0) = \Gamma dl + \Gamma(V/c^2) dl \cdot v'$, and it is $= dl/\Gamma(1 - vV/c^2)$, (the point E in Fig. 2.). It means that in S' n electrons, which built together with the ions l_i to n_i the element of length dl between A and B in S, are not more on the (simultaneously determined) contracted length of the "wire" dl' , (i.e., of the lattice of ions), between points A and B (in S'). In S' these n electrons are on another length of the "wire" $dl'_- = dl/\Gamma(1 - \vec{v}\vec{V}/c^2)$, (for legs parallel to $\vec{V}$). In Fig. 2. this is the distance between points A and E measured simultaneously in S' , (at $t'=0$ in Fig. 2.). Hence the number density of the electrons in S' is $n'_- = n/dl'_- = (n/dl')(1 - \vec{v}\vec{V}/c^2)$, which obviously is not equal to n'_+ . The lack of the electrons appears on the

length dl' of the leg for which $\vec{v}$ and $\vec{V}$ are parallel, while there is an excess of the electrons on dl' of the leg for which $\vec{v}$ and $\vec{V}$ are in the opposite directions. Thus, the length dl containing $n_- = n$ moving electrons, and $n_+ = n$ stationary ions in S , becomes $dl' = dl/\Gamma$ in S' , which contains n'_- electrons and n'_+ ions, where

$$n'_- = n'_+ dl' = (n/dl) dl' = n(1 - \vec{v}\vec{V}/c^2), \text{ and } n'_+ = n. \quad (3)$$

The fact that n'_- is not equal to n can be also inferred from Fig. 2. "counting" the number of world-lines of the electrons and the ions which cross the hypersurfaces $t=0$ in S between points A and B , and the hypersurface $t'=0$ in S' between points A and B (in S'). We see that **the element of length of a wire with current, between two points on the wire, does not contain the same number of particles, i.e., the same matter, when measured by different observers, and consequently it is not the same object.** Therefore in SDL the length of a part of a wire with current is incorrectly defined quantity. This is a fundamental result, not explicitly achieved earlier, which will imply many consequences in almost all branches of physics.

However, despite of it, the total length L' of a moving closed loop with current will contain the same total number of electrons N_e as L in S contained, i.e., as L_0 in the same stationary loop without current contained. When going along the moving closed loop with current (at some t' in S') the usually defined lengths, (SDL), of some parts of a closed CCC will contain an excess of the electrons, while the lack of the electrons will appear at the other parts. The sum of such "lengths" of all parts gives that N'_e and N'_i for the whole length L' of the moving wire are unchanged by motion of the loop, $N'_i = N_i$, $N'_e = N_e$. Hence one could say that

the total length of a closed loop with current has a definite physical sense. But it is not so since the "correctness" of L results from the summation of the incorrectly defined quantities. Furthermore in many cases one is interested not in the whole length but only in the length of a part of a CCC, and such a length has no definite physical interpretation. This consideration explicitly shows that the usual SDL is meaningless not only for a CCC but for all physical systems consisting of relatively moving subsystems. Namely the obtained result (3) depend only on the existence of a relative motion of subsystems in a given IFR and not on the particular nature of these subsystems.

Since SDC is based on SDL one expects the similar ambiguities for the amounts of the macroscopic charges and also for the charge neutrality of a CCC. As shown above there are n ions on dl' in S' . Hence $\delta Q'_+ = ne$, or it is $= \lambda'_+ dl' = \delta Q_+$, where $\lambda'_+ = en'_+ = e(n/dl') = \Gamma \lambda_+$. The positive charge on dl is thus unchanged by motion, but it is not a Lorentz scalar. Obviously, the total positive charge for the whole current loop will also be unchanged by uniform motion of that loop, $Q'_+ = \int_{t'=b} \lambda'_+ dl' = Q_+ = Q_{+,0}$. However, as proved above, in S' n electrons in the legs parallel to $\vec{V}$ are not on the contracted length dl' of the "wire" but on the length $dl'_- = dl / \Gamma(1 - \vec{V}\vec{V}/c^2)$. Hence $\lambda'_- = -en'_- = -e(n/dl'_-)$, or written explicitly $\lambda'_- = \Gamma \lambda_- - \Gamma \lambda_- \vec{V}\vec{V}/c^2$. (This procedure represents an unusual but exact way of the derivation of the transformation law for the time component of the four-current density, which is already exposed in [10], Eq.(7).) Then according to (3) and the property (vii) of SDC the negative and positive charge on dl' of the "wire" are

$$\delta Q'_- = \lambda'_- dl'_+ = -e(n/dl'_-) dl'_+ = \delta Q_- + \delta Q_c, \text{ and } \delta Q'_+ = \delta Q_+, \quad (3a)$$

where $\delta Q_c = -e\Gamma(\vec{V}\vec{J}/c^2)(dl/\Gamma)$ is the change in δQ_- caused by the existence of the conduction current in the proper frame for the ions, (see [7], [10], and [1] Sec.7.5.). Thus δQ_- is changed by motion of the loop. Eq.(3a) once again reveals that in SDL and SDC the electrons and ions are not treated in a symmetrical way.

We now turn our attention to the charge neutrality of a CCC. In SDC the local charge neutrality in the rest frame of the "wire" is defined by the requirement that the sum of positive and negative charges on the length dl is zero, $\delta Q = (\lambda_+ + \lambda_-)dl = 0$. But dl is not correctly defined for a CCC. Therefore one expects that this usual definition of charge neutrality in SDC is incorrect as well. Really in S' the length of that part of the wire is dl' according to SDL. From (3a) one obtains that the total charge $\delta Q'$ on dl' is different from zero, $\delta Q' = \delta Q'_+ + \delta Q'_- = \delta Q_c$. In a simple case of a rectangular loop with current δQ_c of different signs will appear in the opposite legs which are parallel to $\vec{V}$. This is in agreement with previously discussed features of the length, defined by SDL, of a part of a CCC. Since $\delta Q' \neq 0$ the moving wire with current is locally charged and $\mathbf{E}_{ext.}^{(2)}$ appears. Obviously this result for the dependence of the charge δQ of a part of a CCC on the observer is the direct consequence of Eq.(3), i.e., of the inadequacy of SDL for physical systems consisting of relatively moving subsystems.

However, one concludes from (3a), and from the discussion of the length of the whole closed loop with current, that the integration of $\delta Q'_-$ over any closed loop yields that the total negative charge Q'_- will be unchanged, $Q'_- = \int_{t'=b} \lambda'_- dl' = Q_- = -Q_{+,0}$. As a consequence the total charge of any closed loop is also unchanged, $Q' = Q'_+ + Q'_- = Q_0 = 0$. This is an explicit proof that Q in SDC does not change by uniform motion of the loop. At the

same time this procedure reveals that neither δQ , the charge on the length dl of the wire with current, nor Q , the total charge on the length L of the closed loop with current, are properly defined quantities. This follows from the fact that the length dl and L of the wire are incorrectly determined for a CCC.

From Eq. (3) one also concludes that the length dl contains different mass of the moving electrons in S and S' , while the mass of the ions on dl is unchanged. Really if $\delta m_e = n m_e$, $\delta m_i = n m_i$, are the masses of the electrons, ions, on dl in S , and $\delta m'_e = n' m_e$, $\delta m'_i = n m_i$, are their masses on dl' in S' , then from (3) follows

$$\delta m'_e = \delta m_e (1 - \vec{v}\vec{v}/c^2), \text{ and } \delta m'_i = \delta m_i, \quad (3b)$$

whence the mass of the element of length dl' in S' is $\delta m' = \delta m - \delta m_e \vec{v}\vec{v}/c^2$.

3. The usual covariant definitions of length and charge

Next we discuss the usual covariant definition of length and charge. Both the length and the charge of a macroscopic body are **nonlocal** physical quantities connected with an extended body. The manifestly covariant formulation of any physical quantity requires that - (r) **the considered quantity transforms as a tensor quantity**. In order that a **nonlocal** physical quantity connected with an extended body transforms as a tensor **the same set of events in space-time**, (or at least the same set of boundary events), **must be considered in every reference frame**. But the question is which reference frame, with the set of (boundary) events in that frame, has to be taken as the relevant one? The answer is that - (s) **it is the rest frame of the body, and the boundary events have to be taken simultaneously in that frame**. The reason will be explained below. Accordingly the covariant definition of a **nonlocal** physical quantity

involves the usual description by simultaneity only in the rest frame of the object. The relativity of simultaneity causes that the events which are simultaneous in the rest frame of the object, (as required by (s)), are not simultaneous in any other reference frame, and the description is necessarily asynchronous in all other IFRs. (These general features of the covariant definitions of **nonlocal** physical quantities are partially exposed only for the CDL, e.g., [14], but not for the CDC.)

Let us now express the CDL in mathematical terms (we restrict ourselves to IFRs). The distance four-vector in some IFR S' l'^{μ} between two spatial points A and B on the (moving) object is the difference between two position four-vectors $x'_{A,B}{}^{\mu}$, $l'^{\mu} = x'_{B}{}^{\mu} - x'_{A}{}^{\mu}$. Then, by virtue of (r), the invariant length, (the Lorentz scalar), of the distance four-vector is constructed in the form $l' = (l'^{\mu} l'_{\mu})^{1/2}$. This l' is equal to its rest length $l = (l^i l_i)^{1/2}$, measured by simultaneity in S , the rest frame of the object, (the time component of the distance four-vector is zero in S , $x_B^0 - x_A^0 = l^0 = 0$). Thus according to the CDL the length between two spatial points A and B on the (moving) object is

$$l'^2 = (x'_{B}{}^{\mu} - x'_{A}{}^{\mu})(x'_{B\mu} - x'_{A\mu}) = l^2 = (x_B^i - x_A^i)(x_{Bi} - x_{Ai}), \quad (4)$$

with $x_B^0 - x_A^0 = 0$ in the rest frame of the object. For a linear object along the common x, x' -axis, $l'^2 = (x'_B - x'_A)^2 - c^2(t'_B - t'_A)^2$, and it is $= l^2 = (x_B - x_A)^2$. In general the position four-vectors $x'_{A,B}{}^{\mu}$ and the components of l'^{μ} in some improper IFR S' are obtained by the LT from the components in the rest frame of the body S , $l'^{\mu} = (\Gamma l_x, \Gamma l_y, \Gamma l_z, (\Gamma V/c) l_x)$, where $l_x = x_B - x_A$, etc.. The time component of l'^{μ} is $\neq 0$, which means that the space distance between spatial points A and B is taken in S' for pair of nonsimultaneous events. Hence the length l' in S' is $l' = (\Gamma^2 l_x^2 + l_y^2 + l_z^2 - \Gamma^2 l_x^2 V^2/c^2)^{1/2}$, and it is equal to the rest length l of the body, $l = (l_x^2 + l_y^2 + l_z^2)^{1/2}$. Comparing with (1) one

concludes that CDL agrees with SDL only in the rest frame of the object. Thus the length of the (moving) body as a nonlocal physical quantity is defined in the CDL as the spatial distance between two points on the (moving) body, as measured by simultaneity in the rest frame of the body, (the same holds for the volume within the spatial boundaries), see e.g. [14]. All this is illustrated in Fig.3. The "end" events $\mathcal{A}$ and $\mathcal{B}$ are taken simultaneously in S , the rest frame of the wire with current, and their coordinates are $\mathcal{A}(x=0, t=0)$ and $\mathcal{B}(x=dl, t=0)$. The length of that part of the wire in S is $dl = x_B - x_A$. The observers in other reference frame look at the same events $\mathcal{A}$ and $\mathcal{B}$. The coordinates of these events in S' are $\mathcal{A}(x'=0, t'=0)$ and $\mathcal{B}(x'=\Gamma dl, t' = -\Gamma(V/c^2)dl)$. The time component of $\mathcal{B}$ in S' is different from zero, $t' \neq 0$. The asynchronously determined length of that part of the wire in S' is $dl' = \Gamma dl(1 - V^2/c^2)^{1/2}$, and it is $= dl$.

In difference to the SDC the CDC is based on the covariant definition of length, or the volume of the (moving) object. The usual covariant definition of charge, see e.g. [2], Secs. 28. and 29., and [3], is

$$Q_{\delta H} = (1/c) \int_H d\sigma_\mu j^\mu, \quad (5)$$

(written for IFRs). $Q_{\delta H}$ is the charge within a boundary δH of an arbitrary hypersurface H , and it denotes the sum of all elementary charges whose world-lines cross the hypersurface H over which the integration is performed. In another IFR S' $Q'_{\delta H} = (1/c) \int_{H'} d\sigma'_\mu j'^\mu$. The hypersurface H can be

deformed and thereby in S' another hypersurface H' can be chosen, but the boundary δH must be always kept fixed; it is the same for every reference frame. The same number of world lines of elementary charges crosses the hypersurface H' in S' and the hypersurface H in S if δH is the same, i.e.,

if the boundary events are always the same. (This feature of CDC is not properly understood in my paper [5].) Fig.3. nicely illustrates that for the same boundary events $\mathcal{A}$ and $\mathcal{B}$ the same number of world-lines of ions and electrons crosses the hypersurface $t=0$ in S , and the hypersurface H' in S' . H' can be taken as the line from $\mathcal{A}$ to $\mathcal{B}$ along the x -axis, or, e.g., the line from $\mathcal{A}$ to P along the ct' -axis, and then from P to $\mathcal{B}$ along the line parallel to x' -axis. This demonstrates that the charge defined by CDC is then an invariant charge, a Lorentz scalar, $Q'_{\delta H} = Q_{\delta H}$. It has to be noted that in many textbooks and papers the usual CDC and the usual SDC are not clearly distinguished, and the properties of CDC are not explicitly exposed.

From the general properties (r) and (s) of the covariant definition of nonlocal physical quantity one concludes that it would be more appropriate to write the CDC, (5), in the form which explicitly takes into account the feature (s). Suppose that the rest frame of the object is S , while S' is an improper frame. Then

$$Q'_{\delta H} = (1/c) \int_{H'} d\sigma'_\mu j'^\mu = Q_{\delta H} = (1/c) \int_{t=\text{const.}} j^0 d^3x, \quad (5a)$$

where δH is the spatial boundary of the body taken simultaneously in S . j'^μ and $d\sigma'_\mu$ in (5a) are contravariant and covariant four-vectors respectively in S' , and they are obtained as Lorentz transformations of the corresponding four-vectors $(j^0, 0)$ and $(d\sigma_0 = d^3x, 0)$, from S , the rest frame of the object. Thus the total charge of a collection of elementary particles building a macroscopic (moving) body is defined in CDC as the charge within the spatial boundaries of that body measured simultaneously in the rest frame of the body. It has to be noted that the simple choice

of a hypersurface perpendicular to the time axis in S , (in the last term in (5a)), does not equalize the charge defined by CDC with the charge defined by SDC. In SDC the charges are determined simultaneously in the rest frame of the observer. dV in (2) does not transform as a tensor quantity, and dV' is not the LT of dV , while in CDC the charges are determined simultaneously only in the rest frame of the body, and $d\sigma'_\mu$ are the LT of $d\sigma_\mu$ in S . This is an essential difference which is often misunderstood in the literature. It automatically refutes all the "proofs" of the invariance of charge which utilize the invariance of the continuity equation, and then the CDC, but at the end conclude that the charge defined by (2), (where dV is the LC of the proper spatial volume), is an invariant, e.g. [1], [2], [9], etc..

Thus we see that the length defined by CDL and the charge defined by CDC are invariants, the Lorentz scalars in IFRs. The length is always equal to the rest length of the body, $dl'=dl$, the charge is always equal to the charge in the rest frame of the object, $\delta Q'=\delta Q$, and the same holds for the mass of that length, $\delta m'=\delta m$. It is explicitly shown in [3] that the charge on a section of the wire with current between two simultaneously determined, in the rest frame of the "wire", points A and B is unchanged by motion of the wire, (in contrast to δQ in SDC). The mentioned boundary points A and B on the "wire" are not simultaneous in other IFRs. Hence, in an improper frame S' the parts of the length between points A and B are gathered at different moments of time. The charges and the masses on that length are also **asynchronously** collected in S' . According to the above discussion **the main features of the CDL and the CDC are that they are**, in difference to the SDL and SDC, **asynchronous definitions, and the rest frame of the "object" is a privileged frame**, because the CDL and CDC involve the usual description by simultaneity only

in that frame.

Let us explain why in covariant definitions of **nonlocal** physical quantities the rest frame of the object must be chosen as the relevant one. Suppose that the observer O' in some improper frame S' for the loop with current performs his own measurement of the lengths of the legs and of the charges on the legs of the moving loop. Then, in principle, O' **can obtain any value for the lengths of the legs and for the charges on them** by the appropriate choice of the moments of his time, which are associated with boundary spatial points of the legs. The observers in other reference frames can take his/her data as reference data considering the same boundary events in space-time. Again, the considered quantities, the lengths and the charges, will be invariants, but too much arbitrariness is associated with such a procedure; the choice of the frame S' and the choice of the boundary events in S' . Furthermore, there is no connection with the common, synchronous, definitions of the length and charge of the (moving) object. To avoid the mentioned arbitrariness one is forced to choose the rest frame of the body as the relevant frame, in which the physical quantities are synchronously determined.

If one deals with a nonconducting material, or a conductor without current, then the rest frame of the object is unambiguously defined. However when the physical system consists of two subsystems, which are in a relative motion in a given reference frame, then it is not quite clear what has to be chosen as the object with its associated rest frame in which the nonlocal physical quantities have to be defined in a synchronous way. In the case of a wire with current the usual choice for the rest frame of the "object" is - the rest frame of the "wire", i.e., of the lattice of ions, which means that **in the usual CDL and CDC, again as in SDL and SDC, the ions and the electrons are not symmetrically treated.**

Thus for our loop with current n stationary ions l_i to n_i , which are on the rest length of the "wire" dl in S , will be "counted" in every other IFR, as also seen from Fig.3.. But for the moving electrons, which are contained in dl , the S frame is not the rest frame. Therefore, in principle, one could assume that the length dl between A and B contains r electrons l_e to r_e with $r \neq n$. The charge on dl will be in that case $\delta Q = ne - re \neq 0$. The asynchronously determined length (by CDL) and the charge (by CDC) in any other frame will be again equal to those ones in S , since both requirements (r) and (s) for the covariant definition of a nonlocal quantity are fulfilled. But, as it is said, in the usual CDL and CDC the agreement with the values obtained in SDL and SDC in the rest frame of the "wire" is required, whence $r=n$ is adopted, see Fig.3.. As a consequence, **in CDC the electric field outside stationary wire with current is zero** $\vec{E}_{\text{ext.}} = 0$ as in SDC. However, one can pose the following question: Why in CDL and CDC is assumed that the rest frame of the body for a CCC is the rest frame of the ions and not the rest frame of the electrons? Neither ions nor electrons can **independently** exist as coherent dynamical systems, which means that the choice of the lattice of ions for the "body" has no advantage; the masses of the particles of the charged subsystems cannot play the decisive role in this choice. All this shows that the usual CDL and CDC are also inappropriate for physical systems consisting of relatively moving subsystems. Moreover from the experimental point of view it is not clear **how to measure the asynchronously determined nonlocal physical quantities.**

4. The alternative synchronous definitions of length and charge

The preceding consideration reveals that it would be important to formulate the theoretical definition of length and of charge with the

following properties: (a) it contains the common description by simultaneity in the rest frame of **the observer**, as in SDL and SDC, - (b) it always defines the length of an object in such a way that it contains the same matter, and the charge to be independent of its motion, i.e., independent of the system of coordinates, (c) it treats symmetrically the ions and the electrons. To achieve this goal we first redefine SDL for physical systems consisting of relatively moving (as a whole) subsystems in a given reference frame; it will be called the alternative synchronous definition of length, ASDL. It is shown in previous sections that there is no physical sense in the assertion: The length, defined either by the usual SDL or the usual CDL, of a part of a **wire with current** is 2 cm. Namely, for such systems, (e.g., a CCC, or a charge neutral plasma with a current channel, etc.) there are actually two "objects", not one. Instead of speaking about one system - a wire with current, with its length l , or volume, and the charges δQ_+ , δQ_- , and δQ on that l , we treat it as two systems with their lengths l_+ and l_- , and with the charges on these lengths ΔQ_+ and ΔQ_- respectively. **The lengths, (volumes), of these "objects", i.e., the relatively moving subsystems, are defined by ASDL as the spatial distances between two points on each subsystem measured by simultaneity in the rest frame of the observer separately for each "object".** Thus in an IFR S the two lengths can be written as

$$l_+^2 = (x_{B+}^i - x_{A+}^i)(x_{B+,i} - x_{A+,i}), \quad l_-^2 = (x_{D-}^i - x_{C-}^i)(x_{D-,i} - x_{C-,i}), \quad (6)$$

with $x_{B+}^0 - x_{A+}^0 = x_{D-}^0 - x_{C-}^0 = 0$. In S' the same equation holds with primed quantities replacing the unprimed ones, (the points A, B, C, D, remain unchanged). If a linear object is considered, which is along the common x, x' -axis, then the two lengths become $l_+ = x_{B+}(t) - x_{A+}(t)$, $l_- = x_{D-}(t) - x_{C-}(t)$.

The lengths l_+ and l_- and also the volume elements dV_+ and dV_- (in IFRs), (or $\kappa^{1/2} d^3x_{+,-}$ in curved space-time), are again, as in SDL, synchronously determined, but now they depend on the states of motion of charged subsystems in a given reference frame. They are not equal for physical systems like a CCC, while for nonconducting materials and for conductors without current $l_+=l_-=l$, $dV_+=dV_-=dV$, and the ASDL, (6), agrees with SDL, (1). If one knows the coordinate transformations between the proper reference frames (it can be the AFR as well) for positive (negative) charges and an arbitrary reference frame S' in which the positive (negative) charges perform an ordered motion (but not generally the same motion for "+" and "-" charges), then dV'_+ (dV'_-), (for IFRs), will be Lorentz contracted volumes relative to the rest volumes. They contain the same number of "+", ("-"), charges as their proper volumes contained in their rest frames; it will be explicitly shown below, and also can be seen for a part of the wire with current from Fig.4.. This fact that the lengths, (volumes), defined by ASDL contain always the same matter, in difference to the usual SDL, ensures the validity of ASDL.

The general alternative synchronous definition of charge ASDC, which satisfies the above mentioned requirements (a)-(c), and which is based on ASDL, is

$$Q=Q_++Q_-, \quad Q_{+,-} = \int_{t=a} \rho_{+,-} dV_{+,-}, \quad (7)$$

or in the curved space-time $Q_{+,-} = \int_{t=a} |g|^{1/2} (j_{+,-}^0/c) d^3x_{+,-}$. For a piece of matter the charges ΔQ_+ and ΔQ_- contained in dV_+ and dV_- , and already introduced in [7], are $\Delta Q_{+,-} = \rho_{+,-} dV_{+,-}$. (It is asserted in my paper [7] that the charges $\Delta Q_{+,-}$ are Lorentz scalars, but it is not so, as can be inferred from the discussion in this paper. However, since ASDC is based

on ASDL, these charges are actually independent of their motion; this will be also shown below. But the ASDL is a synchronous definition, and thus not a covariant one, which means that the charges $\Delta Q_{+,-}$ are not Lorentz scalars.) Apparently the ASDC, (7), agrees with SDC, (2), for nonconducting materials and for conductors without current since for them $dV_+ = dV_- = dV$, but the two definitions are essentially different for CCCs. The global charge neutrality in the ADC means that the charge defined by Eq. (7) for bounded system is zero, $Q=0$; the local charge neutrality means that ΔQ_- in dV_- is equal to $-\Delta Q_+$ in dV_+ , and thus that the sum of so determined charges $\Delta Q = \Delta Q_+ + \Delta Q_- = 0$. In the case of a CCC such definition of charge neutrality is substantially different than that one in SDC.

Let us show that the ASDL and ACDC have the required properties (a)-(c) considering a closed loop with current. First we consider the lengths of the lattice of ions in different IFRs. For the positive charges of the whole loop the S frame, in which the "wire" is at rest, is the proper frame. The ASDL requires that in S dl_+ is $= dl_0$ for the same stationary wire **without** current, and of course dl_+ contains n ions l_1 to n_1 as dl_0 contained, $n_+ = n$, and the number density of the positive charges is $n_+ = n/dl_0$. This length dl_0 , (the distance $\overline{AB}$ (in S), in Fig.4. in which $S' = S_e$ and $\Gamma = \gamma$ is adopted), containing n ions in S becomes $dl'_+ = dl'_0 = dl_0/\Gamma$, (the distance $\overline{AB}$ (in S_e) in Fig.4.), when measured simultaneously in S' , see Sec.2.. It again contains n ions. The number density of the ions in S' is enhanced $n'_+ = \Gamma n_+$. Instead of dl_0 one can choose any other length of the lattice of ions in S. Then by the same kind of reasoning as in Sec.2. we obtain that, e.g., the length $dl_1 = \Gamma dl_0$ in S, with $r = n_+ (\Gamma dl_0) = \Gamma n$ ions, becomes contracted in S' $dl'_1 = (\Gamma dl_0)/\Gamma = dl_0$. The contracted length dl'_1 contains r moving ions in S' , $r = n'_+ dl'_1 = \Gamma n$, as the corresponding rest length dl_1 contained in S. From (7), and n_+ , one finds that the positive charge

on dl_+ is $\Delta Q_+ = \lambda_+ dl_+ = e(n/dl_0)dl_0 = \Delta Q_{+,0}$, and also Q_+ of the whole loop is $Q_+ = Q_{+,0}$. Similarly in S' $\Delta Q'_+ = \Delta Q_+$, and also $Q'_+ = Q_+$. The charges ΔQ_+ and Q_+ defined by ASDC are the same as δQ_+ and Q_+ defined by SDC, which is expected since SDL and SDC preferred the ions' subsystem. The mass of the ions on dl_0 in S is $\delta m_+ = (n_+ m_+) dl_0 = n m_+$, and there is the same mass of the ions on the contracted length dl' in S' , $\delta m'_+ = \delta m_+$. Obviously the length, (measured simultaneously in the rest frame of the observer), of the ion subsystem of the wire is correctly defined quantity; it always, for every observer, contains the same number of ions. The positive charge and the mass of the ion subsystem which are on that length are unchanged by motion of the lattice of ions.

Next we examine the lengths of the electron subsystem of the current loop in different IFRs. In the stationary wire **without** current there are also n stationary electrons on dl_0 . According to the requirement (c) one has to treat the electrons in their proper frame S_e , (notice that different S_e frames are associated with different parts of the loop), in the same way as the ions are treated in their proper frame S . It is natural then to suppose that for the same wire with current the simultaneously determined element of length $dl_{-,e}$ in S_e , in which the electrons on $dl_{-,e}$ are at rest, is equal to dl_0 , $dl_{-,e} = dl_0$, and also that $dl_{-,e}$ contains the same number n of the electrons as dl_0 does. Thus $n_{-,e} = n$, and the number density of the electrons in S_e is $n_{-,e} = n/dl_0$. There is no physical argument against such an assumption. Therefore $\lambda_{-,e} = -e(n/dl_0) = -\lambda_{+,0}$, as in the stationary wire without current. The negative charge $\Delta Q_{-,e}$ on $dl_{-,e}$ in S_e is from (7) $\Delta Q_{-,e} = \lambda_{-,e} dl_{-,e} = \Delta Q_{-,0}$. We now apply the method from [10] to find the lengths defined by ASDL, and the charges defined by ASDC, for the negatively charged subsystem of the legs in S , the rest frame of the ions. It is an improper frame for the

electrons since in S they are moving along the loop. First the leg parallel to x axis, in which the electrons in S are moving with v in the $+x$ direction, will be considered. Suppose that at $t_e=0$, measured in S_e , the stationary electron l_e , (the point C in in Fig.4.), coincides with the ion l_i , (moving with $-v$ in S_e), and the stationary electron n_e , (the point D in S_e) in Fig.4.), coincides with the moving ion r_i . The coordinates of these two simultaneous events in S_e are $(x_e=0, t_e=0)$ and $(x_e=dl_0, t_e=0)$, respectively. In S , the ions' rest frame, these events are not simultaneous and their coordinates are $(x=0, t=0)$ and $(x=\gamma dl_0, t=\gamma(v/c^2)dl_0)$, respectively. (At the moment $t=0$ in S the ion r_i will be at the position $x_{r_i}(t=0)=\gamma dl_0$, since the ions are at rest in S . It shows once again that there are r ions on the simultaneously determined element of length γdl_0 of the ion subsystem. Since the number density of the ions in their rest frame S is $n_+ = n/dl_0$, we conclude that r must be equal to γn , $r=\gamma n$, as already obtained above.) Further, at $t=0$ in S the moving electron n_e will be at the position $x_{n_e}(t=0)=\gamma dl_0 - \gamma(v/c^2)dl_0 \cdot v = dl_0/\gamma$, (the point D in S) in Fig.4.), and thus there are n moving electrons on the contracted length $dl_- = dl_0/\gamma$ in S . This result reveals that the rest length of the electrons $dl_{-,e} = dl_0$, which contains n electrons in S_e , becomes contracted when measured simultaneously in S , the improper frame for the electrons. The contracted length $dl_- = dl_0/\gamma$ contains the same number n of the electrons in S as the rest length $dl_{-,e}$ contained in S_e . Hence the number density of the electrons in S is enhanced

$$n_- = n/(dl_0/\gamma) = \gamma(n/dl_0) = \gamma n_{-,e} = \gamma n_+. \quad (8)$$

By the same kind of reasoning one can show that an arbitrary rest length of the electrons, e.g., the length $dl_{1,e} = \gamma dl_0$ in S_e , (the distance $\overline{AE}$ in Fig.4.), with $r = n_{-,e}(\gamma dl_0) = \gamma n$ electrons, becomes contracted in S

$dl_{-,1} = (\gamma dl_0)/\gamma = dl_0$, (the distance $\overline{AB}$ (in S) in Fig.4.). The contracted length $dl_{-,1}$ contains r moving electrons in S , $r = n_- dl_0 = \gamma n$, as the corresponding rest length $dl_{1,e}$ contained in S_e . All these conclusions are independent of the chosen moments of time t in S , or t_e in S_e . From (8) follows that the negative charge density λ_- for moving electrons in S is enhanced $\lambda_- = -en_- = -\gamma\lambda_0$. Then (7) and (8) give that the negative charge on the contracted length of the moving electron subsystem dl_- in S is $\Delta Q_- = \lambda_- dl_- = -e(n/dl_-)dl_- = \Delta Q_{-,0}$. Similarly the mass of the electrons on dl_0 in S_e is $\delta m_{-,e} = (n_{-,e} m_e) dl_0 = n m_e$, and there is the same mass of the electrons on the contracted length dl_- in S , $\delta m_- = \delta m_{-,e}$. Thus both the negative charge and the mass of the electron subsystem on the length defined by ASDL for the electrons are unchanged by motion of that subsystem. This proves that the length of the electron subsystem, (taken separately from that one for the ion subsystem), measured by simultaneity in the rest frame of the observer is correctly defined quantity as well.

Then according to (7) $\Delta Q = \lambda_+ dl_+ + \lambda_- dl_- = 0$, and the stationary wire with current is locally charge neutral (in ΔQ sense).

By the similar procedure applied to the other legs of the loop one obtains that **the whole length L_- of the electron subsystem in that stationary loop is contracted relative to its rest length $L_{-,e} = L_0$** , (i.e., relative to the length of the stationary ions in S), $L_- = L_0/\gamma$. The total negative charge of the moving electrons Q_- is situated on the contracted length $L_- = L_0/\gamma$ in S , but with enhanced charge density λ_- , and it is $Q_- = Q_{-,0}$. Hence the total charge Q , from (7), is $Q = Q_0 = 0$.

The above consideration implies an important result, which is already obtained in [7]. Namely we find that **there are n stationary ions and $r = \gamma n$ moving electrons on the simultaneously determined length dl_0 between stationary points A and B on the "wire", when dl_0 is measured by the**

apparatus stationary in S. It means that the total charge δQ on the simultaneously determined length dl_0 of the "wire" is $\delta Q = e(n_+ + n_-)dl_0 = e(1-\gamma)n$, or $\delta Q = (1-\gamma)\lambda_{+,0}dl_0$, which is Eq.(4) in [7]. If one deals with an infinite wire with current then this charge δQ induces the external electric field

$$\vec{E}_{\text{ext.}} = [(1-\gamma)\lambda_{+,0}/2\pi\epsilon_0 r] \hat{r}, \quad (9)$$

where $\hat{r}$ is the unit vector in r direction. It is Eq.(5) in [7].

Using (8) one concludes that there is the change in the mass of that length dl_0 of the infinite wire with current; $\delta m = (n_+ m_i + n_- m_e)dl_0 = nm_i + \gamma nm_e$. The external gravitational field will differ from that one obtained by the use of SDL in the determination of the length of the wire with current, (where $\delta m = n(m_i + m_e)$). The first nonvanishing term in the difference will be of the second-order in v/c .

For a wire of finite cross section in the form of a ring with current, which is stationary in S, the moving electron subsystem will shrink to the parts of the ring with smaller radius. Such unsymmetrical distribution of charges in the ring can be modeled by an outer ring with length L_0 and with the positive charge of the stationary ions $Q_{+,0}$, and by an inner ring of length L_0/γ and with the negative charge of the moving electrons $Q_- = -Q_{+,0}$, as in [15]. Although the total charge Q is zero such configuration of charges induces $\vec{E}_{\text{ext.}}^{(2)}$ in contrast to the Clausius hypothesis. The potential of N_e moving electrons on the inner ring is evaluated in [15] as that they are stationary. This is allowed since according to Baker [16], (see also [8] Prob. 14.13, and [17]), no radiation is emitted by a system of N_i stationary ions and of N_e equally spaced electrons moving with constant speed v in an arbitrary closed path;

their electric and magnetic fields are time independent, i.e., they are the usual static values. The appearance of $\vec{E}_{\text{ext.}}^{(2)}$ is an essential, measurable, difference relative to the SDC and CDC. Apparently the definition of charge neutrality in the ASDC gives clear and unambiguous meaning to the assertion that a non-zero electric field appears outside the charge neutral (ΔQ sense) wire with current, or in a charge neutral (ΔQ sense) plasma with current channel. Actually there is no need to emphasize that a CCC is charge neutral in ΔQ sense, since other definitions of charge neutrality, the SDC and CDC, are inappropriate for a CCC.

In [18] it is simply assumed that in S_e $\lambda_{-,e} = -\lambda_{+,0}$, as in ASDC. However [18] deals with the usual SDC, and hence with the usual definition of the charge neutrality. Then it can be easily shown that with such an assumption a CCC cannot be charge neutral in the usual sense, i.e., in the SDC, in any frame of reference. Only the introduction of ASDL and ASDC and the consequent redefining of charge neutrality (ΔQ sense) resolves this problem, and correctly describes the appearance of an electric field in a charge neutral plasma with current.

The gravitational field for such a ring with current, when the length is determined by ASDL, will differ from that one which uses the usual SDL in the determination of the length. The first nonvanishing term in the difference will be again $\propto v^2/c^2$.

Both $\vec{E}_{\text{ext.}}^{(2)}$ and the additional second-order gravitational fields for the systems consisting of relatively moving subsystems may be important in many physical interesting cases including high plasma currents and the relativistic cosmology.

5. Experiments

It is interesting that the second-order term in the expression (9) is obtained in the old action-at-a-distance Weber's theory, see [19]. Notice that the recent investigations, (see [19] and references therein), consider Weber's law for the force between two interacting charges as only an approximation valid up to second order in $\dot{r}/c$. The old Weber's, and Riemann's theories predict the existence of $\vec{E}_{\text{ext.}}^{(2)}$ not only for an infinite wire with current but for any **stationary** closed CCC. Edwards et al. in [13] tried to check experimentally the existence of $\vec{E}_{\text{ext.}}^{(2)}$ for current-carrying superconducting coils. In the theoretical part of [13] the authors expanded the usual retarded potentials $\phi(\vec{r},t)$ and $\vec{A}(\vec{r},t)$ in Taylor series to order $1/c^2$ and showed that all second-order terms cancel exactly when the integration around the closed path is performed, (they are perfect differentials), and only the Coulomb term remained. Then, they supposed, as it is usual, both in [13] and [20], that in a charge-neutral circuit $\rho=0$, which led them to the conclusion that $\vec{E}_{\text{ext.}}^{(2)}=0$. Thereby it is argued in [13] that Maxwell's theory predicts that $\vec{E}_{\text{ext.}}^{(2)}=0$ for constant conduction currents in closed circuits. When their derivation is carefully analyzed one concludes that it is not Maxwell's theory which predicts $\vec{E}_{\text{ext.}}^{(2)}=0$ for a closed CCC, but it is the usual definition of charge neutrality of a CCC, i.e., the use of SDC. Really in SDC, as explained before, $\delta Q=0$, whence $\rho=0$ follows. Maxwell's equations describe the evolution of the electromagnetic field if the charge-current distribution is given; for the chosen sources of the fields the equations give the electromagnetic field tensor. Then from $\rho=0$ Maxwell's equations give that $\vec{E}=0$. (See [21], where it is also shown that the objections to the theory with ASDC, which are raised in [22], are groundless.) Of course if instead of the SDC one adopts the ASDC then Maxwell's equations will give that

$\vec{E}_{\text{ext.}}^{(2)} \neq 0$, as in [15]. From the point of view of the present paper, where the close connection between the definition of charge neutrality of a CCC and the definition of length for such systems is revealed, we conclude that the experiments [13] and [20] test, in fact, which of the definitions of length and charge, the SDL, and hence the SDC, or the ASDL and the ASDC are valid. There is also another interpretation of [13] and [20] as the tests of the dependence of an electron's charge on its velocity. However numerous experiments confirm the invariance of an elementary charge e , see [23]. Then, taking the invariance of e for granted we can indeed interpret [13, 20] as the tests of SDL and SDC. As explained in Sec. 2. the global charge neutrality of a stationary CCC, and the use of only one length scale in SDC, lead to δQ on dl is $\delta Q=0$, and consequently $\vec{E}_{\text{ext.}}^{(2)}=0$. Since the CDC agrees with SDC in the rest frame of the wire then the same holds for CDC. In ASDC there are two length scales and $\vec{E}_{\text{ext.}}^{(2)} \neq 0$ is predicted even for a **stationary** closed loop with current. For all definitions the total charge of a current loop is zero and consequently all three definitions predict that there is no signal (potential) in a Faraday-cage configuration, which is in agreement with experiments [20]. The whole theoretical discussion for the potential ϕ (the expression (3) in [20]) involving the parameter κ , (representing "the deviation from conventional electromagnetic theory", i.e., from Maxwell's theory), is irrelevant for our interpretation of the experiments [13, 20]. Simply, for the SDC, (and CDC), the potential ϕ must be always zero, while for the ASDC ϕ has to be $\neq 0$ for non-Faraday cage configurations. The results of the experiments [13,20] were very surprising for the supporters of SDC. **Potentials were observed in all individual runs for non-Faraday cage configurations when type-II superconductors NbTi and Nb were used, but not for type-I superconductor Pb.** The functional dependence of the observed potentials

was $\phi \propto LI^2$, where L is the length of the wire, and I is the current in the coil. The observed potentials showed large magnitude differences and even sign changes. The authors of [20] offered the following explanation for the observed potentials: "...stray charges on Teflon insulators induced a potential difference when conductors moved slightly as the result of magnetic forces on the conductors which changed when the coil current was changed." Accordingly the I^2 dependence of ϕ is explained to be caused by the I^2 dependence of the magnetic force between nearby wire sections. Further it is also supposed that the total effect of adding and canceling of magnetic forces is proportional to the wire length. The differences in magnitude of ϕ and its sign changes are explained by the dependence of the induced electrostatic potential change upon the magnitude (and sign) of the charge isolated on nearby insulators. Although it is asserted in [20] that this mechanism accounts for all of the observed features of the potential, it is not explained why such a mechanism is not effective for a type-I superconductor. Probably it is supposed that the allowed currents in Pb are too small to give the required magnetic fields which can produce a force capable of distorting the coil sufficiently to induce the observed signals.

In our opinion the mentioned mechanism [20] with stray charges accounts for the magnitude differences in the observed potentials and their sign changes, but does not account for the functional dependence $\phi \propto LI^2$ and also for the negative results with Pb wire. Namely we argue that from the proportionality of the magnetic force between adjacent wires with I^2 does not follow that $\phi \propto I^2$. Really, suppose that the force between nearby sections of the wire is proportional to I^2 . Any loop in the coil, except the terminal ones, is surrounded by the two neighboring loops. In these loops the directions of the currents are opposite than in that one

between them. (The wire is wound in bifilar turns with 2 current-opposing loops per turn.) It means that any loop will be repelled by its neighbors. (We note that the final conclusion is not dependent on the manner in which the wire is wound.) In order that Φ is $\propto I^2$ **the change in the positioning of all loops of the coil relative to fixed positions (in a run) of stray charges would need to be proportional to the force between any two adjacent loops**, (notice that the stray charges have accidental positions, accidental magnitudes and signs). Obviously, such a requirement for the proportionality cannot hold for quantities which actually are not directly connected. Let us proceed further taking for the moment that such proportionality exists, so the change in the positioning of all loops is $\propto I^2$. Then the next step for $\Phi \propto I^2$ would be that **the change in the distribution of the charges induced on loops by the electric fields of stray charges must be proportional to the change in the positioning of all loops relative to accidentally situated stray charges, (which also have accidental magnitudes and signs)**. It is indeed very unlikely that any of these steps be fulfilled. Of course, the same steps would need to occur for $\Phi \propto L$. Hence, by the same sort of arguments one can show that, contrary to the assertion in [20], it is quite unreasonable that the total effect of adding and canceling of the forces between adjacent wires can be $\propto L$. Thus, this consideration clearly brings out the untenability of the explanation given in [20] for the functional dependence of Φ .

However, the functional dependence of Φ observed in experiments [13,20] immediately comes about when the ASDL and ASDC are adopted. In [15] it is shown how it works for the ring with current; the sum of the potentials $\Phi = \Phi^+ + \Phi^-$ is obtained to be the product of two terms, one of which is $(\mu_0/4\pi\rho A)I^2L$, while the other term contains the r and ϑ dependence. (In Eq.(6) in [15] there is a misprint; $K(k)$ has to be in the

numerator. Further, the average value of $\langle \phi^+ + \phi^- \rangle$ over the sphere is zero, which is in agreement with null result for Faraday-cage configurations in [20].) Thus the mechanism from [20] can be only responsible for the magnitude differences and sign changes in the observed potentials, since that mechanism with stray charges can be seen as superposed on the more fundamental one predicted, at first, in [7]. It remains only to explain the negative result for Pb wire from the point of view of the theory with ASDC. One reason could be the relatively smaller values of the allowed currents for a type-I superconductor. The other more subtle reason refers to the difference between superconducting wires used in experiments and wires with negligible resistance and homogeneous current density over any section of the wire, which are considered in the theory. The contraction of the length (volume) of the electron subsystem, i.e., the shrinking of the electron subsystem in a ring with current, [15], is supposed to occur in the whole volume of the wire. It means that the current density is assumed to exist over the whole cross-section of the wire. However in a type-I superconductor any current always flows only inside the penetration depth λ . Therefore the shrinking of the electron subsystem can occur only within λ . But then the effect of the shrinking in Pb wires can be expected to be much smaller than for the theoretically considered wire. Thereby the induced potentials probably are not detectable in experiments [20]. From the theoretical point of view in the case of Pb wires one cannot model the ring with current by two rings whose lengths are L and L/γ . On the other hand in type-II superconductors in the mixed state the current flows not just at the surface, as in a type-I superconductor, but throughout the whole body of the metal. The shrinking and the induced potentials will be detectable in the experiments, and the experimentally obtained functional dependence for ϕ will correspond in that case to the theoretical results

with ASDC, as shown in [15].

The experiments which also support the validity of ASDC are some "unexplained electromagnetic experiments", particularly the exploding-wire phenomenon. This is investigated in [24], and discussed in [21] and [22], and will not be repeated here.

Recently I have come across to an interesting phenomenon appearing in rotamak discharge when d.c. currents are induced in a plasma by high frequency fields, [25] and [26]. In both experiments the usual diamagnetic currents and unexpected, and unexplained by the known theories, negative current rings were observed. In [26] it is speculated that "the negative current ring could arise from a continuous flow of plasma from the outer regions of the discharge vessel towards the center." Then this "...flow leads to a $(\vec{v} \times \vec{B})$ e.m.f. which could drive negative current." Such an explanation for the appearance of the negative current rings could be accepted, but the question is how does arise the postulated flow of charges. I speculate that the mentioned flow of plasma from the outer regions of the discharge vessel towards the center is the phenomenon of the same kind as it is the shrinking of the electron subsystem in a stationary ring with current considered in [15]. At this point we use another picture of special relativity, that one developed at length by L. Jánossy in his book [27], and also by J.S. Bell, [28], to better explain the mentioned phenomena; this picture will be called Jánossy-Bell (J-B) picture. (The similar ideas have been utilized in [29].) In the generally accepted picture of special relativity the contraction of length of the meter stick and the time-dilation effect merely arise in the measurement process, while the physical processes and structures underlying clocks and rulers do not actually change. The Lorentz transformations have a passive role simply connecting the data obtained by the observers and their

equipment in different IFRs; LT transforms among equivalent descriptions of the same physical processes. In the J-B picture the physical system changes in an objective way when acceleration has taken place. The contraction and the dilation of coherent dynamical systems really happen as a result of actual changes in physical processes and structures when such systems are moving. The LT has an active interpretation, (Jánossy calls it the Lorentz deformation), connecting the state of an object at rest in an IFR S with the state of the same but moving object again in S ; only one IFR and the equipment in that frame are involved. As Bell, [28], says: "... the laws of physics in any **one** reference frame account for all physical phenomena, including the observations of moving observers." Thus, in accord with J-B picture of special relativity we interpret the shrinking of the electron subsystem in a stationary wire with current, and the above mentioned flow of charges in experiments [26], as the consequences of an actual Lorentz contraction of the moving electron subsystems. In the same way as Jánossy, [27], Ch. V, and Bell, [28], described the contraction of the moving body by physical changes in orbits of the atoms one could describe and explain the shrinking of the electrons in a wire with current, but this will not be done here.

In addition let us mention that the experimental consequences of the appearance of $\vec{E}_{\text{ext.}}^{(2)}$, (but obtained in a different way than in ASDC), are also discussed, e.g., in the anomalous diffusion in plasmas [19], and in the problems of stability of e.m. currents in arcs and plasmas, [30].

Conclusions

The consideration in this paper shows in an exact way that the usual definitions of length and charge are not appropriate for such physical systems as are a CCC, or a current channel in a plasma, etc.. In the SDL

the length of a part of a CCC contains different number of particles for different observers, which means that it is not properly defined. As a consequence the charge δQ of that part of a CCC changes with its motion. In the CDL the length of the object is, in fact, its rest length, but for a CCC the rest frame of the stationary ions is not at the same time the rest frame of the moving electrons. Since the CDL agrees with SDL in the rest frame of the "wire" it again in the same way as SDL does not treat in a symmetrical way the ions and the electrons in a CCC. In the ASDL the lengths of relatively moving charged subsystems in a CCC are determined separately and they always contain the same number of particles. Therefore the charges defined by ASDC, which is based on ASDL, are independent of their motion. The charge neutrality of a CCC is redefined in ASDC. In difference to SDC the ASDC predicts the second-order electric fields outside steady currents already for stationary CCCs. The analysis of the existing experiments shows that they can be explained by ASDL and ASDC.

At the end we emphasize that the description of physical systems consisting of relatively moving subsystems, (not necessarily charged), always require the use of ASDL and not the usual SDL. This will also induce the changes in, e.g., the usually determined gravitational fields for such systems. We believe that these changes in gravitational fields will be very important for astrophysics and cosmology.

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Figure captions

Fig. 1. The wire without current in S , the rest frame of the wire, and in S' , which is moving with V relative to S along the common x, x' -axis, considered in SDL and SDC. Solid vertical lines are world-lines of $n(=6)$ ions, whereas dashed vertical lines are world-lines of n electrons. The length $\overline{AB}(\text{in } S) = dl_0$, taken at $t=0$ in S , contains n stationary ions and n stationary electrons. In S' , at $t'=0$, the contracted length $\overline{AB}(\text{in } S') = dl'_0 = dl_0/\Gamma$ contains n moving ions and n electrons moving with the same velocity V as the ions. Since the length measured by different observer contains the same matter it is correctly defined quantity. The charge δQ_0 and the mass δm_0 on that length are unchanged by motion of the wire.

Fig. 2. The wire with current in the ions' rest frame S and in the rest frame of the electrons $S_e = S'$, considered in SDL and SDC. Solid vertical lines are world-lines of n ions, whereas dashed lines are world-lines of n electrons. The length $\overline{AB}(\text{in } S) = dl = dl_0$ of the "wire", taken at $t=0$ in S , contains n stationary ions and n electrons moving with V to the right. The charge δQ and dl in S is $=\delta Q_0=0$, and the mass $\delta m = \delta m_0$. The contracted length of the "wire" $dl' = dl/\Gamma$, taken at $t'=0$ in S' , contains again n ions, but not n electrons. The n electrons are on the length $\overline{AE} = dl'_-$. Since the length of a part of a wire with current contains different matter for different observers it is incorrectly defined quantity. Both the charge δQ and the mass δm on such a length are changed by motion of the wire.

Fig. 3. The wire with current in S and S' . In the covariant formulation the same end events A and B are considered by all observers. The length of that part of the wire is an invariant in CDL, and it is equal to the length dl between the spatial end points taken simultaneously in S , the rest frame of the ions, $dl'=dl$. In S' the events A and B are not simultaneous and the length dl' is determined asynchronously. The charge δQ defined by CDC, and the mass δm on that length of the wire are also invariants, and they are always equal to the charge and the mass determined simultaneously in S , the rest frame of the ions. In S' both the charge and the mass are collected asynchronously.

Fig. 4. The wire with current in S and in the electrons' rest frame S_e considered in ASDL and ASDC. The simultaneously determined ($t=0$) length $\overline{AB}(\text{in } S)=dl_0$ of the stationary ion subsystem in S contains n stationary ions. The simultaneously determined ($t_e=0$), contracted length $\overline{AB}(\text{in } S_e)=dl_0/\gamma$ of the moving ion subsystem in S_e contains again n ions. The simultaneously determined ($t_e=0$) length $\overline{CD}(\text{in } S_e)=dl_0$ of the stationary electron subsystem in S_e contains n stationary electrons. The simultaneously determined ($t=0$) contracted length $\overline{AD}(\text{in } S)=dl_0/\gamma$ of the moving electron subsystem in S contains again n electrons. The charges and the masses on the length $\overline{AB}$ for the ions and $\overline{CD}$ for the electrons are unchanged by motion of these subsystems. Particularly on the length $\overline{AB}(\text{in } S)=dl_0$ in S there are n stationary ions and $r=\gamma n$ moving electrons.