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Dissipative 2D electron on a torus in external magnetic field: transport properties and magnetic translations ¹

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Abstract

The effect of dissipation on the electron ground state wave function on a torus in the presence of an external transverse magnetic field is analyzed on large time scales ($t > \frac{1}{\eta}$). Its extension to the multiparticle center of mass wavefunction is also given. The novel transport properties of the system are then studied by applying Laughlin gauge argument with the use of the magnetic translation operators.

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1. Introduction

In a recent paper (ref.[1], hereafter to be referred to as [I]) we proposed a way of studying the transition region between plateaus for a Quantum Hall Fluid by analyzing, the effect of a dissipative term $\eta\dot{q}$ (η being the viscosity constant and $\dot{q}$ the velocity) on the motion of a 2d electron in the presence of an external transverse magnetic field $\vec{B}$. As a result dissipation provides the phenomenon of the shrinking of the Gaussian describing the Lowest Landau Level (LLL) state of the electron and a corresponding change in the transport properties (on large time scales, $t \gg 1/\eta$. In particular the longitudinal conductance is different than zero: such a phenomenon is characteristic of the transition region between two generic plateaus ([2]).

In this paper our approach is applied to the case of a non-trivial 2D geometry, the torus; then the properties of the magnetic translations group and their representations are analyzed, when the dissipation is present, so to understand better the nature of the phenomenon of the shrinking of the Gaussian and the relationship between the electron charge and the magnetic flux associated with it.

Further the multiparticle case is presented and the compressibility property of the system is also discussed.

The paper is organized as follows:

in section 2 we present a gauge-independent formalism in terms of the coherent variable ξ ; then the evolution on large time scales of a single LLL state in the asymmetric gauge is given.

In section 3 we construct the finite magnetic translation operators and study a finite dimensional representation of their algebra for the one electron case.

In section 4 we briefly discuss the multiparticle generalization of the center of mass wave function on the torus for the $N_e/N_s = 1/m$ case and the Hall conductance, σ_H , and the longitudinal one, σ_L , are derived by using Laughlin argument with the help of the finite magnetic translations for the N_e particle system (see ref.[3]).

In section 5 we realize that the ground state is not anymore stable and we loose the incompressibility of the Hall fluid as one would suspect; some comments on the role of dissipation on the position of Fermi surface are also given.

2. Asymptotic LLL states in the Landau gauge

In this section we review some results, given on the Corbino disk in [I], and extend them to the infinite cylinder case.

2a. The $\eta = 0$ case

In [I] it was found that the effect of dissipation on large time scales on the LLL states of an electron, resulting in a “squeezing” of the gaussian, can be described in a simple way by multiplying the $z(=x+iy)$ “coordinate” by a factor $\lambda = (\omega - i\eta)/\omega$ (ω being the cyclotron frequency), that is:

$$z \rightarrow \frac{\omega - i\eta}{\omega} z \quad (1)$$

which is a transformation composite of a dilatation and a rotation. Then all the asymptotic LLL wave functions and the asymptotic operators projected on the LLL can be obtained in a straightforward way by applying the above transformation to the variable z appearing in the unperturbed $\eta = 0$ corresponding quantities.

We are now going to show that the effect of dissipation on large time scales in an arbitrary gauge is still described by eq.(1) after replacing z with the coherent state coordinate ξ .

Let us define the coherent state variable ξ starting with the $\eta = 0$ case by introducing the following operators:

$$\begin{aligned}\hat{b}^\dagger &= i\sqrt{\frac{\omega}{2}} \left[z + \frac{\dot{z}}{i\omega} \right] = i\omega \hat{z}_c \\ \hat{b} &= -i\sqrt{\frac{\omega}{2}} \left[z^\dagger - \frac{\dot{z}^\dagger}{i\omega} \right] = -i\omega \hat{z}_c^\dagger\end{aligned}\tag{2}$$

where $\hat{z}_c$ and $\hat{z}_c^\dagger$ are the orbit center operators, with the following commutation relation between them:

$$[\hat{b}, \hat{b}^\dagger] = 1\tag{3}$$

The coherent state $|\xi\rangle$ is then defined by:

$$|\xi\rangle = \exp\left(-\frac{|\xi|^2}{2}\right) \exp(\hat{b}^\dagger \xi) |0\rangle\tag{4}$$

with the vacuum state $|0\rangle$ annihilated by $\hat{b}$

$$\hat{b}|0\rangle = 0\tag{5}$$

In order to write a given LLL state $|\psi\rangle$ in the coherent representation it is sufficient to evaluate its projection $\langle \xi | \psi \rangle$ on the coherent state $|\xi\rangle$. This operation is equivalent to define a function $G_S(\xi, z)$ such that:

$$\psi(\xi) = \langle \xi | \psi \rangle = \int d^2 z G_S(\xi, z) \psi(z).\tag{6}$$

G_S may be obtained by inserting an unity partition $I = \int d^2 z |z\rangle \langle z|$ in the scalar product which defines $\psi(\xi)$.

The explicit form of G_S in the symmetric gauge has been given in [I] for the general $\eta \neq 0$ case. For $\eta = 0$ it becomes:

$$G_S(\xi, z) = \frac{\omega}{2\pi} \exp\left[-\frac{1}{2}|\xi|^2 + \sqrt{\frac{\omega}{2}}\bar{z}\xi - \frac{\omega}{4}|z|^2\right].\tag{7}$$

By choosing for $\psi(z)$ in eq.(6) the generic LLL state $\psi_j(z)$ of angular momentum j :

$$\psi_j(z) = \sqrt{\frac{\omega}{\pi}} \frac{1}{\sqrt{j!}} (z\sqrt{\frac{\omega}{2}})^j \exp(-\frac{\omega}{4}|z|^2)\tag{8}$$

we obtain the corresponding state $\psi_j(\xi)$ in the ξ representation (where, from now on, $\bar{\xi}$ denotes the eigenvalue of $\hat{z}_c^\dagger$ and ξ its complex conjugate):

$$\psi_j(\xi) = \psi_j(z = \xi) = \sqrt{\frac{\omega}{\pi}} \frac{1}{\sqrt{j!}} \left(\sqrt{\frac{\omega}{2}} \xi \right)^j \exp\left(-\frac{\omega}{4} |\xi|^2\right) \quad (9)$$

that is, the variable z is simply replaced by the variable ξ .

In order to construct the corresponding function $G_A(\xi, z)$ in the asymmetric gauge, i.e. on the cylinder, we notice that the operators $\hat{b}$, $\hat{b}^\dagger$ defined in eq.(2) are now given by:

$$\begin{aligned} \hat{b} &= \sqrt{\frac{\omega}{2}} \frac{\partial}{\partial z} + \sqrt{\frac{\omega}{8}} z + \sqrt{\frac{\omega}{8}} \bar{z} \\ \hat{b}^\dagger &= -\sqrt{\frac{\omega}{2}} \frac{\partial}{\partial \bar{z}} + \sqrt{\frac{\omega}{8}} z + \sqrt{\frac{\omega}{8}} \bar{z} \end{aligned} \quad (10)$$

It turns out that $G_A(\xi, z)$ has the following expression:

$$G_A(\xi, z) = \exp \left[-\frac{\omega}{4} (|\xi|^2 - 2\xi\bar{z}) - \frac{\omega}{2} y^2 - \frac{\omega}{4} \bar{z}^2 \right] \quad (11)$$

Being the electron wavefunction periodic in the x direction, that is $\psi(x+L, y) = \psi(x, y)$, $G_A(\xi, z)$ must be properly periodicized (see ref.[4]), obtaining:

$$G_A^{per}(\xi, z) = \exp \left[-\frac{\omega}{2} \left(y^2 + \frac{|\xi|^2}{2} \right) \right] \sum_{r=-\infty}^{\infty} \exp \left[\frac{\omega}{4} (2\xi(\bar{z} + rL) - (\bar{z} + rL)^2) \right] \quad (12)$$

By using G_A^{per} in eq.(6) for the LLL single state wavefunction on the cylinder $\psi_k(z)$ given by:

$$\psi_k(z) = \sqrt{\frac{1}{\pi L \sqrt{\omega}}} \exp \left[\frac{2\pi i k}{L} x - \frac{\omega}{2} \left(y + \frac{2\pi k}{\omega L} \right)^2 \right] \quad (13)$$

we obtain the corresponding state $\phi_k(\xi)$ in the ξ representation:

$$\phi_k(\xi) = \sqrt{\frac{1}{\pi L \sqrt{\omega}}} Q(\xi, \bar{\xi}) \exp \left[\frac{2\pi i k}{L} \xi - \frac{2\pi^2}{\omega L^2} k^2 \right] \quad (14)$$

where:

$$Q(\xi, \bar{\xi}) = \exp \left[-\frac{\omega}{4} (-\xi^2 + |\xi|^2) \right] = \exp \left[-\frac{\omega}{2} \xi_y^2 - i \frac{\omega}{2} \xi_x \xi_y \right] \quad (15)$$

By comparing eq.(14) with eq.(9) we notice that the “gauge” factor $\exp(-i\omega/2\xi_x\xi_y)$ appears when going from the Corbino disk to the infinite cylinder geometry. We will comment on the role of this factor when the magnetic translation generators in the coherent representation will be derived.

2b. The $\eta \neq 0$ case

In the presence of η it has been shown in [I] that one can describe the evolution of a LLL electron state in terms of the operators $\hat{b}'$, $\hat{b}'^\dagger$ given by:

$$\begin{aligned}\hat{b}' &= \left(\bar{z} + \frac{\dot{\bar{z}}}{\eta - i\omega} \right) \sqrt{\frac{\eta^2 + \omega^2}{2\omega}} \\ \hat{b}'^\dagger &= \left(z + \frac{\dot{z}}{\eta + i\omega} \right) \sqrt{\frac{\eta^2 + \omega^2}{2\omega}}\end{aligned}\quad (16)$$

By constructing again the coherent state representation relative to the new operators $\hat{b}'$, $\hat{b}'^\dagger$ one gets for the function G_S^η in the presence of dissipation:

$$G_S^\eta = \sqrt{\eta^2 + \omega^2} \exp \left[-\frac{\omega - i\eta}{4} |z|^2 + \frac{\omega - i\eta}{2} \bar{z}\xi - \frac{\omega^2 + \eta^2}{4\omega} |\xi|^2 \right] \quad (17)$$

which allows for the large time scale evolution of the generic LLL state in the coherent state representation given by eq.(9). In fact by using eq.(6) one obtains, for the asymptotic state:

$$\psi_j^{t \rightarrow \infty}(\xi) = \sqrt{\frac{\eta^2 + \omega^2}{\pi\omega}} \frac{1}{\sqrt{j!}} \left(\xi \frac{\omega - i\eta}{\omega} \right)^j \exp \left[-\frac{\eta^2 + \omega^2}{4\omega^2} |\xi|^2 \right] \quad (18)$$

Let us comment that, being $\hat{z}_{t \rightarrow \infty} = \hat{z}_c$, there is no difference between the coordinate operator eigenvalue z and the coherent coordinate ξ . For such a reason we can use indifferently z or ξ on large time scales ($t \gg 1/\eta$).

By comparing eq.(18) with eq.(9) we notice that the full effect of the presence of the dissipation in the symmetric gauge is to make the following replacement on the variable ξ :

$$\xi \rightarrow \frac{\omega - i\eta}{\omega} \xi \quad (19)$$

The physical interpretation of this “composite transformation” in terms of the transport properties of the system have been given in [I].

Now we will show that the above equation continues to be valid in the asymmetric gauge for the large time scales evolution of the system.

In order to do so let us write the operators $\hat{b}'$, $\hat{b}'^\dagger$ in the asymmetric gauge:

$$\begin{aligned}\hat{b}' &= \sqrt{\frac{\eta^2 + \omega^2}{2\omega}} \left(-\frac{2i}{\eta - i\omega} \frac{\partial}{\partial z} + \frac{\bar{z}}{2} + \frac{\omega + i\eta}{\omega - i\eta} \frac{z}{2} \right) \\ \hat{b}'^\dagger &= \sqrt{\frac{\eta^2 + \omega^2}{2\omega}} \left(-\frac{2i}{\eta + i\omega} \frac{\partial}{\partial \bar{z}} + \frac{z}{2} + \frac{\omega - i\eta}{\omega + i\eta} \frac{\bar{z}}{2} \right)\end{aligned}\quad (20)$$

and construct, as done previously, the function $G_A^\eta(z, \xi)$ for the case $\eta \neq 0$. It turns out that:

$$G_A^\eta(z, \xi) = \sum_{r=-\infty}^{\infty} \exp \left[-\frac{\omega - i\eta}{2} (y^2 - (z + rL)\xi) \right] \exp \left[-\frac{\omega^2 + \eta^2}{4\omega} |\xi|^2 - \frac{\omega}{4} (z + rL)^2 \right] \quad (21)$$

and that, by using the evolution equation (6), we obtain for the generic LLL state given by eq.(14) asymptotically:

$$\phi_k^{t \rightarrow \infty}(\xi) = Q \left(\frac{\omega - i\eta}{\omega} \xi, \frac{\omega + i\eta}{\omega} \bar{\xi} \right) \exp \left[\frac{2\pi i k}{L} \frac{\omega - i\eta}{\omega} \xi - \frac{2\pi^2}{\omega L^2} k^2 \right] \quad (22)$$

By comparing eq.(22) with eq.(14) we see that the effect of η on the large time scales evolution of the system is fully described by the transformation given in eq.(19). We will use extensively this result in the following.

2c. Asymptotic canonical operators in the coherent state representation

It is now straightforward to evaluate the asymptotic canonical operators x, y, p_x, p_y in the coherent representation. Infact one can evaluate their matrix elements between LLL states in the coherent representation and impose the condition that they are equal to the corresponding ones calculated in coordinate representation ([I]).

In this way, as a consequence of the appearance of a “gauge factor” in eq.(14), we obtain:

$$\begin{aligned} \hat{x} &\rightarrow \xi_x, & -i \frac{\partial}{\partial x} &\rightarrow -i \frac{\partial}{\partial \xi_x} + \frac{\omega}{2} \xi_y \\ \hat{y} &\rightarrow \xi_y, & -i \frac{\partial}{\partial y} &\rightarrow -i \frac{\partial}{\partial \xi_y} + \frac{\omega}{2} \xi_x. \end{aligned} \quad (23)$$

The coherent form of the velocity operators $\hat{v}_x, \hat{v}_y$ is then:

$$\hat{v}_x = -i \frac{\partial}{\partial \xi_x} - \frac{\omega}{2} \xi_y; \quad \hat{v}_y = -i \frac{\partial}{\partial \xi_y} + \frac{\omega}{2} \xi_x \quad (24)$$

For the infinitesimal generators of the magnetic translations defined by

$$\pi_x \equiv \hat{v}_x - \omega y; \quad \pi_y \equiv \hat{v}_y + \omega x \quad (25)$$

we get their coherent realization:

$$\pi_x = -i \frac{\partial}{\partial \xi_x} + \frac{\omega}{2} \xi_y; \quad \pi_y = -i \frac{\partial}{\partial \xi_y} - \frac{\omega}{2} \xi_x. \quad (26)$$

With the help of eq.(19) we can obtain the corresponding asymptotic operators: $\hat{v}_x^\eta, \hat{v}_y^\eta$ in the presence of dissipation. Infact, being $\hat{v}$ a vector operator, we have (see [I]):

$$\begin{aligned} \hat{v}_x^\eta &= -i \frac{\partial}{\partial \xi_x^\eta} - \frac{\omega^2 + \eta^2}{2\omega} \xi_y^\eta \\ \hat{v}_y^\eta &= -i \frac{\partial}{\partial \xi_y^\eta} + \frac{\omega^2 + \eta^2}{2\omega} \xi_x^\eta \end{aligned} \quad (27)$$

In the same way we get the asymptotic form of the generators of the magnetic translations, π_x^η, π_y^η :

$$\begin{aligned}\pi_x^\eta &\equiv -i\frac{\partial}{\partial\xi_x^\eta} + \frac{\omega^2 + \eta^2}{2\omega}\xi_y^\eta \\ \pi_y^\eta &\equiv -i\frac{\partial}{\partial\xi_y^\eta} - \frac{\omega^2 + \eta^2}{2\omega}\xi_x^\eta\end{aligned}\quad (28)$$

We notice that the commutator of π_x^η and π_y^η , is given by:

$$[\pi_x^\eta, \pi_y^\eta] = i\frac{\omega^2 + \eta^2}{\omega} \quad (29)$$

In the following we will use the operators defined above to construct a basis for the LLL wave functions on the torus in the case $\eta \neq 0$ by providing finite-dimensional representations of the magnetic translation group for it.

3. Magnetic translations in the presence of dissipation and their representations

3a. Finite magnetic translations on a torus and their representations

Let us now derive the generic wave function on a torus obeying the periodic boundary conditions along the “rotated” axes by noticing that as $t \rightarrow \infty$ the coherent variable $\tilde{\xi}$ is just the eigenvalue of the coordinate operator $\hat{z}_\infty^\dagger$.

In the absence of dissipation the wave functions on a $L \times L$ square torus are constructed by finding the finite-dimensional representations of the magnetic translations group (see ref.[3]). By using eq.(26) we get the coherent-state representation for $\mathcal{S}_L$ and $\mathcal{T}_L$:

$$\begin{aligned}\mathcal{S}_L &= \exp(iL\pi_x(\xi)) = \exp\left(L\frac{\partial}{\partial\xi_x}\right) e^{iL\omega\frac{\xi_y}{2}} \\ \mathcal{T}_L &= \exp(iL\pi_y(\xi)) = \exp\left(L\frac{\partial}{\partial\xi_y}\right) e^{-iL\omega\frac{\xi_x}{2}},\end{aligned}\quad (30)$$

which obey the “braid” relation:

$$\mathcal{S}_L\mathcal{T}_L = \mathcal{T}_L\mathcal{S}_L \exp(i\omega L^2) \quad (31)$$

We know that there exist finite dimensional representations of the magnetic translations group given above only when the flux piercing the area L^2 is rational in units of the magnetic quantum flux $\Phi_0 = hc/e (= 2\pi$ in our units).

In the presence of dissipation the corresponding operators $\mathcal{S}_{L_\eta}^\eta, \mathcal{T}_{L_\eta}^\eta$ are given by:

$$\begin{aligned}\mathcal{S}_{L_\eta}^\eta &= \exp\left(L_\eta\frac{\partial}{\partial\xi_x^\eta}\right) e^{iL_\eta\frac{\eta^2+\omega^2}{\omega}\frac{\xi_y^\eta}{2}} \\ \mathcal{T}_{L_\eta}^\eta &= \exp\left(-L_\eta\frac{\partial}{\partial\xi_y^\eta}\right) e^{-iL_\eta\frac{\eta^2+\omega^2}{\omega}\frac{\xi_x^\eta}{2}}\end{aligned}\quad (32)$$

which obey the braid relation:

$$\mathcal{S}_{L_\eta}^\eta \mathcal{T}_{L_\eta}^\eta = \exp\left(i \frac{\eta^2 + \omega^2}{\omega} L_\eta^2\right) \mathcal{T}_{L_\eta}^\eta \mathcal{S}_{L_\eta}^\eta = \exp(i\omega L^2) \mathcal{T}_{L_\eta}^\eta \mathcal{S}_{L_\eta}^\eta \quad (33)$$

We notice that, due to the presence of the factor $(\eta^2 + \omega^2)/\omega^2$, the size of the elementary cell gets reduced by the factor $\rho \equiv \sqrt{\frac{\eta^2 + \omega^2}{\omega^2}}$ in order to allow for finite dimensional representation of the magnetic translations, that is: $L \rightarrow L_\eta \equiv \rho^{-1} L$.

We can comment on equations (29) and (33) by picturing the effect of η as:

1. a modification of the field lines describing the external magnetic field (see fig. 1);
2. a shrinking of the size of the elementary cell in such a way to keep constant the total magnetic flux through the torus:

$$\frac{\omega^2 + \eta^2}{\omega} L_\eta^2 = \omega L^2 \quad (34)$$

Looking at eq.(33) we immediately notice that interesting cases are the ones in which $\frac{\eta^2 + \omega^2}{\omega} L_\eta^2 = 2\pi N_s$, where N_s is an integer. In this case we obtain $[\mathcal{S}_{L_\eta}^\eta, \mathcal{T}_{L_\eta}^\eta] = 0$. Then we realize that a finite subgroup of the magnetic translations $\mathcal{S}_{L_\eta/N_s}^\eta, \mathcal{T}_{L_\eta/N_s}^\eta$, of elementary step L_η/N_s , can be defined, with commutator given by:

$$\mathcal{S}_{L_\eta/N_s}^\eta \mathcal{T}_{L_\eta/N_s}^\eta = \exp(2\pi i/N_s) \mathcal{T}_{L_\eta/N_s}^\eta \mathcal{S}_{L_\eta/N_s}^\eta \quad (35)$$

It is easy to give a set of functions Φ_k^η invariant under the action of $\mathcal{S}_{L_\eta}^\eta$ and $\mathcal{T}_{L_\eta}^\eta$, obeying at the same time the correct double “quasi periodic” boundary conditions $\mathcal{S}_{L_\eta}^\eta \Phi_l^\eta = \mathcal{T}_{L_\eta}^\eta \Phi_l^\eta = \Phi_l^\eta$.

It turns out that a finite representation of the subgroup of the magnetic translations $\mathcal{S}_{L_\eta/N_s}^\eta, \mathcal{T}_{L_\eta/N_s}^\eta$, with step L_η/N_s , is given by the set of the N_s functions (see ref.[3] for the $\eta = 0$ case):

$$\Phi_l^\eta(\xi_x^\eta, \xi_y^\eta) = Q \left(\sqrt{\frac{\eta^2 + \omega^2}{\omega^2}} \xi^\eta, \sqrt{\frac{\eta^2 + \omega^2}{\omega^2}} \bar{\xi}^\eta \right) \Theta \left[\begin{matrix} \frac{l}{N_s} \\ 0 \end{matrix} \right] \left(\frac{\xi N_s}{L_\eta} | i N_s \right) \quad (36)$$

$l = 1, \dots, N_s$

and their action on the Φ_l^η is expressed by:

$$\begin{aligned} \mathcal{S}_{L_\eta/N_s}^\eta \Phi_l^\eta &= e^{2\pi i \frac{l}{N_s}} \Phi_l^\eta \\ \mathcal{T}_{L_\eta/N_s}^\eta \Phi_l^\eta &= \Phi_{l+1}^\eta \end{aligned} \quad (37)$$

We notice that the functions Φ_l^η may be obtained from the corresponding ones in the case $\eta = 0$ by the following replacements:

$$\xi \rightarrow \rho \xi^\eta, \quad L \rightarrow \rho L_\eta. \quad (38)$$

4. The N_e electrons center of mass wave function and its topological properties

It is now straightforward to construct the N_e electrons center of mass wave functions Φ_l^η on the torus in the “rotated” system with the external flux condition $N_S = mN_e$. Infact let us consider the total magnetic translation operators defined by

$$\begin{aligned} S_{\frac{L_\eta}{m}}^{(\eta)} &= \Pi_{i=1}^{N_e} S_{\frac{L_\eta}{m}}^{i(\eta)} \\ T_{\frac{L_\eta}{m}}^{(\eta)} &= \Pi_{i=1}^{N_e} T_{\frac{L_\eta}{m}}^{i(\eta)} \end{aligned} \quad (39)$$

then we ask that Φ_l^η is a m -dimensional representation of a finite subgroup of the total magnetic translation group generated by $S_{\frac{L_\eta}{m}}^{(\eta)}, T_{\frac{L_\eta}{m}}^{(\eta)}$ which satisfy the algebra

$$S_{\frac{L_\eta}{m}} T_{\frac{L_\eta}{m}} = e^{2\pi i/m} T_{\frac{L_\eta}{m}} S_{\frac{L_\eta}{m}}, \quad (40)$$

in analogy with the $\eta = 0$ case (which has been presented in detail in ref.[3]). Then the center of mass ground state wave function on a torus is obtained as

$$\Phi_l^\eta(\mathcal{W}^\eta, \bar{\mathcal{W}}^\eta) = Q \left(\sqrt{\frac{\eta^2 + \omega^2}{\omega^2}} \mathcal{W}^\eta, \sqrt{\frac{\eta^2 + \omega^2}{\omega^2}} \bar{\mathcal{W}}^\eta \right) \Theta \left[\begin{matrix} l/m \\ 0 \end{matrix} \right] \left(\frac{\mathcal{W}^\eta m}{L_\eta} |im \right) \quad (41)$$

where $\mathcal{W}^\eta \equiv \sum_{i=1}^{N_e} w_i^\eta$ is the center of mass variable and Θ is the Jacobi function with characteristic $1/m$. We should notice that the new step L/m appears in eq.(39) for the total magnetic translations operators denoting that the N_e “highly correlated” electrons appear as “independent” particles each carrying a flux $m\phi_0$ (associated with the electron cell) (see ref.[3]).

We will now derive the transport properties of the system by using a topological argument which, based on the Laughlin gauge argument, uses the finite magnetic translation operators $\mathcal{S}_{L_\eta/m}$ and $\mathcal{T}_{L_\eta/m}$ introduced in eqs.(39).

By extending a previous argument ([3]) one can see that, by making an adiabatic flux variation $\Delta\Phi = \Phi_0$ along the torus axis, the transformed wavefunction $\Phi_l^{\eta(\Delta\Phi)}$, which in general obeys new boundary conditions, is given by:

$$\Phi_l^{\eta(\Delta\Phi)} = \Phi_{l+1}^\eta(\mathcal{W}^\eta, \bar{\mathcal{W}}^\eta) \quad (42)$$

That is, Φ_l^η may be obtained by applying the finite translation $\mathcal{T}_{L_\eta/m}$ of step L_η/m to the original function. As a result the charge of the electron has moved by “a step” L_η/m after the insertion of the flux Φ_0 , obtaining for the topological invariant quantity σ'_H and for σ'_L in the “rotated” system:

$$\sigma'_H = \frac{1}{m}, \quad \sigma'_L = 0. \quad (43)$$

Without going into further details we are now ready to evaluate the longitudinal and Hall conductance of our physical system by noticing that, when the electron moves by a step L_η/m , in the original frame the electron moves by the steps:

$$\left(\frac{L_\eta}{m} \right)_x = \frac{L_\eta}{m} \frac{\eta\omega}{\eta^2 + \omega^2}, \quad \left(\frac{L_\eta}{m} \right)_y = \frac{L_\eta}{m} \frac{\omega^2}{\eta^2 + \omega^2} \quad (44)$$

along x, y respectively. That is, as a result of the flux variation, one gets charge transfer in the y (“Hall”), as well as in the x (“Longitudinal”) direction. Then we obtain the following values for the conductivities:

$$\begin{aligned}\sigma_H^\eta &= \frac{\omega}{m} \frac{\omega}{\eta^2 + \omega^2} \\ \sigma_L^\eta &= \frac{\omega}{m} \frac{\eta}{\eta^2 + \omega^2}\end{aligned}\tag{45}$$

which agree with the results presented in [1] being the magnetic length squared $\lambda^2 = m/\omega$ in our units.

We notice that the conductivities σ_H, σ_L transform as a two-dimensional vector under the $U(1)$ transformation contained in eq.(1).

This analysis can be easily extended to the general rational flux condition $N_s = (m/p)N_e$ ([8]) which should be considered as the generic case.

5. Comments and conclusions

The results expressed in eq.(45) which provide us with the new transport properties of the Hall fluid on a torus geometry agree with our previous investigation on the behaviour of the electron on the Corbino disk [1]. The appearance of a longitudinal conductance σ_L different than zero suggests on one hand that we are working in a region away from the plateau, on the other hand the values of σ_L, σ_H given by eq.(45) reproduce exactly the classical ones obtained by a simple Langevin approach (see, for example, ref.[5]). More explicitly these values correspond to the slope middle points between plateaus, which correspond to rational values of the filling factor ν .

The system of N_e electrons in the presence of dissipation appears at a first sight to share similar properties to the $\eta = 0$ case. In fact in the rotated frame we recover the “usual” topological interpretation for σ_H' and a finite dimensional representation of the magnetic translation group expressed in eq.(41) ([6,2]); furthermore the delocalization properties expressed by the fact that the zeroes of the Jacobi functions cover the entire fundamental region are still valid in such a frame ([7]).

However one should notice that the neutrality condition expressed by $N_s = mN_e$ is now not realized by the Hall states only. Infact one has:

$$\left(\frac{\omega^2}{\eta^2 + \omega^2} + \frac{\eta^2}{\eta^2 + \omega^2}\right)N_e m = (n_H + n_L)N_e m = N_s$$

where $n_H(n_L)$ are the density of Hall (Longitudinal) states. An important consequence of this is the lack of stability for the Laughlin plasma related to the fact that an infinitesimal change of the external magnetic field ω can modify the vortex magnetic charge associated with the electron as expressed by the previous equation.

As a consequence the Hall fluid in these points of the transition region does not behave as an incompressible one.

It would be very interesting to further investigate on other physical properties of the Hall fluid at these special points, as for example the symmetry properties of the

system in the context of a dynamical description of the evolution of the system and on the role played by 2D CFT in the description of the system.

One further comment must be addressed in relation to eqs.(29), (33). In according to them, the phenomenon of localization of the Gaussian describing the electron LLL state (see [1]) can be viewed in terms of a scale reduction of the physical system accompanied by a squeezing of the external flux lines in such a way to keep constant the total flux through the $L \times L$ torus. Such a description of the phenomenon of localization would then suggest the appearance of a radial component of the electron current in the framework of a hydrodynamic model for a Hall quantum droplet, in agreement with the compressibility of the Hall liquid.

For the multiparticle case, we comment on the band structure of our system ([9]). Reminding that when the Fermi level lies in a gap separating occupied from unoccupied states there is only a Hall contribution to the conductance, we remark that the presence of impurities in the system make electron bound states appear near the band edges. These electron bound states make more probable the transition between the valence and the conduction band. In our case the impurities seem to be distributed on the sites of a sublattice, with a correlation between them and consequently the mobility gap separating the band edge from the localized states is zero. Infact there is a longitudinal conductance different than zero even in the case of an infinitesimal electric field.

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Figure captions

Fig. 1: Modification of the field lines for $\eta \neq 0$.

References

- [1] G. Cristofano, D. Giuliano, G. Maiella, L. Valente, *to appear in Int. Jou. Mod. Phys. B*(1995).
- [2] For a review on the subject, see for example chapters 1,3 of “Quantum Hall Effect”, M. Stone ed., World Scientific, Singapore, 1992.
- [3] G. Cristofano, G. Maiella, R. Musto, F. Nicodemi, *Mod. Phys. Lett. A***6** (1991) 1779; see also G. Cristofano et al., *Nucl. Phys. B***33** (Proc. Suppl.)C(1993)119.
- [4] H. Azuma, S. Iso, *Phys. Lett. B***331**(1994)107.
- [5] Y. S. Wu, Topological Aspects of the Quantum Hall Effect “Physics, Geometry and Topology” 1989 Banff Proceeding, H. C. Lee ed..
- [6] Q. Niu, D. J. Thouless, Y. S. Wu. *Phys. Rev. B***31**(1985)3322.
- [7] D. P. Arovas, R. W. Blatt, F. D. M. Haldane, P. B. Littlewood, P. Rammal, *Phys. Rev. Lett.* **60**(1988)619.
- [8] G. Cristofano, G. Maiella, R. Musto, F. Nicodemi, *Mod. Phys. Lett. A***7**(1992)2589.
- [9] R. B. Laughlin, M. L. Cohen, J. M. Kosterlitz, H. Levine, S. B. Libby, A. M. Pruisken, *Phys. Rev. B***32**(1985)1311.
- [10] M. Stone, *Phys. Rev. B***42**(1990)212; X. G. Wen, *Int. Jou. Mod. Phys. B***6**(1992)1711.