Using High-Fold Data from the New Generation of $\gamma$-ray Detector Arrays

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Abstract

The new generation of high efficiency gamma-ray spectrometers allow the collection of high fold gamma-ray coincidence data. Some problems associated with the sorting and subsequent analysis of this data are discussed. A method is presented to allow the creation of statistically correct spectra when selection criteria are placed on gamma-ray energies.
1 Introduction

The new generation of high efficiency $\gamma$-ray detector arrays, such as the Eurogam [1], Gammasphere [2] and Gasp [3] arrays, offers the exciting possibility of taking data in which very many $\gamma$-rays associated with long rotational cascades are observed in coincidence. This is referred to as high-fold coincidence data. Some of the problems associated with the subsequent sorting and analysis of this data are examined in this article.

2 The Unfolding Procedure

In a typical experiment with the Eurogam Phase 2 array using a heavy-ion fusion-evaporation reaction the most probable suppressed coincidence fold is three to four. In other words the energies of three or four Compton suppressed germanium detectors are recorded as a single coincidence event. Because of the large detection efficiency of such an array much higher fold events (with over ten energies per event) are also observed. Although these very high fold events occur with steadily reducing probability they do, however, make a non negligible contribution to the data set. The question arises in the subsequent analysis as to how to sort such high fold coincidence data in order to produce spectra of various fixed dimensions under various selection criteria.

Sorting methods usually involve decomposing (combinatorially) each $n$-fold event into a number of lower $m$-fold subevents. This procedure is commonly referred to as unfolding or unpacking. For example, a typical analysis might be based on 4-fold coincidences. In this case each four fold event is used once. However, each 5-fold event can be unfolded to form five 4-fold subevents, each 6-fold to 15 4-fold subevents and so on. For Eurogam Phase 2 this procedure can lead to an increase in the number of 4-fold ‘events’ by a factor of greater than 10 overall, the exact number depending on the distribution of folds in the data. In general each $n$-fold event generates

$$\binom{n}{m} = \frac{n!}{m!(n-m)!} = \frac{n(n-1)\ldots(n-m+1)}{1.2.3\ldots m}$$ (1)

$m$-fold subevents where each subevent contains $m$ energies from the original event. Each $m$-fold subevent can be used once to increment the contents of the corresponding channel of an $m$-dimensional spectrum. In this way each set of $m$ energies (referred to as an $m$-tuple) in the original event corresponds to one count in an $m$-dimensional spectrum (one to one correspondence).

3 The Traditional Method of Analysis

In some traditional sorting methods the unfolded $m$-fold subevents are assumed to be independent for the purposes of further analysis, i.e the assumption is that the subevents could be further decomposed into still lower fold ‘sub’-subevents, (or almost equivalently that lower dimensional projections could be made from $m$-dimensional spectra). This assumption of independence is not correct and has important consequences for spectra produced from high-fold data sets as outlined below.
The problem becomes apparent when selection criteria (for example gamma-ray energy windows or 'gates') are placed on the data during an event-by-event sort. Consider the situation where an m-dimensional spectrum is to be produced by an event-by-event sort of the data. However, before incrementing the spectrum a minimum of p energies, in the n-fold event must satisfy certain selection or gating criteria. This is commonly referred to as p-fold gating. In the traditional sorting method each n-fold event is first unfolded combinatorially into (m+p)-fold subevents. Each (m+p)-fold subevent is then further independently unfolded into m-fold 'sub'-subevents. The channel in the spectrum corresponding to the m-tuple energies is incremented whenever all of the remaining p energies in the original subevent satisfy the selection criteria. The one-to-one correspondence between the m-tuples in the original event and the counts in the spectrum is often lost during this second unfolding. The procedure is illustrated in the following example.

Suppose that a number of gating conditions are set on the data of which at least 3 must be satisfied (p = 3) before incrementing a one-dimensional spectrum (m = 1). This is typical of the situation which occurs when studying for example superdeformed rotational cascades. Thus we require at least four energies in an event before the spectrum can be incremented. Parameters in the event which pass a gating condition will be referred to as \( g_1, g_2, \ldots \) while parameters which do not pass a gating condition will be referred to as \( x_1, x_2, \ldots \). Each n-fold event is unfolded into

\[
\binom{n}{4} = \frac{n(n-1)(n-2)(n-3)}{1.2.3.4} \tag{2}
\]

4-fold subevents. Each 4-fold subevent is then further unfolded into four 1-tuples (singles). The spectrum is incremented for each single only if the remaining three energies in the subevent satisfy the gating criteria. For the case of n=4 there are two possibilities for incrementing the spectrum:

1. Exactly three parameters meet the gating condition and one does not; \( g_1, g_2, g_3, x_1 \). In this case only \( x_1 \) will be incremented into the spectrum.

2. All four parameters satisfy a gate condition \( g_1, g_2, g_3, g_4 \). Then one can see that all four parameters will be used to make one increment each.

These cases do not present a problem. However, problems emerge for folds \( n \geq 5 \). For example, when \( n=5 \) there are three possibilities to consider.

1. Three parameters meet gating conditions and two do not \( g_1, g_2, g_3, x_1, x_2 \). In this case the two \( x \) parameters will each cause one increment while the \( g \) parameters will not be incremented.

2. Four parameters satisfy gating conditions while the remaining energy does not; \( g_1, g_2, g_3, g_4, x_1 \). In this case four increments will be made with the \( x_1 \) parameter while each of the \( g \) parameters will be incremented once.

3. Finally, all five energies meet the gating conditions; \( g_1, g_2, g_3, g_4, g_5 \). In this situation one can see that each parameter will be used to make four increments.
Thus we see that a quite non-statistical weighting is given to certain parameters in high fold events depending on the details of the gating conditions. With increasing values of the fold $n$ and increasing numbers of parameters which satisfy gating conditions the weighting factors get progressively larger. If we consider an extreme situation of an event of fold $n=10$, of which nine parameters meet the gating conditions then $g$ parameters will each make 56 increments and the $z$ parameter 84 increments! Although comparatively rare such events do make contributions to a spectrum. The practical consequence of this problem is illustrated in figure 1A. Sharp spikes are apparent both in the peaks and the surrounding background. It must be emphasised that these spikes are each due to one parameter usually from just a single event. It should also be emphasised that only (rare) very high fold events contribute large spikes to the spectrum. Therefore such spikes were usually not a such a significant problem with lower efficiency arrays.

4 An Alternative Method

To avoid these spikes appearing in the spectra each m-fold correlation can only be used once in a spectrum incrementation. In particular, unfolded m-tuple subevents must not be further unfolded and independently incremented into a spectrum. We now present several examples of an alternative method for generating multi-dimensional spectra with the correct statistical weighting given to each event and which accounts properly for the correlations between the energies in the original event.

4.1 One-dimensional Spectra with Selection Criteria

Firstly the example is considered of creating a one-dimensional spectrum when a number of gating conditions have been placed on the data. Suppose that at least three gating conditions from a larger gate list must be satisfied before incrementing the spectrum.

In the new method an n-fold event is unfolded directly into 1-tuple subevents (singles). Each single is incremented if and only if at least three energies in the remainder of the event pass the selection criteria.

For the case of $n=4$ the spectrum incrementation is exactly the same as outlined above for the traditional method.

For folds $n \geq 5$ we have two possibilities.

1. Exactly the correct number of parameters satisfy the selection criteria; $(g_1, g_2, g_3, x_1, \ldots x_{n-3})$. In this case (as for the traditional sort) $x_1, \ldots x_{n-3}$ are each incremented once in the spectrum.

2. If more than the minimum number of gating conditions are satisfied, $(g_1, g_2, \ldots g_l, x_{1l} \ldots x_{n-l})$, then there are always sufficient selection criteria satisfied in the remainder of the event whichever parameter we are examining. Thus in this case all parameters make one and only one increment in the spectrum.

Figures 1A and 1B show spectra obtained for the yrast superdeformed band of $^{149}$Gd using (A) the traditional sorting procedure and (B) the new method outlined above. The change is dramatic and is of great importance when measuring gamma-ray energies and
intensities and in identifying very weak transitions. For example in studying the phenomenon known as $C_4$ staggering [4] in superdeformed rotational bands it is necessary to determine the energies of the $\gamma$-rays to better then 0.1 keV. In the case of the 1558 keV transition, shown in Fig. 1, the presence of the spike on the high energy side of the peak in the unfolded data (A) leads to a shift in the centroid of this peak of 0.5 keV!

The number of counts in the peaks of figure 1B are approximately a factor of two less than in figure 1A, but this is entirely due to the fact that each parameter in an event only contributes one count to the spectrum in figure 1B. The peak intensities in gated one-dimensional spectra will be further discussed below. Figure 2, showing a higher energy region of the $^{149}$Gd spectrum, provides an even more dramatic illustration of the problems associated with unfolding events.

### 4.2 Two-dimensional Spectra

When creating gated two dimensional spectra (matrices) the method is similar. Each event is directly unfolded into 2-fold subevents (doubles). For each double the matrix is incremented only if at least the minimum number ($p$) of energies in the remainder of the event satisfy the selection criteria ($p$ equals three in the above example).

We again have several possibilities for the number of spectrum increments per event depending on the number of parameters which satisfy the selection criteria.

1. Exactly the minimum number of gating conditions are met; $(g_1, \ldots, g_p, x_1, \ldots, x_{n-p})$. In this case only doubles extracted from the $x$ parameters are incremented into the spectrum.

2. At least two more than the minimum number of gating conditions are satisfied ($\geq p + 2$); $(g_1, \ldots, g_{p+2}, x_1, \ldots, x_{n-p-2})$. In this case when testing any double extracted from the data one always has sufficient parameters in the remainder of the event to satisfy the selection criteria. Thus all doubles are incremented once into the matrix.

3. Exactly $p+1$ gating conditions are satisfied. For a two-dimensional matrix this is the intermediate case. Only doubles which involve one or no gating parameter are incremented into the matrix as doubles which include two gating parameters do not satisfy the selection criteria with the remainder of the event.

### 4.3 General m-dimensional Spectra

The procedure for creating a general m-dimensional spectrum follows from the above examples. Each event is directly unfolded into m-fold subevents (m-tuples). For each m-tuple the m-dimensional spectrum is incremented only if at least the minimum number ($p$) of energies in the remainder of the event satisfy the selection criteria. It should be pointed out here that the selection criteria could involve the option of the same gate being used more than once in a single event if for example there are two unresolved $\gamma$-rays in the selected cascade.

There are three possibilities for the number of spectrum increments.
1. Exactly \( p \) gating conditions are satisfied. Only \( m \)-tuples from the \( x \) parameters are incremented into the spectrum.

2. At least \( p+m \) gating conditions are satisfied. Then \( m \)-tuples unfolded from all of the parameters are incremented.

3. If \( p+k \) gating conditions are satisfied, where \( k < m \). Only \( m \)-tuples which involve \( k \) or less gating parameters are incremented into the spectrum.

5 Continuity at the Gate Borders

Certain prescriptions for gated spectrum incrementation can lead to discontinuities in the spectrum at the gate boundary channels. For example, consider the results of a spectrum incrementation prescription in which all the energies in an event are incremented if at least \( p \) parameters satisfy the energy gates. In this example if \( p - 1 \) other energies satisfy the energy gates then a particular transition is incremented if its detected energy is inside a gate but not incremented if its detected energy is outside a gate. The resultant spectrum is (considerably) higher inside the gates than outside. In general, such discontinuities can only appear when the incrementation of an energy in an event depends on whether it is inside or outside a gate window.

The new sorting method described here will produce spectra which are continuous across the gate boundaries because the spectrum incrementation prescription depends only on the energies in the remainder of the event and not on the energies of the particular \( m \)-tuple involved in the incrementation. In particular, it is not correct to unilaterally reject entire events simply because two or more gamma-ray energies in the event fall inside the same gate window. This rejection will result in a spectrum which is not continuous at the gate boundaries because the incrementation prescription depends on the energies in the whole event and not only on the energies in the remainder of the event (contrary to the definition of the new method)! However, the requirement that more than one gamma-ray energy from the remainder of the event cannot fall inside the same energy gate is a valid selection criterion and would produce a spectrum which is continuous at the gate boundaries.

6 Similarities and differences Between the two methods

The two methods give exactly the same results when creating spectra (of any dimension) when no selection criteria are set on the event-by-event sort. However, there are significant differences between spectra created with the two methods when selection criteria have been applied during the event-by-event sort.

6.1 Statistical Uncertainty in the Spectra

As can be seen in Figures 1 and 2 gated spectra created with the new method contain less counts in a particular channel than the equivalent spectra created with the traditional
method from the same data set. This is due to the fact that each parameter in the event is used once and only once in the new method and it does not imply any loss of statistical accuracy. In fact it is shown in the appendix that the relative statistical error in the channel counts is smaller for the new sorting method. Furthermore, the new method produces the best possible statistical accuracy (the least relative error) for the number of counts in a particular channel of a spectrum. Therefore, this is the method which enables the most accurate energy and intensity measurements.

To elaborate on these statements it should be pointed out that when using the traditional method the same parameter can be incremented more than once in the same event. Consider again the example of a gated one dimensional spectrum created with p-fold gating. A series expansion for the number of counts C in a particular channel of the spectrum gives,

\[ C = \sum_{k \geq p} \binom{k}{p} C_k \]  

where \( C_k \) is the number of events which contain this channel and which also contain k other energies within the gate set. Therefore, for the traditional method it is not correct to assume that the uncertainty in a particular channel counts is the square root of the number of counts. In fact this underestimates the real error (see appendix).

For the case of a spectrum created with the new sorting method and with the same selection criteria the same analysis gives

\[ C' = \sum_{k \geq p} C_k \]  

In this case we see that the statistical uncertainty in the number of counts in a particular channel can be assumed to be the square root of the number of counts.

### 6.2 Peak Intensities in Gated One-dimensional Spectra

In a gated spectrum the relative intensities of peaks which were not used as part of the selection criteria will be the same in both methods.

It is useful, but non-trivial, to estimate the relative intensities between gate and non-gate photopeaks in a spectrum. To do this we consider a somewhat idealised situation where all of the transitions in a cascade are assumed to have the same intensity I but different energies (doublets in this case are not allowed). Furthermore, the gates in the gate set are assumed to be wide enough to cover the entire width of all the peaks. Finally, the detection efficiency is assumed to be independent of gamma-ray energy.

Under these conditions, for a spectrum created with the traditional method, the ratio of peak intensities is

\[ I_{\text{gate}} / I_{\text{nongate}} = \frac{d - p}{d} \]  

where \( d \) is the number of elements \((d > p)\) in the gate list and \( p \) is the minimum number of gates required before incrementing the spectrum.

For a similar spectrum created with the new sorting method the number of counts in a non-gate peak is

\[ I_{\text{nongate}} = I \sum_{k=p}^{d} (\frac{d}{p})^{k+1} (1 - e)^{(d-k)} \]  

and in a gate peak is

\[ I_{\text{gate}} = \sum_{k=p}^{d-1} (d-1)! e^{k+1} (1 - e)^{(d-k-1)} \]  

(7)

In these equations \( e \) is the peak detection efficiency of the array.

It is interesting to consider two extreme cases

1. For very small values of the peak detection efficiency such that only the minimum number of gating conditions \( p \) are met. In this case only the first terms in equations 6 and 7 are important and the ratio of gated to non gated intensities becomes the same as the unfolding method. This is to be expected since the two methods are identical in this limit.

2. For very large values of the peak detection efficiency (\( e \to 1.0 \)) such that all the gates in the gate list are satisfied. Then only the last term in the expansion is important and the ratio of intensities tends to unity.

It is apparent that the first case most nearly describes the situation in current arrays. However, in the new sorting method there is no simple relation for the intensity ratio and in practice this value is expected to rise a little above that for the unfolding method because of the presence of higher terms in the expansion.

7 Subsequent Gating on m-dimensional spectra

Additional gates are often placed on multi-dimensional spectra in order to produce a final one-dimensional spectrum for analysis. For example, one may set multiple gates on two axes of a three-dimensional spectrum (a cube) in order to produce a one-dimensional projection of the third dimension. This projected-spectrum is often used for measurements of gamma-ray energies, coincidence intensities etc. However, setting gates on the cube is essentially the same as a second independent unfolding of the data and, therefore, does not produce statistically correct spectra. In other words spikes will be present in the projected-spectrum whether the original spectrum was produced with the traditional or new sorting methods. The only exception is when a single gate-pair is placed on the cube. In this case a statistically correct projection will result. It appears to us that in general a statistically accurate spectrum can only be generated by a direct event-by-event sort of the data. Therefore, gamma-ray intensity and energy measurements should be made only from such spectra or, alternatively, directly by fitting peaks in the multi-dimensional spectrum itself (for example 3-dimensional fits to peaks in a cube, or two dimensional fits in a matrix such as performed by analysis programs such as Radware [5]). However, the inherent ‘spikiness’ of the projected-spectra is reduced when using gated matrices, cubes, etc created using the new sorting method. For example, when setting gates on a \( p \)-fold gated two-dimensional matrix the magnitude of the spikes in the projected spectrum varies linearly with the number of gate transitions in the new sorting method compared with combinatorially with the traditional sorting method. It should be noted that in this case also an approximately statistically correct spectrum can be obtained (using the new sort method) by dividing each channel of the projected spectrum by the gate fold number plus one, (i.e. \((p + 1\) fold gating).
8 Contributions to the Spectrum from Different fold Events

Finally, it is interesting to consider the contribution to the spectra from the various fold events in the data stream for a typical study of superdeformed bands in the mass $A \sim 150$ region.

Table 1 shows the normalised fold distribution following a Eurogam Phase 2 experiment which populated high-spin states in $^{152}$Dy and which had a threshold trigger requirement of a coincidence of five unsuppressed germanium detectors. For each fold the data are further subdivided according to the number of energies in the event which pass a set of 15 gates set on the yrast superdeformed band in $^{152}$Dy. From this data it is possible to calculate the contribution of each fold to a gated one-dimensional spectrum. The results are presented in figure 3.

As can be seen the raw fold distribution is peaked around four, but with significant amounts of higher fold events contributing to the data stream (the peak of the raw fold distribution is determined by the threshold trigger condition in the experiment). The remaining curves in figure 3 show the percentage contribution of each fold to the spectrum when at least two, three or four gates, from a gate-list of fifteen energies in the yrast superdeformed band, are required in an event before spectrum incrementation. For the case of three gates (requiring at least a four fold event to make a spectrum increment) one can see that most of the spectrum increments are due to fold seven events (~25%) while only ~3% of the spectrum increments are due to four fold events. However, more than 50% of the raw data stream events contain four or fewer energies. Therefore, the data rate to tape (often the limiting rate in an experimental situation) can be reduced by more than a factor of two with only a 3% loss of events in the triple-gated spectrum.

9 Conclusions

In this paper we have demonstrated the problems associated with the double unfolding of events containing many parameters such that spikes are produced in the final spectra. A technique has been developed for applying gating conditions on complete events to generate statistically correct one- and higher-dimensional spectra which are free of spikes.

However, the best analysis to discover new gamma-ray cascades in a data set is either to sort data into spectra of fixed dimension (commonly 2 or 3) or into an ordered list of addresses of fixed fold (commonly $\leq 6$) [6] and then setting gates. In either of these procedures it is necessary to decompose events of varying fold into structures of fixed dimensions such that knowledge is lost of the correlation between the $m$-tuples which are derived from the complete event. The procedures described in this paper can be used to produce statistically correct multi-dimensional spectra but the resulting one-dimensional spectra obtained by applying further gating conditions to the multi-dimensional spectra will contain spikes, though of reduced magnitude.

It is important in the long term to develop fully spike free methods which have the advantage of rapidly changing the selection criteria as is possible with matrices or ordered lists. This may imply storing data as complete events and an ordered list mode with
no fixed fold limitation offers a solution, since the memory requirements do not become excessive. To the best of our knowledge no such software currently exists. Clearly it would require highly sophisticated formats to minimise storage requirements and keep access time, when setting gates, to a minimum.

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10 Appendix

In equations 3 and 4 the \( C_k \) numbers are random variables obeying Poisson statistics. Therefore, the corresponding statistical uncertainties \( \Delta C_k \) are equal to \( \sqrt{C_k} \). Using the error propagation formula for the case of the traditional sorting the statistical uncertainty in \( C \) is

\[
\Delta C = \sqrt{\sum_{k \geq p} \frac{\binom{k}{p}^2 C_k}{C_k}} = \sqrt{\sum_{k \geq p} \frac{\binom{k}{p}^2}{C_k}} \quad (8)
\]

and the relative error of \( C \) is

\[
\frac{\Delta C}{C} = \frac{\sqrt{\sum_{k \geq p} \binom{k}{p}^2 C_k}}{\sum_{k \geq p} \binom{k}{p} C_k}.
\]

For the case of a spectrum created with the new sorting method with the same selection criteria the relative error of counts in the same channel is

\[
\frac{\Delta C'}{C'} = \frac{\sqrt{\sum_{k \geq p} C_k}}{\sum_{k \geq p} C_k}.
\]

Now we show that the right hand side of equation 9 is larger than the right hand side of equation 10 and, in general,

\[
\frac{\sqrt{\sum_{k \geq p} C_k}}{\sum_{k \geq p} C_k} \geq \sqrt{\frac{\sum_{k \geq p} a_k^2 C_k}{\sum_{k \geq p} a_k C_k}}.
\]

for any non negative \( a_k \) and \( C_k \). The equality applies only if all the \( a_k \) values are equal. Equation 11 implies that any incrementation method which gives different weights to the \( C_k \) terms than the new method (i.e. if not all of the \( a_k \) values are equal to one) will give a larger relative error for the number of counts. In the special case where all the \( a_k \) values are the same but not equal to one the spectrum is essentially the same (multiplied by \( a_k \)) as the spectrum created with the new method.

To prove equation 11 start with the trivial equation

\[
\sum_{k \geq p} \sum_{j \geq p} (a_k - a_j)^2 C_k C_j \geq 0.
\]

From this equation it follows that
\[
\sum_{k \geq p} \sum_{j \geq p} a_k^2 c_k c_j \geq \sum_{k \geq p} \sum_{j \geq p} a_k a_j c_k c_j
\]

(13)

and

\[
(\sum_{k \geq p} a_k^2 c_k)(\sum_{k \geq p} c_k) \geq (\sum_{k \geq p} a_k c_k)^2.
\]

(14)

After rearranging this equation and taking the square root of both sides we arrive at equation 11.

References


Table 1: Percentage contribution of various fold events to the Eurogam Phase 2 data stream for a typical experiment which populated high-spin states in the mass $A \sim 150$ region. The reaction was $^{34}\text{S} + ^{124}\text{Sn} \rightarrow ^{152}\text{Dy} + 6\text{n}$ at a beam energy of 182 MeV. For each fold the data have been further subdivided according to the percentage of the data which pass a set of 15 gates set on the yrast superdeformed band in $^{152}\text{Dy}$.

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Figure Captions

Figure 1. Spectra obtained from the reaction $^{124}\text{Sn}(^{30}\text{Si},5\text{n})^{149}\text{Gd}$ at a bombarding energy of 162 MeV. The data was sorted for a minimum coincidence fold of 5 requiring at least 4 gates to be satisfied within a list of 20 gates. The upper spectrum (A) is created by the traditional sorting procedure while the lower spectrum (B) makes use of the new sorting procedure discussed in the text.

Figure 2 The spectra are created in an identical manner to Fig. 1, but correspond to the higher energy region which is dominated by a smooth $\gamma$-ray continuum.

Figure 3 Figure showing the percentage contribution of each fold to the raw event stream (open circles) and to a gated one dimensional spectrum when at least two (open square), three (closed squares) or four (closed circles) gates are required from a gate list of fifteen gates on the yrast superdeformed band in $^{152}\text{Dy}$. The data are from a Eurogam Phase 2 experiment which populated $^{152}\text{Dy}$ following the reaction $^{124}\text{Sn}(^{34}\text{S},6\text{n})$ at 182 MeV.
Fig. 1
Fig. 3

Percentage Contribution

Fold Distribution

Contribution To Gated Spectra

- Raw fold
- Two Gates
- Three Gates
- Four Gates

Fold

0 10 20 30

0 2 4 6 8 10 12 14