Heisenberg Model and Supersymmetric t - J Model with Two-Band Structure in One Dimension

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We study critical properties of the one-dimensional integrable Heisenberg model and supersymmetric t - J model with two-band structure. We apply conformal field theory analysis to the Bethe-ansatz solution of these models, and compute bulk susceptibilities and critical exponents. The universal formulas obtained for critical exponents can apply for nonintegrable spin and electron models with two-band structure.

There has been much interest in two-band models for highly correlated electron systems, such as the d - p model for high- T_c superconductors and the Anderson lattice model for heavy fermions. As a first step to understand two-band systems, one-dimensional (1D) models have been studied intensively by exact solutions [1], variational methods [2], numerical calculations [3,4], bosonization methods [5], etc. In this paper, we study critical properties of the 1D Heisenberg model and t - J model with two-band structure, by combining conformal field theory (CFT) with the exact solution. Though the present model is rather special, the main results obtained for scaling dimensions can also apply for nonintegrable models. We depict our strategy with the Heisenberg model, and then discuss the two-band supersymmetric t - J model.

Let us start with the Heisenberg spin chain with internal structure,

$$\mathcal{H}_H = J \sum_{i=1}^N \left[\sum_{\sigma, \sigma'=\uparrow, \downarrow} \sum_{m, m'=1}^2 (c_{i\sigma}^{m\dagger} c_{i\sigma'}^{m'} c_{i+1\sigma'}^{m'\dagger} c_{i+1\sigma}^m - n_i n_{i+1}) - \Delta_B (n_i^{(1)} - n_i^{(2)}) \right], \quad (1)$$

where $c_{i\sigma}^{m\dagger}$ is the creation operator for electrons. Both of lower ($m=1$) and upper ($m=2$) bands, separated by the band splitting Δ_B , have two spin states ($\sigma=\uparrow, \downarrow$). The interband charge fluctuation is allowed under the local constraint: the number of electrons being unity at each site, i.e. $n_i = \sum_{m=1}^2 n_i^{(m)} = 1$. One notes that the intraband interaction is of the ordinary Heisenberg type $S_i^m \cdot S_j^m$ with $SU(2)$ symmetry, while the interband interaction takes a rather special form. For these specific couplings, the model coincides with the integrable four-component spin model solved by Sutherland [6]. We proceed our discussions with the specific model (1), and then show that the obtained results can apply more generally for nonintegrable models.

The exact solution of the model (1) was obtained in a set of the algebraic equations for rapidities λ_j^α for $\alpha=1, 2, 3$ [6],

$$N \Theta_2(\lambda_j^\alpha) \delta_{\alpha 1} = 2\pi J_j^\alpha + \sum_{i=1}^{N_\alpha} \Theta_1(\lambda_j^\alpha - \lambda_i^\alpha) - \sum_{s=\pm 1} \sum_{i=1}^{N_{\alpha+s}} \Theta_2(\lambda_j^\alpha - \lambda_i^{\alpha+s}) \quad (2)$$

with $N_0 = N_4 = 0$, where $\Theta_n(x) = 2 \tan^{-1}(nx)$, and the quantum number J_j^α specifies the excitations. The total energy is $E = J[N - \sum_{j=1}^{N_1} (\lambda_j^2 + 1/4)^{-1}] + \Delta_B(N_2 - N_1)$. We denote the

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excitations labeled by $\alpha = 1, 2, 3$ as spinons for simplicity. Following standard techniques [6], we first compute the spin susceptibilities for lower and upper bands χ_m ($m = 1, 2$) and the susceptibility for interband fluctuations defined by $\chi_\Delta = d(n^{(1)} - n^{(2)})/d\Delta_B$,

$$\chi_m = \frac{q_m}{2\pi v_m} \quad (m = 1, 2), \quad \chi_\Delta = \frac{2\xi^2}{\pi v_\Delta}, \quad (3)$$

where $q_m = 1$. The dressed charge $\xi \equiv \xi(B)$ reads

$$\xi(\lambda) = 1 + \int_{-B}^B \frac{d\lambda'}{2\pi} G(\lambda - \lambda') \xi(\lambda'), \quad (4)$$

where the integral kernel in Fourier space is given by $\hat{G}(\omega) = 2\pi[1 - e^{-|\omega|}]/[[1 + e^{|\omega|}]]$, and B is the cut-off parameter related to interband fluctuations (see also eq.(8)). Here v_m ($m = 1, 2$) is the spin velocity of each band, and v_Δ is the velocity of interband excitations. These formulas include some information for conformal properties [7]. The spin susceptibility χ_m with $q_m = 1$ is characteristic of level-1 SU(2) current algebra, while the interband susceptibility χ_Δ with the coefficient $2\xi^2$ reflects U(1) current algebra [8]. We shall confirm this explicitly by conformal dimensions calculated below.

Let us discuss critical properties exploiting the CFT analysis of the spectrum [9,10]. To this end, we first compute finite-size corrections to the ground-state energy,

$$E = E_0 - \frac{\pi}{6N}(v_1 + v_2 + v_\Delta). \quad (5)$$

We can see that excitations are described by three $c = 1$ CFTs [11], which may be referred to as generalized Tomonaga-Luttinger liquids [12-16]. The corrections to the excitation spectrum are similarly computed in a form which satisfies conformal invariance [11], $E = E_0 + (2\pi/N) \sum_j v_j (\Delta_j^+ + \Delta_j^-)$, where $j = 1, 2, \Delta$. As a result we can determine the scaling dimensions for various primary fields, defined by $x_c = \sum_j 2(\Delta_j^+ + \Delta_j^-)$,

$$x_c = \frac{1}{2} \overrightarrow{dN}^t \begin{pmatrix} 2 & -1 & 0 \\ -1 & 1 + \xi^{-2} & -1 \\ 0 & -1 & 2 \end{pmatrix} \overrightarrow{dN} + \overrightarrow{dl}^t \begin{pmatrix} 1 + \frac{\xi^2}{2} & \xi^2 & \frac{\xi^2}{2} \\ \xi^2 & 2\xi^2 & \xi^2 \\ \frac{\xi^2}{2} & \xi^2 & 1 + \frac{\xi^2}{2} \end{pmatrix} \overrightarrow{dl}, \quad (6)$$

where $(\overrightarrow{dN})_\alpha$ is the change of particle number of α spinons, $(\overrightarrow{dl})_\alpha$ is the number of spinons transferred from $-K_F^\alpha$ to K_F^α , and $K_F^\alpha (= \pi N_\alpha/N)$ is the Fermi momentum of α spinons. This scaling dimension is characterized by the momentum transfer, $P = \sum_{\alpha=1}^3 2K_F^\alpha (\overrightarrow{dl})_\alpha$. Note that if we denote the first matrix in (6) as $\mathbf{D}$, then the second reads $\mathbf{D}^{-1}$ reflecting the modular invariance [7,9]. It is seen from (6) that the intraband excitations are described by two independent SU(2) CFTs, which are characterized by the diagonal scaling dimensions $D_{11} = D_{33} = 2$ (the value for the self-dual point). On the other hand, interband excitations are governed by Gaussian CFT with the continuously varying dimension $D_{22} = 1 + \xi^{-2}$. For $\Delta_B = 0$, the model reduces to the SU(4) Heisenberg model, and hence the matrix $\mathbf{D}$ becomes the SU(4) Cartan matrix with $\xi = 1$ [17].

We are now ready to discuss critical properties of the correlation functions. For example, we show a few terms of the asymptotic spin correlator for the lower band,

$$\langle S_Z^{(1)}(x) S_Z^{(1)}(0) \rangle = \frac{A_1}{x^{\alpha_1}} \cos(2k_F^{(1)} x) + \frac{A_2}{x^{\alpha_2}} \cos(4k_F^{(1)} x) + \dots, \quad (7)$$

where $k_F^{(1)} = \pi n^{(1)}/2$ and $k_F^{(2)} = \pi n^{(2)}/2$. We omit for simplicity other terms such as $2k_F^{(2)}$ oscillating one with exponent α_1 and also terms with larger exponents. Taking the quantum numbers $(\overrightarrow{dl})_j = \delta_{1j}, \delta_{2j}$ [16], we obtain α_1 and $\alpha_2 (= 2x_c)$ as

$$\alpha_1 = 1 + \xi^2/2, \quad \alpha_2 = 4(\alpha_1 - 1), \quad (8)$$

where the second formula is the scaling relation for exponents. All the other correlation exponents can be similarly evaluated: for instance the asymptotic behavior of $\langle S_Z^{(2)}(x) S_Z^{(2)}(0) \rangle$ is determined by the same exponents as $\langle S_Z^{(1)}(x) S_Z^{(1)}(0) \rangle$. Furthermore, we can obtain the universal relation between correlation exponents and bulk quantities,

$$\alpha_1 = \frac{3\tilde{\gamma} - \tilde{\chi}_1 - \tilde{\chi}_2 + \tilde{\chi}_c/2}{3\tilde{\gamma} - \tilde{\chi}_1 - \tilde{\chi}_2}, \quad (9)$$

where $\tilde{\gamma}$ is the specific heat coefficient, and all the quantities are normalized at $\Delta_B = 0$. We emphasize here that although the present model has special couplings, the universal formulas obtained for critical exponents (6), (8) and (9) can apply more generally for nonintegrable

two-band spin models with $U(1) \times SU(2) \times SU(2)$ symmetry, for which the $U(1)$ sector describes interband fluctuations. In Fig. 1(a), we show correlation exponents as a function of Δ_B . We observe non-monotonic behaviors in the correlation exponents, which somehow reflect the dispersion relation in the 1D spectrum. Note that the system has three massless spinons for $\Delta_B/J < \log 2$, while for $\Delta_B/J > \log 2$, the orbital degeneracy is completely lifted ($k_F^{(2)} = 0$, $k_F^{(1)} = \pi/2$), and the low-energy physics can be described by the $S = 1/2$ spin model. Therefore at $\Delta_B/J = \log 2$ the critical exponent α_1 changes from 1.5 to 1, while the α_2 term disappears. It has been claimed for heavy fermion systems that the orbital degeneracy is essential to suppress the magnetic order [18]. This can be confirmed in our model: for $\Delta_B/J < \log 2$, the magnetic order is suppressed by rather large exponents due to the orbital effects, but once two bands are completely separated for $\Delta_B/J > \log 2$, the magnetic order becomes more favorable due to the enhanced antiferromagnetic spin correlations with the exponent $\alpha_1 = 1$.

We now turn to the two-band t - J model upon doping holes into Heisenberg model (1),

$$\mathcal{H}_{t-J} = -t \sum_{i=1}^N \sum_{\sigma=1}^2 \sum_{m=1}^2 (c_{i\sigma}^{m\dagger} c_{i+1\sigma}^m + c_{i+1\sigma}^{m\dagger} c_{i\sigma}^m) + \mathcal{H}_H. \quad (10)$$

with the local constraint $n_i < 1$. For $t = J$, the model reduces to the solvable supersymmetric model with two-band structure [6]. Bulk properties of the model were discussed in [1]. We discuss conformal properties here. In this case, there appear additionally the charge degrees of freedom, and consequently the critical behavior is described by two $SU(2)$ CFTs and two $U(1)$ CFTs. Following the above procedures, we have computed scaling dimensions by extending (6) to the 4×4 matrix. For example, the scaling dimension in the region close to the insulator (low dopings) is described by the matrix,

$$\mathbf{D} = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 1 + \xi^{-2} & -1 & \frac{1}{2} - \frac{\xi_B}{\xi^2} \\ 0 & -1 & 2 & -1 \\ 0 & \frac{1}{2} - \frac{\xi_B}{\xi^2} & -1 & \frac{3}{2} + \frac{\xi_B^2}{\xi^2} \end{pmatrix}, \quad (11)$$

where $\xi_B = 1 - n^{(1)}$. In (11), the fourth row and column are related to the charge degrees of freedom. Using this formula, we can compute the critical exponents for correlation functions. For example, $\langle S_Z^{(1)}(x) S_Z^{(1)}(0) \rangle$ has the asymptotics similar to (7), but the corresponding critical exponents are modified by charge fluctuations,

$$\tilde{\alpha}_p = \sum_{i=1}^4 (\mathbf{D}^{-1})_{pi}, \quad p = 1, 2. \quad (12)$$

Note that the matrix formula (11) for scaling dimensions is applicable generally for highly correlated two-band electron models close to the insulating phase. Also, the scaling relation (8) can apply for this case. In Fig.1(b), we show the critical exponents as a function of Δ_B . By comparing Fig. 1(a) and (b), one can see that the spin exponents increase discontinuously by doping holes into the insulator, which may suppress the tendency to the magnetic order: for $\Delta_B = 0$, we have $\alpha_1 = 3/2 \rightarrow \tilde{\alpha}_1 = 13/8$, $\alpha_2 = 2 \rightarrow \tilde{\alpha}_2 = 5/2$, whereas for $\Delta_B/J > \log 2$, $\tilde{\alpha}_1$ takes $3/2$ like ordinary one-band electron models [13]. We have checked that the superconducting correlation can not be dominant in the present supersymmetric model even if we change the band-splitting. Therefore in order to obtain enhanced superconducting correlations [5], we may take into account interband interactions which were dropped in the present model.

In summary we have studied critical properties of the solvable spin and electron models with two-band structure. Though the models are special, the main formulas obtained are general, which can apply even for nonintegrable models.

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FIGURES

FIG. 1. Critical exponents for spin correlation functions: (a) Heisenberg chain, (b) supersymmetric t - J model close to the insulator

Fig. 1