HOW FAR ARE PHYSICAL MESONS FROM CRITICAL MESONS ?

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ABSTRACT

The mass dependence of the chiral phase transition is studied in the linear $SU(3) \times SU(3)$ sigma model to leading order in a $1/N$ -expansion. The chiral transition is washed out for an average pseudoscalar meson octet mass ≥ 203 [MeV]. The corresponding ratio of critical to physical light quark condensates $m_{u,d}^{\text{crit}}/m_{u,d}$ is estimated as ~ 30 %, while it is only ~ 3 % in a mean-field approximation. A ratio close to 1 would be desirable for visible effects in RHIC-experiments. For physical meson masses we find a rapid crossover in the normalized light quark condensate for $181.5 \leq T \leq 192.6$ [MeV]. The entropy density increases by $5.5 \pm 0.8 \cdot 10^{-3}$ [GeV³] in the same temperature interval. Due to large errors in the transition region we cannot exclude a finite latent heat $\Delta L \leq 0.2$ [GeV/fm³]. This bound is only 10 % of the prediction from the bag model equation of state.

1. Introduction

Spontaneous symmetry breaking is frequently used as driving force of phase transitions in QCD. In this talk we focus on the chiral transition at finite temperature and zero baryon number density. In particular we consider the dependence of the order of the chiral transition on the current quark masses of the light and strange quarks.

The order of a phase transition is a basic thermodynamic concept. A phase transition is said to be of first order, if at least one of the first derivatives of a thermodynamic potential Ω with respect to an external field shows a finite discontinuity in the infinite volume limit. It is called to be of second order, if at least one of the second derivatives of Ω diverges with power law singularities in the infinite volume limit. If no non-analyticities occur in Ω , we call the (smooth) conversion from one phase to the other a *crossover phenomenon*.

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The phenomenological consequences of phase transitions strongly depend on the order of the transitions. In applications of finite temperature transitions in QCD a number of signatures has been proposed that should be visible in relativistic heavy-ion collisions (RHIC-experiments). A first order transition with supercooling effects could lead to a prolongation of lifetime of the hot fireball which should be measurable with pion-interferometry¹. Strong supercooling would lead to pronounced fluctuations in the multiplicity distributions in rapidity space dN/dy which lie beyond the statistical noise².

In the other extreme case of a smooth crossover phenomenon it is difficult to think of signatures which are specific for the extended region of phase conversion, unless the crossover phenomenon is still sharp enough in a certain range of temperatures, where it resembles a first order transition. If the rapid change occurs over a temperature interval of say $\Delta T \sim 10$ [MeV], it may still make sense to discuss the experimental manifestations of a “gap” in entropy and energy densities. This is the reason why we have been interested in a quantitative measure for the strength of the crossover phenomenon.

From a theoretical point of view the most appealing situation occurs for a second order chiral/deconfinement transition. Since the singular behaviour in thermodynamic quantities is universal, i.e. it does not depend on microscopic details of the system, a complicated theory like QCD may be replaced by a much simpler model which shares the universality class with QCD. As typical experimental signatures for such a transition in RHIC-experiments clusters of charged and neutral pions have been proposed to be measurable in the detectors, when “disoriented chiral condensates” collapse to the “right” (i.e. temperature adequate) vacuum^{3,4}.

2. Mass Sensitivity of the Chiral Transition

From numerous investigations over two decades it is known that the order of QCD transitions is rather sensitive to the approximation scheme. It depends on the number of colours (N_c), the number of flavours (N_f), the bare coupling g (in the staggered fermion formulation of lattice QCD), the size of the volume and last but not least on the current quark masses in the Lagrangian. Recall that one is ultimately interested just in one case: the physical case of 3 colours, 2 light and 1 heavier flavour ($m_{u,d} \sim 10$ [MeV], $m_s \sim 150$ – 200 [MeV]) in the space-time-continuum for large or infinite volumes.

Much is known about less realistic limiting cases of QCD, in which the quark masses m_q are sent to zero or infinity. In the chiral limit of N_f massless flavours QCD is invariant under $SU(N_f) \times SU(N_f)$ chiral transformations. The phase transition is then driven by the spontaneous breaking of $SU(N_f) \times SU(N_f)$ to $SU(N_f)$. For the particular case of 3 massless flavours one expects from the renormalization group analysis of Pisarski and Wilczek⁵ that the chiral transition is of first order.

Conjectures of this type are no longer conclusive, if the deviations from the symmetric limits of zero (or infinite) quark masses are large. One may argue that the light quark masses $m_{u,d}$ are small compared to the scale set by T_c ($T_c \sim 150\text{--}250$ [MeV]), which is the relevant scale in thermodynamic processes, while $m_{c,b,t}$ are large with respect to T_c . But the strange quark mass m_s is just of the order of T_c and certainly non-negligible in the thermodynamics of strongly interacting gases of hadrons or quarks and gluons.

To get an idea about the influence of finite quark masses on the phase structure of QCD it is instructive to compare QCD with more familiar systems of statistical physics. In Table 1 we give a dictionary between an “external field” and its conjugate order parameter variable for a generic liquid/gas system, a ferromagnet, the chiral transition in QCD and in an effective model for QCD, the linear sigma model.

Table 1. Dictionary of conjugate fields

liquid/gas	ferromagnet	QCD	linear sigma model
p	H	m_q	$\varepsilon_0, \varepsilon_8$
V_s	$-M$	$\langle \bar{q}q \rangle, \langle \bar{s}s \rangle$	$\langle \sigma_0 \rangle, \langle \sigma_8 \rangle$

The pressure p belongs to the specific volume V_s , which serves as an order parameter in a liquid/gas transition. Beyond a critical value for the pressure the liquid/gas transition turns into a crossover phenomenon with smooth changes in all thermodynamic quantities. Similarly the magnetization M is the order parameter associated with an external magnetic field H . Beyond a critical magnetic field it may happen (it does happen for a 3-dimensional-3-state-Potts model) that the first order transition is of second order at the critical field strength, but is completely washed out for larger values of H . A similar role to p and H is played by the current quark masses m_q in QCD. In an effective model for QCD we will use external fields $\varepsilon_0, \varepsilon_8$, which induce finite quark masses on the level of QCD or finite meson masses in the effective model, cf. section 3. As order parameters we have chosen the light and strange quark condensates $\langle \bar{q}q \rangle, \langle \bar{s}s \rangle$ for the chiral transition in QCD and two meson condensates $\langle \sigma_0 \rangle, \langle \sigma_8 \rangle$ in the sigma model. Therefore the question arises as to whether the physical values for the quark/meson masses are still small enough for the chiral transition to persist under the presence of finite masses.

There are essentially two ways in which one can address the question of mass sensitivity of finite temperature transitions in QCD. The first one are Monte Carlo simulations in lattice QCD, the second one are effective models for QCD in the space-time-continuum. The lattice approach has certainly the advantage of starting directly from QCD in its discretized version. On the other hand it stays notoriously difficult in spite of the progress during the last decade to simulate QCD on a lattice which

Fig. 1. Meson mass diagram (m_K over m_π) for the order of the chiral transition. The shaded area corresponds to a mass range leading to 1st order chiral transitions, the dashed line to 2nd order transitions. The physical mass point (open circle) lies in the crossover region. Most part of the diagram is hypothetical.

is fine-grained enough to be close to the continuum limit and large enough to avoid finite size effects.

Effective models in the continuum are not rigorously derived from QCD. Usually they are limited to a certain range of energies and temperatures. The description is restricted to a few degrees of freedom which are thought to be the most relevant ones for the specific aspect of interest.

3. The $SU(3) \times SU(3)$ Linear Sigma Model

In our application we assume that the restoration of the spontaneously broken $SU(3) \times SU(3)$ symmetry is the driving mechanism for the chiral phase transition. The deviations in the spectrum from the idealized octet of pseudoscalar Goldstone bosons are parametrized by terms which break the $SU(3) \times SU(3)$ symmetry explicitly. The assumption in our decimation is that only mesons associated with the $SU(3) \times SU(3)$ multiplets are important for the phase transition. The criterion is chiral symmetry (rather than the size of the meson masses, otherwise one should include ρ -mesons or others as well).

We have focused on the aspect of mass sensitivity of the chiral transition in the framework of effective models. Fig. 1 shows a generic phase diagram for meson masses with and without strange quarks (m_K and others over m_π). The shaded area indicates a region of first order transitions around the chiral limit. The dashed line are mass values which lead to second order transitions. The corresponding mesons are called *critical* mesons. The point with physical values for m_K , m_π (open circle) is depicted in the crossover region not too close to the second order transition boundary, the location anticipates our final results⁷. Up to a few mass combinations (m_K , m_π) the diagram is conjectural similarly to the famous Columbia plot⁶ for bare quark masses.

As a quantitative measure for the distance (in mass parameter space) between the physical and critical meson masses we consider the ratio of the associated critical

to physical light quark masses $m_{u,d}^{\text{crit}}/m_{u,d}$. If this ratio turns out to be much smaller than 1, physical masses lie deeply in the crossover region, and it is difficult to imagine any signatures specific for the phase conversion from the plasma to the hadron phase. The most attractive possibility is a ratio of the order 1. In that case the physical masses are almost “critical”, effects due to a large correlation length should be visible. Non-universal features of the sigma model should be negligible and the decimation of QCD to an effective model in the same universality class is an allowed simplification. As long as the deviations from the ideal second order behaviour are perturbatively small, they are theoretically under control (see e.g. the finite mass scaling analysis of Boyd et al.⁸ or Rajagopal and Wilczek⁴).

We have chosen the $SU(3) \times SU(3)$ linear sigma model as an effective model for the low temperature phase of QCD ($T \leq T_c$) in the vicinity of the chiral limit⁷. In the low temperature phase quarks are confined to hadrons, and chiral symmetry is spontaneously broken. The meson spectrum reflects some remnants of this symmetry breaking, cf. section 2. The linear sigma model has been extensively discussed in the renormalization group analysis of Pisarski and Wilczek⁵ as the most general renormalizable effective model sharing the chiral symmetry properties with QCD. More recently the $SU(2) \times SU(2)$ version of this model has been studied by Wilczek and Rajagopal and Wilczek^{9,4} to discuss theoretical and experimental consequences of a second order chiral transition.

We use the $SU(3) \times SU(3)$ version in a similar spirit as in Refs. 9, 4. Here again it is instructive to recall an analogous concept in statistical physics. Landau’s free energy functional F has been constructed in terms of an order parameter field ϕ to describe the phase structure of a system that admits a second order transition. First order transitions can be described as well, if ϕ^3 - or ϕ^6 -terms are included in F . The $SU(3) \times SU(3)$ linear sigma model can be regarded in a similar way. Its action is constructed in terms of QCD’s chiral order parameter field ϕ , where ϕ is a complex 3×3 -matrix, parametrized as

$$\phi =_{N_f=3} \frac{1}{\sqrt{2}} \sum_{\ell=0}^8 (\sigma_\ell + i \pi_\ell) \lambda_\ell. \quad (1)$$

Here λ_ℓ denote the Gell-Mann matrices, π_ℓ are the pseudoscalar mesons, σ_ℓ are the scalar mesons. The mesonic order parameter field is a bilinear in the left-handed and right-handed quark fields $\phi_{ij} := \langle \bar{q}_i^L q_j^R \rangle$. In the sigma model the quark structure is ignored by construction for all temperatures $T \geq 0$. For low temperatures this approximation is certainly justified. In the vicinity of T_c it becomes questionable, and for temperatures above T_c the sigma model clearly fails as description for the quark-gluon plasma. In terms of ϕ the Lagrangian reads

$$L = \int d^4x \left\{ \frac{1}{2} \text{Tr}(\partial_\mu \phi \partial_\mu \phi^\dagger) \right\} - \frac{1}{2} \mu_0^2 \text{Tr}(\phi \phi^\dagger) + f_1 (\text{Tr} \phi \phi^\dagger)^2 + f_2 \text{Tr}(\phi \phi^\dagger)^2 +$$

$$+ g(\det \phi + \det \phi^+) - \varepsilon_0 \sigma_0 - \varepsilon_8 \sigma_8 \quad (2)$$

Note that there are 2 independent quartic terms with couplings f_1 and f_2 . The det-terms are cubic in the components of ϕ , g is the “instanton” coupling, it takes care on the right η - η' -mass splitting, μ_0^2 is the coupling of the quadratic term. The external field ε_0 gives a common mass to the (pseudo)scalar meson octet, while ε_8 accounts for the right mass splitting inside the (pseudo)scalar meson octet.

4. Tree Level Parametrization at Zero Temperature

The parameters μ_0^2 , f_1 , f_2 , g , ε_0 , ε_8 of the Lagrangian (2) should be chosen such that the model reproduces the experimental values of the (pseudo)scalar meson masses. The parametrization of the sigma model is not unique^{10,11,12}. Here we are interested in a tuning of meson masses in terms of a few parameters. Suitable parameters are the external fields ε_0 , ε_8 . They induce finite quark masses according to

$$\begin{aligned} -\varepsilon_0 &= \alpha(2\hat{m} + m_s) \\ -\varepsilon_8 &= \beta(\hat{m} - m_s), \end{aligned} \quad (3)$$

where $\hat{m} \equiv (m_u + m_d)/2$, α and β are constants. Eq. (3) follows from an identification of terms in the Lagrangians for quarks and mesons, which transform identically under $SU(3) \times SU(3)$. The meson masses are determined for given ε_0 , ε_8 , once the couplings μ_0^2 , f_1 , f_2 , g are specified and the condensates $\langle \sigma_0 \rangle_{T=0}$, $\langle \sigma_8 \rangle_{T=0}$ are calculated for given μ_0^2 , f_1 , f_2 , g . Thus we vary the quark and meson masses by varying ε_0 and ε_8 . The chiral limit is obtained for $\varepsilon_0 = 0 = \varepsilon_8$. The couplings μ_0^2 , f_1 , f_2 , g are then determined from the mass input in the chiral limit, i.e. $m_\pi = m_K = m_\eta = 0$, $m_{\eta'} = 850$, $m_{\sigma_{\eta'}} = 800$, $m_{\sigma_\eta} = 600$, all masses in units of [MeV], $f_\pi = 94$ [MeV]. Next we keep μ_0^2 , f_1 , f_2 , g fixed to their values in the chiral limit and change ε_0 , ε_8 . The choice $\varepsilon_8 = 0$, $\varepsilon_0 \neq 0$ leads to an $SU(3)$ -symmetric case with only one order parameter field σ_0 for which the numerics considerably simplifies. Meson masses with almost experimental values are induced for $\varepsilon_0 = 0.0265$ [GeV³], $\varepsilon_8 = -0.0345$ [GeV³]. We find $m_\pi = 129.3$, $m_K = 490.7$, $m_\eta = 544.7$, $m_{\eta'} = 1045.5$, $m_{\sigma_\pi} = 1011.6$, $m_{\sigma_K} = 1031.2$, $m_{\sigma_\eta} = 1198.0$, $m_{\sigma_{\eta'}} = 749.5$, all masses in units of [MeV]. This way we have constructed a mapping

$$\{m_{u,d}, m_s\} \leftrightarrow (\varepsilon_0, \varepsilon_8) \leftrightarrow \{m_{\text{Meson}}^2\} \quad (4)$$

between quark and meson masses.

It remains to translate the meson condensates at zero temperature to the light and strange quark condensates. In the same way as we obtained the relation (3), we find here

$$\langle \bar{q}q \rangle = \frac{\varepsilon_0}{2\hat{m} + m_s} \langle \sigma_0 \rangle + \frac{\varepsilon_8}{2(\hat{m} - m_s)} \langle \sigma_8 \rangle$$

$$\langle \bar{s}s \rangle = \frac{\varepsilon_0}{2\hat{m} + m_s} \langle \sigma_0 \rangle - \frac{\varepsilon_8}{\hat{m} - m_s} \langle \sigma_8 \rangle . \quad (5)$$

Eqs. (5) are derived at zero temperature. We take these relations as temperature independent and use them to determine $\langle \hat{q}q \rangle(T)$, $\langle \hat{s}s \rangle(T)$ from the measured values for $\langle \sigma_0 \rangle(T)$, $\langle \sigma_8 \rangle(T)$, see below.

5. The Large- N_f -Approach

Ultimately we are interested in the temperature dependence of the meson condensates $\langle \sigma_0 \rangle(T)$, $\langle \sigma_8 \rangle(T)$. The condensates are determined as the minima of the constrained free energy density $U_{\text{eff}}(\sigma_0, \sigma_8)$, i.e. the free energy density under the constraint on the average values

$$\frac{T}{V} \int_0^{1/T} d\tau \int d^3x \sigma_{0,8}(\vec{x}, \tau) = \bar{\sigma}_{0,8} , \quad (6)$$

where $\bar{\sigma}_{0,8}$ are prescribed values for the background fields, while the same average values should vanish for σ_ℓ , $\ell = 1, \dots, 7$ and π_ℓ , $\ell = 0, \dots, 8$. The relation to the full partition function is given by

$$\begin{aligned} Z &= \int d\sigma_0 \int d\sigma_8 \exp\{-\beta \cdot V U_{\text{eff}}(\sigma_0, \sigma_8)\} = \\ &= \int d\sigma_0 \int d\sigma_8 \int \mathcal{D}\phi \text{ constraint} \cdot \exp\left\{-\int d^4x \mathcal{L}[\phi]\right\} , \end{aligned} \quad (7)$$

with ordinary integrals for the $\sigma_{0,8}$ -integrations.

As a next step we quadratize the quartic part of the interaction by applying a Hubbard-Stratonovich transformation^{13,14}. In the simpler case of a scalar ϕ^4 -theory it reads

$$\text{const} \cdot \exp\{-\alpha(\phi^2(x))^2\} = \int_{c-i\infty}^{c+i\infty} \mathcal{D}\Sigma(x) e^{\Sigma^2(x) - \Sigma(x)\phi^2(x)2\sqrt{\alpha}} . \quad (8)$$

The path integral over the auxiliary field $\Sigma(x)$ is evaluated in a saddle point approximation. We replace $\Sigma(x)$ by $\text{sad} \cdot \text{diag}(1,1,1)$. The saddle point approximation corresponds to the leading term in a $1/N$ -expansion in an $O(N)$ -model¹⁵. The $SU(3) \times SU(3)$ linear sigma model reduces to an $O(18)$ -model for $g = f_2 = 0$. The $N = 18$ mesonic modes correspond to $N_f = 3$ flavours ($N = 2N_f^2$). Therefore we call our approximation “large- N_f ”.

The advantage of the large- N_f -approximation is that we end up with an effectively free field theory. The only remnant of the interaction is hidden in the dispersion relation

$$\omega_Q^2 = p^2 + \text{sad} + \mu_0^2 + m_Q^2 \quad (9)$$

Here $Q = 1, \dots, 8$ labels the particle multiplets, m_Q^2 are the mass squares defined by the quadratic terms in the fluctuating fields, sad is the contribution from the auxiliary field.

The final expression for $U_{\text{eff}}(\sigma_0, \sigma_8, \text{sad})$ contains a classical part U_{class} , a part coming from the quadratization U_{saddle} , a zero point energy part which is dropped, and a thermal part U_{th}

$$\begin{aligned} U_{\text{eff}}(\sigma_0, \sigma_8, \text{sad}) &= U_{\text{class}} + U_{\text{saddle}} + U_{\text{th}} \\ U_{\text{th}} &= \frac{1}{\beta} \cdot \sum_{Q=1}^8 g(Q) \int \frac{d^3 p}{(2\pi)^3} \ln(1 - e^{-\beta \omega_Q}) , \end{aligned} \quad (10)$$

where $g(Q)$ are the multiplicities of the multiplets, e.g. $g(1) = 3$ for 3 pions etc. The full expression for U_{eff} can be found in Ref. 7.

The expression for $U_{\text{eff}}(\sigma_0, \sigma_8, \text{sad})$ is evaluated in a high temperature expansion and—alternatively—fully numerically. The high-temperature expansion for a free field theory is standard. Since we are interested in the low temperature phase ($T \leq T_c$), the most we can expect from a high-temperature expansion are qualitative results. The *numerical* evaluation of U_{eff} looks quite straightforward, but it is plagued by imaginary parts in the effective potential. Although the final maximum in sad and the minima in σ_0, σ_8 turn out to lie in the region of real valued U_{eff} , the routines encounter imaginary parts in intermediate steps. The choice of initial values for sad, σ_0, σ_8 has to be optimized to make the numerical evaluation tractable.

Two independent error sources enter the calculation of U_{eff} . The first one is the numerical determination of the saddle point value sad*. We estimate $\Delta \text{sad}^* \sim \pm 0.006 [\text{GeV}^2]$ for $T \sim 190 [\text{MeV}]$ leading to an induced error in $\Delta \langle \sigma_{0,8} \rangle \leq 10^{-3} [\text{MeV}]$. The second one comes from the flat shape of U_{eff} in the transition region. We have

$$\begin{aligned} \Delta \langle \sigma_{0,8} \rangle &\sim 4.3 [\text{MeV}] && \text{for } \varepsilon_0 = 2.5 \cdot 10^{-4} [\text{GeV}^3] \\ \Delta \langle \sigma_{0,8} \rangle &\sim 1.3 [\text{MeV}] && \text{in the chiral limit .} \end{aligned} \quad (11)$$

The numbers in Eq. (11) reflect that the potential becomes more and more flat in the transition region as the external fields approximate their critical values.

6. Critical Meson Masses in Mean-Field

The large- N_f -approximation goes beyond mean-field as it corresponds to a re-summation of a certain subclass of Feynman diagrams¹⁶. Thus it is instructive to compare our results with a pure mean-field calculation, where the full effective potential is replaced by the classical part in terms of two constant background fields σ_0, σ_8 . For simplicity we consider here only the $SU(3)$ -symmetric case, where $\varepsilon_8 = 0 = \sigma_8$, $\varepsilon_0 \neq 0$, σ_0 denotes a constant background field. The effect of a finite (high) temperature in a mean-field calculation is a renormalization of the quadratic term in the Lagrangian. Thus a finite temperature can be mimicked by tuning μ_0^2 while keeping the other couplings f_1, f_2, g fixed. For a critical field $\varepsilon_0^{\text{crit}}$, the first order transition just disappears, and so does the cubic term in U_{class} . At $\varepsilon_0^{\text{crit}}$ U_{class} starts with a term

proportional to $(\sigma_0 - \sigma_0^{\text{crit}})^4$, where σ_0^{crit} is the minimum of U_{class} for critical values $\mu_0^{2\text{crit}}, \varepsilon_0^{\text{crit}}$. Thus we have

$$U_{\text{class}}(\sigma_0) = -\frac{1}{2}\mu_0^2\sigma_0^2 + \frac{2g}{3\sqrt{3}}\sigma_0^3 + (f_1 + \frac{f_2}{3})\sigma_0^4 - \varepsilon_0\sigma_0 \quad (12)$$

$$U_{\text{class}}|_{\text{crit}}(\sigma_0) = \frac{1}{4!} \frac{\partial^4 U_{\text{class}}}{\partial \sigma_0^4} \Big|_{\text{crit}} (\sigma_0 - \sigma_0^{\text{crit}})^4 + O(\sigma_0^5) \quad (13)$$

Here $|_{\text{crit}}$ means “evaluated for critical parameters”. Note that U_{class} in Eq. (12) takes the same form as a free energy functional for a liquid/gas system. It supports the analogy between a liquid/gas system and the chiral transition in QCD as mentioned in the dictionary of Table 1. The vanishing of the first three derivatives in Eq. (13) determines $\sigma_0^{\text{crit}}, \mu_0^{2\text{crit}}$ and $\varepsilon_0^{\text{crit}}$ as functions of f_1, f_2, g . In the physical case of $\varepsilon_0 = 0.0265 [\text{GeV}^3], \varepsilon_8 = -0.0345 [\text{GeV}^3]$ we obtain

$$\frac{m_{u,d}^{\text{crit}}}{m_{u,d}} \sim 0.03 \pm 0.02. \quad (14)$$

Such a small ratio of 3 % for the critical to physical light quark masses would mean that the chiral phase transition is easily washed out by tiny quark masses, and for physical quark masses one is left with a rather smooth crossover phenomenon.

Our interest in the mean-field result is the order of magnitude of this ratio. Recall that a first order transition can have different origins. A first one is a cubic term in the classical part of the potential. A second one is a ϕ^6 -term which may be needed for stabilization of the free energy when the quartic coupling picks up a negative sign due to renormalization effects. For 2 or more independent relevant couplings a further type of first order transition can be a so called *fluctuation induced* transition. Since the linear $SU(3) \times SU(3)$ sigma model contains two such couplings f_1 and f_2 , the chiral transition may be mainly fluctuation induced. This hypothesis has been recently discussed in Ref. 11, see also Refs. 17, 18, 19.

If the order of magnitude of the ratio (14) changes beyond the mean-field level, it sheds some doubts on the simplified description of Eq. (12) and favours the hypothesis of a fluctuation induced transition.

7. Results and Conclusions

As measure for the mass dependence of the chiral transition we have studied the ratio of critical to physical light quark masses $m_{u,d}^{\text{crit}}/m_{u,d}$. For masses larger than the critical ones the chiral transition has turned into a crossover phenomenon. In the large- N_f -approximation the chiral transition is washed out for an average pseudoscalar octet mass $\geq 203 [\text{MeV}]$. In the same approximation we have found a ratio $m_{u,d}^{\text{crit}}/m_{u,d}$ of $\sim 30 \%$, which lies between our mean-field result of 3 % and an estimate from the lattice of $\sim 50 \%$ ^{6,11}. The mean-field and large- N_f -values refer to a

ratio of $m_s/m_{u,d} \sim 18.2$. The intermediate value for the large- N_f -result is plausible, as “large- N_f ” goes beyond “mean-field” due to resummations of Feynman diagrams as mentioned above, whereas lattice Monte Carlo simulations include all quantum fluctuations at once. A ratio of 30 % is certainly not large enough for predicting visible remnants of a nearby second order chiral transition.

There is some hope that the ratio gets closer to 1 if further fluctuations are included in the effective model and the true nature of the chiral transition is fluctuation induced¹¹. We do not expect large subleading corrections in $1/N$ as $N = 18$ in our case. We will rather change the approximation scheme to check the hypothesis of a fluctuation induced chiral transition.

We have measured the weakening of the first order transition in the $SU(3)$ -symmetric case ($\varepsilon_8 = 0$). The gap in the light quark condensate above $T = 177$ [MeV] slowly decreases for finite meson masses. It has disappeared for an external field strength of $\varepsilon_0 = 6.6 \cdot 10^{-4}$ [GeV³]. Similarly the barrier height between the coexisting minima in the effective potential decreases from $1.4 \cdot 10^{-4}$ [GeV/fm³] in the chiral limit to $2.1 \cdot 10^{-6}$ [GeV/fm³] for $\varepsilon_0 = 2 \cdot 10^{-4}$ [GeV³] and $\varepsilon_8 = 0$.

For physical meson masses we find a sharp crossover phenomenon in the light quark condensate between $T = 181.4$ and 192.5 [MeV], $\langle \bar{q}q \rangle_T$ decreases to 50 % of $\langle \bar{q}q \rangle_{T=0}$ over a temperature interval of $\Delta T = 10$ [MeV]. The strange quark condensate $\langle \bar{s}s \rangle_T$ stays almost constant up to a temperature of ~ 200 [MeV].

Similar results are found for the energy density and the entropy density (Fig. 2). Both quantities behave smoothly as a function of T over the entire temperature range up to large errors in the transition region. From these errors we obtain an upper bound on a finite latent heat of 0.2 [GeV/fm³] that would be compatible with our data. The value is in agreement with Leutwyler’s bound of 0.4 [GeV/fm³]²⁰ which has been obtained from Clausius–Clapeyron-relations in the framework of chiral perturbation theory. Further note that it is only 10 % of the value predicted by the naive bag model equation of state. To our knowledge it is yet unclear, whether 0.2 [GeV/fm³] latent heat are sufficiently large to induce measurable signatures in heavy-ion-experiments.

Furthermore we have calculated the difference $\varepsilon(T_2)/T_2^4 - \varepsilon(T_1)/T_1^4 \equiv \Delta\varepsilon/“T_c^4”$. Here $T_1 = 181.5$ [MeV] and $T_2 = 192.6$ [MeV] denote the temperatures, where the rapid crossover sets in and ends, respectively. In a first order transition with a finite discontinuity $T_1 = T_2 = T_c$. Hence we can compare our value for $\Delta\varepsilon/“T_c^4” = 0.29$ with the ratio for the gluonic energy density $\Delta\varepsilon_{\text{gluonic}}/T_c^4$ on the lattice in a pure $SU(3)$ gauge theory. The lattice results are 2.44 ± 0.24 for 4 time slices and 1.8 ± 0.18 for 6 time slices²¹. The dependence on the number of time slices is a clear sign for finite size effects. Still our bound on the chiral contribution to the total gap in energy density over T_c^4 is by an order of magnitude smaller than the gluonic contribution. It should be taken as a warning not to jump to conclusions from our results in view of RHIC-experiments. So far we have only investigated the contribution from chiral symmetry to the full equation of state for a hot hadron gas. The contribution from

Fig. 2. Entropy density s over T^3 , energy density ε over T^4 , and pressure p over T^4 in the large- N_f -approximation for physical meson masses.

gluonic degrees of freedom has been completely left out. It is a question of relative size, which effects in the full deconfinement/chiral transition are dominant.

For further studies of the chiral transition and its interplay with the deconfinement transition gluonic degrees of freedom should be included as well. Dual Ginzburg-Landau models are effective models for gluonic degrees of freedom which seem to be promising candidates also for a unified description of chiral and deconfinement properties (see the contributions of T. Suzuki and H. Toki, these proceedings).

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