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IMPLICATIONS OF GLUON RADIATION EFFECTS AND SOFT TRANSVERSE MOMENTA FOR THE NUCLEON SUBSTRUCTURE IN QCD

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Abstract

The proton and neutron helicity-conserving form factors in the spacelike region are studied within a convolution scheme which takes into account (i) soft-gluon radiative corrections via Sudakov-type form factors, and (ii) the intrinsic $k_{\perp}$ -dependence of the nucleon wave function. Results are presented for a complete set of nucleon distribution amplitudes which comply with QCD sum rules. Implications for the endpoint behavior of nucleon distribution amplitudes are discussed in the context of Appell decompositions. The calculated leading-order perturbative contribution to both form factors falls short compared to existing data, indicating substantial higher-order perturbative corrections (i.e., a large K factor) and/or sizeable soft contributions, e.g., higher-twist effects, at presently accessible momentum-transfer values.

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1. Introduction

The dynamics of exclusive reactions in the context of Quantum Chromodynamics (QCD), and in particular hadronic form factors, is of fundamental importance but has yet not received a satisfactory, settled formulation, mainly because explicit information on the non-perturbative hadronic wave functions is required. The asymptotic, i.e., large Q^2 -behavior of electroweak form factors can be calculated with perturbative QCD using the renormalization group (RG), once factorization (technically, the operator product expansion (OPE)) of the reaction amplitude has been accomplished. Factorization is the property of cross sections (in inclusive reactions) or amplitudes (in exclusive reactions) that high-momenta (“hard”) and low-momenta (“soft”) regions can be disentangled in such a way that the factorized parts depend only on the dynamics specific for the corresponding scale. Then the whole process is described by a convolution and the evolution of the factorized amplitudes is RG-controlled. More important, the hard part of the process in leading logarithmic approximation becomes computable within perturbative QCD. We will call this approach “the standard convolution scheme” (SCS) of exclusive reactions [1,2]. However, factorization at the amplitude level encounters severe infrared (IR) and mass singularities, rendering the application of the RG equation questionable, unless the isolation and resummation of such contributions can be performed in a consistent way (for a recent review and earlier references, see Ref. [3]).

Having extracted a (process-dependent) hard-scattering amplitude, the remaining soft contributions are considered to be part of (universal) hadron wave functions which contain the complicated bound-state dynamics and thus require the application of nonperturbative techniques. Hadron wave functions and electromagnetic form factors can, in principle, be simulated on the lattice [4,5] and continuous effort has been devoted to improve computer algorithms. A useful method to incorporate nonperturbative effects in the continuum came with the development of QCD sum rules adapted to the light cone [2,6–8]. Studies of this sort (for a recent review see, e.g., Ref. [9]) show that the pion [6] and the nucleon [8,10,11] wave

functions integrated over the transverse parton momenta (alias the hadron distribution amplitudes) may have a much more complex structure than previously anticipated [1]. Indeed, hadron distribution amplitudes derived from QCD sum rules show an asymmetric balance of longitudinal momentum fractions of valence quarks, leading to form factors which have the right magnitude and QCD-evolution behavior [12,9]. Moreover, the novel “heterotic” distribution amplitude for the nucleon [11,13,14] describes several (helicity-conserving) exclusive decays of charmonium states into $p\bar{p}$ in excellent agreement with the high-precision data of the E760 experiment at Fermilab [15]. On the other hand, it was pointed out in Ref. [16,17] that such asymmetric distribution amplitudes enhance the endpoint regions of phase space and that excluding these regions, the leading perturbative contribution to the pion and nucleon form factors, calculated with them, is reduced to only a few percent. If so, this behavior would not only prevent a useful factorization, it would also render perturbative QCD inapplicable even at the highest Q^2 values probed so far by experiment.

The last few years have seen substantial advances in our understanding of exclusive reactions, based on the observation by Li and Sterman [18,19] that taking into account gluon radiative corrections within a modified convolution scheme (MCS), which retains the transverse-momentum dependence of virtual partons, the endpoint contributions to exclusive amplitudes, where the RG fails, get suppressed by Sudakov-type form factors [20]. This improves the self-consistency of the perturbative treatment and reinstates the validity of the RG interpretation (see Fig. 1).

This review focuses on these problems and elaborates on some novel theoretical approaches [21–23] concerning the implementation of the Sudakov effect in the nucleon form factors. It is based on a series of lectures and talks, presented by the author in recent meetings [9,24]. The pion form factor within the MCS scheme was recently discussed by Kroll in Ref. [25].

2. Standard vs. modified convolution scheme

Assuming factorization, the helicity-conserving nucleon (magnetic) form factor G_M within the SCS can be written as the product of a hard-scattering amplitude describing the short-distance quark-gluon interactions, and two soft wave functions corresponding to the intact nucleon in the initial and the final state [1]:

$$G_M(Q^2) = \int_0^1 [dx] \int_0^1 [dx'] |f_N(\mu)|^2 \Phi^*(x', \mu) T_H(x, x', Q, \mu) \Phi(x, \mu), \quad (1)$$

where the dimensionful constant f_N represents the nucleon wave function at the origin of configuration space. Here for the sake of simplicity, the factorization and the renormalization scales have been identified in favor of a single scale μ , and Q is the large external momentum. Recall that the renormalization scale is the subtraction point at which the renormalized coupling constant is defined and that the factorization scale marks the interface between soft and hard physics. The nucleon distribution amplitude, being RG-controlled, has the general form

$$\Phi(x_i, Q^2) = \Phi_{as}(x_i) \sum_{n=0}^{\infty} B_n(\mu^2) \exp \left\{ \int_{\mu^2}^{Q^2} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_F [g(\bar{\mu}^2)] \right\} \tilde{\Phi}_n(x_i), \quad (2)$$

where $\Phi_{as} = 120x_1x_2x_3$ is the asymptotic amplitude over longitudinal momentum fractions of the valence quarks and $\tilde{\Phi}_n$ denote the eigenfunctions of the interaction kernel [1]. The projection coefficients

$$B_n(\mu^2) = \frac{N_n}{120} \int_0^1 [dx] \tilde{\Phi}_n(x_i) \Phi(x_i, \mu^2) \quad (3)$$

depend on the nucleon wave function itself and are therefore uncalculable within perturbative QCD. However, their Q^2 -evolution is governed by perturbative QCD and in leading logarithmic approximation (cf. Eq. (2)) one has

$$B_n(Q^2) = B_n(\mu^2) \left[\frac{\ln(Q^2/\Lambda_{QCD}^2)}{\ln(\mu^2/\Lambda_{QCD}^2)} \right]^{-\gamma_n}. \quad (4)$$

[For a more detailed exposition, see Ref. [9] in conjunction with Ref. [10].] As we first pointed out in Ref. [13] (see also Ref. [26]), it is possible to derive an analytic expression which unambiguously relates the expansion coefficients B_n to the *strict* moments of the mixed-symmetry nucleon distribution amplitude $\Phi_N(x_k, \mu^2)$ [6]:

$$\Phi_N^{(i0j)} = \int_0^1 [dx] x_1^i x_2^0 x_3^j \Phi_N(x_k, \mu^2) . \quad (5)$$

That not all moments $\Phi_N^{(n_1 n_2 n_3)}$ are linearly independent is a consequence of energy-momentum conservation on the light cone, i.e., $\sum_{i=1} x_i = 1$, which implies [7]: $\Phi_N^{(n_1, n_2, n_3)} = \Phi_N^{(n_1+1, n_2, n_3)} + \Phi_N^{(n_1, n_2+1, n_3)} + \Phi_N^{(n_1, n_2, n_3+1)}$. Inverting Eq. (5) subject to the orthogonalization constraint of the eigenfunctions $\tilde{\Phi}_n$, the expansion coefficients become rigorously calculable up to any desired order of polynomial expansion M :

$$\frac{B_n(\mu^2)}{\sqrt{N_n}} = \frac{\sqrt{N_n}}{120} \sum_{i,j=0}^{\infty} a_{ij}^n \Phi^{(i0j)}(\mu^2) \quad (6)$$

with $i + j \leq M$. The projection coefficients a_{ij}^n and the normalization constants N_n up to order $M = 4$ have been tabulated in Ref. [9]. Higher orders are given in Ref. [27]. Hence, evaluating the strict moments of the nucleon distribution amplitude up to a given order, one can determine the corresponding expansion coefficients of the same order and vice versa. The interested reader may consult Ref. [9] for an in-depth discussion of these issues.

In the MCS, which retains the explicit transverse-momentum dependence of virtual partons, soft-gluon exchanges that contribute to radiative corrections can be resummed via RG techniques having recourse to the Grammer-Yennie method [28]. Their contributions accumulate in Sudakov-type exponential factors which contain double logarithms – the latter due to the overlap of soft and collinear regions. Thus, formally, the nucleon wave function gets multiplicatively modified by a finite renormalization factor which includes the effects of soft anomalous dimensions, according to

$$\Psi_N^{(mod)} = e^{-S} \Psi_N , \quad (7)$$

where the factor e^{-S} resums contributions from two-particle reducible diagrams (in the axial gauge), whereas two-particle irreducible diagrams (giving rise to single logarithms) are absorbed in the hard scattering amplitude T_H [18–20]. This modified factorization is illustrated in Fig. 2. It is for the remaining genuine soft interactions, those inside the Φ -box, that we need nonperturbative calculations. Here the momenta vary down to zero, the effective coupling constant becomes strong, perturbation theory is not useful, and so a nonperturbative method is required.

In a nutshell, and with subtleties, arguments and indices suppressed, the proton magnetic form factor can be written in the form [19]

$$G_M(Q^2) = \frac{16}{3} \int_0^1 [dx][dx'] \int [d^2 k_\perp][d^2 k'_\perp] \sum_{j=1}^2 T_{H,j} \left(x, x', \vec{k}_\perp, \vec{k}'_\perp, Q, \mu \right) \times Y_j \left(x, x', \vec{k}_\perp, \vec{k}'_\perp, \mu \right). \quad (8)$$

with $[dx] \equiv dx_1 dx_2 dx_3 \delta \left(\sum_{i=1}^3 x_i \right)$ and $[d^2 k_\perp] = \frac{1}{(16\pi^3)^2} \delta^{(2)} \left(\sum_{i=1}^3 \vec{k}_{\perp i} \right) d^2 k_{\perp 1} d^2 k_{\perp 2} d^2 k_{\perp 3}$. One recognizes that this formula appears as an intermediate step in deriving the hard-scattering expression for the proton form factor within the SCS [1]. Taking Fourier transforms with respect to the transverse configuration space, the analogous expression to Eq. (8) in the MCS is written in the form

$$G_M(Q^2) = \frac{16}{3} \int_0^1 [dx][dx'] \int \frac{d^2 b_1}{(4\pi)^2} \frac{d^2 b_2}{(4\pi)^2} \sum_j \hat{T}_j \left(x, x', \vec{b}, Q, \mu \right) \hat{Y}_j \left(x, x', \vec{b}, \mu \right) e^{-S_j}, \quad (9)$$

where the Fourier transform of $f(\vec{k}_\perp) = f(\vec{k}_{\perp 1}, \vec{k}_{\perp 2})$ is defined by

$$\hat{f}(\vec{b}) = \frac{1}{(2\pi)^4} \int d^2 k_{\perp 1} d^2 k_{\perp 2} \exp \left(-i\vec{b}_1 \cdot \vec{k}_{\perp 1} - i\vec{b}_2 \cdot \vec{k}_{\perp 2} \right) f(\vec{k}) \quad (10)$$

and $\vec{b}_3 = \vec{b}_2 - \vec{b}_1$. The diagrams of hard-gluon exchanges in the MCS can be conveniently combined [19] to give ($\bar{x}_i \equiv 1 - x_i$, $C_F = 4/3$)

$$\hat{T}_1 = \frac{8}{3} C_F \alpha_s(t_{11}) \alpha_s(t_{12}) K_0 \left((\bar{x}_1 \bar{x}'_1)^{1/2} Q b_1 \right) K_0 \left((x_2 x'_2)^{1/2} Q b_2 \right), \quad (11)$$

$$\hat{T}_2 = \frac{8}{3} C_F \alpha_s(t_{21}) \alpha_s(t_{22}) K_0 \left((x_1 x'_1)^{1/2} Q b_1 \right) K_0 \left((x_2 x'_2)^{1/2} Q b_2 \right), \quad (12)$$

where K_0 is the modified Bessel function of order 0 (i.e., the Macdonald function) and b_l denotes the length of the corresponding interquark transverse-separation vector. Since there are only two independent transverse-separation vectors (a consequence of dispensing with the $k_\perp$ -dependence of the quark propagators), and due to the rotational invariance of the system with respect to the longitudinal axis, the form factor given by Eq. (9) is a function of only two independent variables $\vec{b}_1 = \vec{b}'_1$ and $\vec{b}_2 = \vec{b}'_2$. Physically, this means that the external probe (i.e., the photon) mediates only such transitions from the initial to the final proton state which have the same transverse configurations of quark distributions.

The arguments of the running coupling constant, t_{ji} , are defined as the maximum scale of either the longitudinal momentum $\propto Q$ or the inverse transverse separation $\propto 1/b_l$, appearing in the argument of K_0 . They are associated with the virtualities of the exchanged gluons, namely,

$$\begin{aligned} t_{11} &= \max \left[(\bar{x}_1 \bar{x}'_1)^{1/2} Q, 1/b_1 \right], \\ t_{21} &= \max \left[(x_1 x'_1)^{1/2} Q, 1/b_1 \right], \\ t_{12} &= t_{22} = \max \left[(x_2 x'_2)^{1/2} Q, 1/b_2 \right]. \end{aligned} \quad (13)$$

In the transverse configuration space, the wave-function part in Eq. (9) is given by

$$\begin{aligned} \hat{Y}_1 &= \frac{1}{\bar{x}_1 \bar{x}'_1} \left\{ 4\hat{\Psi}_{123}^{*'} \hat{\Psi}_{123} + 4\hat{\Psi}_{132}^{*'} \hat{\Psi}_{132} + \hat{\Psi}_{231}^{*'} \hat{\Psi}_{231} + \hat{\Psi}_{321}^{*'} \hat{\Psi}_{321} \right. \\ &\quad \left. + 2\hat{\Psi}_{231}^{*'} \hat{\Psi}_{132} + 2\hat{\Psi}_{132}^{*'} \hat{\Psi}_{231} + 2\hat{\Psi}_{321}^{*'} \hat{\Psi}_{123} + 2\hat{\Psi}_{123}^{*'} \hat{\Psi}_{321} \right\}, \end{aligned} \quad (14)$$

$$\begin{aligned} \hat{Y}_2 &= \frac{1}{2\bar{x}_2 \bar{x}'_1} \left\{ 3\hat{\Psi}_{132}^{*'} \hat{\Psi}_{132} - \hat{\Psi}_{231}^{*'} \hat{\Psi}_{231} - \hat{\Psi}_{231}^{*'} \hat{\Psi}_{132} - \hat{\Psi}_{132}^{*'} \hat{\Psi}_{231} \right\} \\ &\quad - \frac{1}{\bar{x}_3 \bar{x}'_1} \left\{ 4\hat{\Psi}_{321}^{*'} \hat{\Psi}_{321} + \hat{\Psi}_{123}^{*'} \hat{\Psi}_{123} + 2\hat{\Psi}_{321}^{*'} \hat{\Psi}_{123} + 2\hat{\Psi}_{123}^{*'} \hat{\Psi}_{321} \right\} \end{aligned} \quad (15)$$

for the proton [21] and by

$$\begin{aligned} \hat{Y}_1^n &= \frac{1}{\bar{x}_1 \bar{x}'_1} \left\{ -2\hat{\Psi}_{123}^{*'} \hat{\Psi}_{123} - 2\hat{\Psi}_{132}^{*'} \hat{\Psi}_{132} + \hat{\Psi}_{231}^{*'} \hat{\Psi}_{231} + \hat{\Psi}_{321}^{*'} \hat{\Psi}_{321} \right. \\ &\quad \left. - \hat{\Psi}_{231}^{*'} \hat{\Psi}_{132} - \hat{\Psi}_{132}^{*'} \hat{\Psi}_{231} - \hat{\Psi}_{321}^{*'} \hat{\Psi}_{123} - \hat{\Psi}_{123}^{*'} \hat{\Psi}_{321} \right\}, \end{aligned} \quad (16)$$

$$\begin{aligned} \hat{Y}_2^n = & \frac{1}{\bar{x}_2 \bar{x}'_1} \left\{ \hat{\Psi}_{231}^{*'} \hat{\Psi}_{231} + \hat{\Psi}_{231}^{*'} \hat{\Psi}_{132} + \hat{\Psi}_{132}^{*'} \hat{\Psi}_{231} \right\} \\ & + \frac{1}{\bar{x}_3 \bar{x}'_1} \left\{ 2 \hat{\Psi}_{321}^{*'} \hat{\Psi}_{321} - \hat{\Psi}_{123}^{*'} \hat{\Psi}_{123} + \hat{\Psi}_{321}^{*'} \hat{\Psi}_{123} + \hat{\Psi}_{123}^{*'} \hat{\Psi}_{321} \right\} \end{aligned} \quad (17)$$

for the neutron [22]. The Fourier transform of the proton wave function reads

$$\hat{\Psi}_{123}(x, \vec{b}, \mu) = \frac{1}{8\sqrt{N_c!}} f_N(\mu) \Phi_{123}(x, \mu) \hat{\Omega}_{123}(x, \vec{b}) , \quad (18)$$

where the $k_\perp$ -dependent part is modeled by

$$\hat{\Omega}_{123}(x, \vec{b}) = (4\pi)^2 \exp \left[-\frac{1}{4a^2} (x_1 x_3 b_1^2 + x_2 x_3 b_2^2 + x_1 x_2 b_3^2) \right] . \quad (19)$$

The exponentiation of the probability for no-emission of soft gluons in Eq. (9) is expressed in the Sudakov-type form factors e^{-S_j} , calculated by Botts and Sterman [20] on the basis of previous extensive work by Collins, Soper, and Sterman [29]. These form factors lead to a suppression of the IR gluon radiation by virtual colored lines at large mutual transverse separation. This suppression gets stronger with increasing momentum transfer.

The explicit expressions for the Sudakov exponents, including RG effects and next-to-leading terms, may be found in Refs. [18,19]. They are numerically dominated by the double logarithm

$$\frac{2C_F}{\beta_0} \ln \left(\frac{\xi_i Q}{\sqrt{2}\Lambda_{QCD}} \right) \ln \left[\frac{\ln(\xi_i Q / \sqrt{2}\Lambda_{QCD})}{\ln(1/b\Lambda_{QCD})} \right] , \quad (20)$$

where ξ_i is one of the fractions x_i or x'_i , and $\beta_0 = \frac{33-2n_f}{3}$ is the first-order term of the Gell-Mann and Low function. Expression (20) effects the salient advantage of the Li-Sterman approach, as now the interquark separation scales, collectively denoted by b , may serve as *inherent* IR regulators. This marks also the essential difference to previous approaches which considered gluon radiation from *isolated* quarks. In the small- b region, i.e., for $1/b$ large compared to the longitudinal momentum scales $\xi_i Q$, the gluonic radiative corrections are considered as higher-order terms to $\hat{T}_H$ and are excluded from the Sudakov form factor. Consequently, for $\xi_i \leq \sqrt{2}/bQ$ the Sudakov functions $s(\xi_i, b, Q)$ are set equal to zero. On

the other hand, as b increases e^{-S} decreases, reaching zero at $b\Lambda_{QCD} = 1$. In the pion case, there is only one transverse scale, notably, the quark-antiquark separation b , so that suppression is automatically accomplished. Indeed, when it happens that one Sudakov function $s(\xi, b, Q) = 0$ (meaning that the corresponding exponential is set equal to unity) the other (negative) Sudakov function in the exponent, $s(1-\xi, b, Q)$, diverges, thus providing sufficient suppression.

This behavior makes it possible to choose a renormalization scale which depends on the initial and final longitudinal momenta. It was shown in Ref. [30], within the collinear approximation, that such a subtraction point minimizes the contributions of the next-to-leading order corrections, ensuring dominance of the leading-order perturbative contribution. In the SCS, such a choice leads to singularities in the running coupling constant α_s , unless it is saturated by additional IR regulators; e.g., a dynamical gluon mass [10,31]. Here the modification of α_s becomes superfluous because the Sudakov effect inhibits soft-gluon emission, thus effectively *screening* the α_s -singularities. Physically, the Sudakov suppression enhances the dominance of quark configurations with a small color dipole moment and this enhancement becomes more pronounced as Q^2 increases.

Concerning the nucleon, the situation is more complicated because several transverse scales are involved and therefore a careful IR regularization is required. Now the Sudakov function comprises six terms ($\xi_l = x_l$ or $\xi_l = x'_l$ for $l = 1, 2, 3$), each depending on its own transverse scale and with corresponding QCD-evolution contributions, driven by the anomalous dimensions associated with quark self energy. Recall that the IR cutoff to be used is the factorization scale, below which OPE becomes a poor approximation. It is also understood that genuine nonperturbative momenta below the IR cutoff are implicitly accounted for in the nucleon wave function. In order to ensure IR protection in both the longitudinal and the transverse endpoint region, we have investigated in Ref. [21] some optional IR-cutoff prescriptions. We found that to consistently obviate infinities – while still using the $\overline{\text{MS}}$ scheme – one has to correlate the transverse scales and use a single IR cutoff

that is taken to be the maximum transverse separation (“MAX” prescription):

$$\tilde{b} \equiv \max\{b_1, b_2, b_3\} = \tilde{b}_1 = \tilde{b}_2 = \tilde{b}_3 . \quad (21)$$

The quantities $\tilde{b}_i$ are IR-cutoff parameters, naturally related to, but not uniquely determined by the interquark separations b_i , so that different choices are possible. [Note that the factorization scale is $\min\{1/\tilde{b}_i\}$.] The underlying physical idea of the “MAX” prescription is the following: One expects that because of the color neutrality of a hadron, its quark distribution cannot be resolved by gluons with a wavelength much larger than a characteristic interquark separation scale $\tilde{b}_i$. Thus, gluons with wavelengths large compared to the (transverse) hadron size probe the hadron as a whole, i.e., in a color-singlet state and decouple. As a result, quarks in such configurations act coherently and therefore (soft) gluon radiation is dynamically inhibited. The particular advantage of this approach [21,22] is that (i) it suffices to protect the amplitudes from becoming singular for all possible kinematic configurations, and that (ii) it yields a perturbative contribution to the nucleon form factors which *saturates*, i.e., which is rather insensitive to distances of order $1/\Lambda_{QCD}$. Such distances characterize the soft region and cannot be properly accounted for by perturbative means.

To see that other choices (as the one adopted by Li [19]) may lead to uncompensated α_s -singularities, consider a string-like quark configuration in the region characterized by $x_1 < \sqrt{2}\Lambda_{QCD}/Q$ [where $\exp\{-s(\xi_1, \tilde{b}_1, Q)\}$ is fixed to unity] with $\tilde{b}_1\Lambda_{QCD} \rightarrow 1$ and $\tilde{b}_2\Lambda_{QCD} \simeq \tilde{b}_3\Lambda_{QCD} \simeq 1/2$. Then, none of the other two Sudakov functions vanishes and hence the only suppression is only due to QCD evolution. This, however, is not sufficient to compensate the α_s -singularities in the limit $\tilde{b}_1\Lambda_{QCD} \rightarrow 1$. As a result, the integrand in (9) has singularities behaving as $\sim \ln^{-\kappa} \left(\frac{1}{\tilde{b}_i\Lambda_{QCD}} \right)$. The maximum degree of divergence is given by $\kappa = \frac{1}{\beta_0} \left(\frac{4}{3} + 2\tilde{\gamma}_{max} - 2 \right) + 1$, where the first term $4/3$ comes from the evolution of f_N , $\tilde{\gamma}_{max}$ is the maximum value of the quark anomalous dimension within a truncated expansion of the nucleon distribution amplitude in terms of eigenfunctions (cf. Eq. (2)), the term -2 stems from QCD evolution of the Sudakov exponents, and finally, the term 1 originates from

that $\alpha_s(t_{jk})$ which becomes singular in Eq. (9) (cf. Eq. (13)). Which particular α_s becomes singular, depends on the prescription imposed on the IR cutoff parameters $\tilde{b}_l$. For $\tilde{\gamma}_{max} \geq \frac{1}{3}$, the integral (9) is not well defined. The behavior of the Sudakov function $e^{-s(\xi_l, \tilde{b}_l, Q)}$ at fixed momentum transfer is displayed in Fig. 3.

3. Phenomenology

In order to consider concrete applications of the formalism described in the preceding section, explicit and reliable models for the valence-quark structure of the nucleon are needed. In a recent work [23], we have determined a whole set of nucleon distribution amplitudes which satisfy the existing QCD sum rules [8,7] with comparable degree of accuracy. Still more significant is the fact that all these solutions distribute themselves across a “fiducial orbit”, characterized by a scaling relation between the form-factor ratio $|G_N^n|/G_M^p$ and the expansion coefficient B_4 (cf. Eq. (6)). To account for confinement-size effects, we follow Jakob and Kroll [32], and include the intrinsic (soft) transverse momentum of the nucleon wave function in the form of a Gaussian: $\Omega_{123}(x, \vec{k}_\perp) = (16\pi^2)^2 \frac{a^4}{x_1 x_2 x_3} \exp\left(-a^2 \sum_{i=1}^3 k_{\perp i}^2 / x_i\right)$ which multiplies the distribution amplitudes over longitudinal momentum fractions in the calculation of the nucleon form factors. Note that the total nucleon wave function is normalized to unity from which the oscillator parameter a , appropriate for each particular model, is fixed. The results for the magnetic form factor of the proton and the neutron, obtained this way, are shown in Fig. 4 in comparison with data [33]. The shadowed strip corresponds to the predictions extracted from the amplitudes constituting the fiducial orbit.

It is worth noting that imposing the “MAX” prescription, the nucleon form factors accumulate the bulk of their contributions in IR-safe regions. To be specific, approximately 50% of the results are acquired in regions where $\alpha_s^2 \leq 0.54$ for momentum-transfer values starting at 6 to 10 GeV² – depending on which particular distribution amplitude is used (details are given in Refs. [21,22]).

An apparent consequence of retaining the $k_{\perp}$ -dependence in the hard-scattering amplitude (Li and Sterman [18,19]) and in the nucleon wave function (Jakob and Kroll [32]) is a considerable reduction of the (leading) perturbative contribution. On the other hand, the self-consistency of the perturbative treatment is improved because factorization and hence the RG behavior has been restored. Thus, the Sudakov effect, when properly taken into account, eliminates nonfactorizable contributions from endpoint regions and reinstates the validity of the convolution scheme for the calculation of exclusive amplitudes, like hadronic form factors. For calculations based on the MCS of the proton and the pion form factors in the timelike region, we refer the reader to Ref. [34]. A recent analysis of meson-photon transition form factors, which also explores the implications for the shape of the pion wave function, is given in Refs. [25,35].

To effect that the depletion of the perturbative contribution is not the consequence of truncating the nucleon distribution amplitude at the level of second-order eigenfunctions [23], we show in Fig. 5 predictions for the proton and the neutron magnetic form factors calculated with distribution amplitudes, determined by Schäfer [36], which incorporate Appell polynomials of order three. The solid line shows the result for the amplitude Sch II which deliberately incorporates third-order terms. It is clearly obvious that both form factors G_M^p and G_M^n overshoot the data and have a wrong Q^2 evolution. Besides, their saturation behavior deteriorates, for now they still increase at Q^2 as large as 50 GeV^2 and insensitivity to the soft region $b_l \Lambda_{QCD} \rightarrow 1$ sets in at $Q^2 \approx 17 \text{ GeV}^2$ which is much larger compared to the scale found for the second-order amplitudes, notably, $Q^2 \leq 10 \text{ GeV}^2$. As we have discussed in detail in previous publications [9–12], the moment sum rules are not stringent enough to exclude this sort of amplitudes. Additional criteria have to be imposed to filter out physically meaningful solutions. In order that the truncation of the infinite eigenfunctions expansion, given by Eq. (2), is computationally useful, we have to ensure dominance of the lowest-order contributions. Given this premise, the guiding principle is to minimize the influence of higher-order terms which have been discarded. This procedure is analogous to

the optimization of the renormalization-scheme dependence of physical quantities computed in a perturbative scheme. Without enough understanding of the underlying nonperturbative dynamics, it remains a challenge to develop a method of computing hadron distribution amplitudes as a *whole*. For the time being, we must content ourselves with an *effective* description in terms of truncated polynomial expansions, which (i) comply with the QCD sum rules and (ii) are RG-controlled, i.e., satisfy the evolution equation. So if one is satisfied with a given accuracy relative to existing data, reconstructing an analytic representation of higher and higher polynomials is unnecessary – maybe even irrelevant.

Along similar lines of thought, Schäfer [36] has determined a collection of amplitudes which, although including third-order Appell polynomials, effectively resemble those of polynomial order two. The dotted line in Fig. 5 exemplifies the form-factor predictions obtained with such amplitudes. To be specific, the underlying amplitude (denoted Sch IVb) was determined under the proviso of a “smoothness” criterion with the purpose of excluding spurious oscillations by suppressing contributions from higher-order (Appell) polynomials. This criterion is quite restrictive and as one sees the obtained results have the same overall quality as those computed with amplitudes truncated after taking into account bilinear combinations of longitudinal momentum fractions. This indicates clearly that the truncation at the second polynomial order is justifiable, since the shape of the corresponding amplitudes is a characteristic property of the *entire* series and that the errors (and amount of cut-off dependence involved) are of subleading importance if properly incorporated. For completeness, Fig. 6 shows also form factors, computed with an amplitude (termed Sch III) which exhibits unphysical oscillations. The form-factor ratio $-G_M^n/G_M^p$ as a function of the momentum transfer Q^2 for the three amplitudes Sch II, Sch III, and Sch IVb is shown in Fig. 6.

Figure 7 illustrates the influence of gluonic radiative corrections and soft transverse momenta on the fiducial orbit [23]. Implementing these effects, the coherent structure of the orbit is not destroyed but changed in a global way. The solid and dashed lines correlate

to the calculations within the MCS [22] at two different scales, namely, at $Q^2 = 30 \text{ GeV}^2$ and $Q^2 = 10^3 \text{ GeV}^2$, respectively, whereas the dashed-dotted line is the original fiducial orbit. As Q^2 increases and gluon radiation effects become less and less important, the modified orbit approaches more and more the original one. This observation is double-edged: it means not only that the orbit structure is stable and can be used also in the MCS to exclude unphysical solutions, for example, Sch III, but also supports the view that in the axial gauge e^{-S} operates like a finite wave-function renormalization factor as outlined above. This conception may be used reversely to model hadron distribution amplitudes with an improved, i.e., “softened” endpoint behavior relative to those determined by QCD sum rules alone. Work in this direction will be reported elsewhere.

4. Conclusions

It is realistic to believe that the modified convolution scheme – albeit yet insufficient for a complete analytical understanding of the hadron form factors at presently accessible momentum transfers – represents a major development, both conceptually and technically, towards a deeper understanding of exclusive reactions in QCD. To this end, the calculation of higher-order perturbative contributions to the hard-scattering amplitude (expressed in terms of the so-called K-factor) and estimation of higher-twist effects (e.g., inclusion of higher Fock-state components of the hadron wave functions) would be valuable. The conjunction of different approaches – such as QCD sum rules, and the Li-Sterman factorization scheme – offers a promising perspective for a deeper theoretical understanding. The importance of these issues for phenomenological applications makes such conjunctions imperative.

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FIGURES

Fig. 1. Flow chart illustrating two possible ways of realizing factorization in a convolutive description of exclusive processes within QCD.

Fig. 2. Modified factorization of soft-gluon contributions tantamount to a finite renormalization of the nucleon wave function.

Fig. 3. Exponential of the Sudakov function $s(\xi_l, \bar{b}_l, Q)$ vs ξ_l and $\bar{b}_l \Lambda_{QCD}$ for $Q = 30\Lambda_{QCD}$. In the hatched area the Sudakov function is set equal to zero according to Li's requirement [19].

Fig. 4. Proton (lhs) and neutron (rhs) magnetic form factor vs Q^2 with $\Lambda_{QCD} = 180$ MeV. Open circles indicate F_1^p data, adapted from Ref. [33]. The theoretical results are from Refs. [21,22]. They were obtained under the proviso of the “MAX” prescription, including RG evolution and the soft transverse momentum of the nucleon wave function – the latter being normalized to unity. The shaded bands contain the predictions derived for the set of nucleon distribution amplitudes determined in Ref. [23]. The curves correspond to selected model amplitudes: Chernyak-Ogloblin-Zhitnisky (COZ) [8] (solid line), heterotic [11] (dashed line), and optimized COZ [23] (dotted line).

Fig. 5. Plots for the proton and neutron magnetic form factor within the MCS calculated with nucleon distribution amplitudes, determined by Schäfer [36], which include third-order Appell polynomials. As in the previous figure, the “MAX” prescription and the value $\Lambda_{QCD} = 180$ MeV were used.

Fig. 6. Ratio of the nucleon magnetic form factors computed with the Schäfer amplitudes vs the momentum transfer Q^2 .

Fig. 7. Pattern of form-factor ratio $-G_M^n/G_M^p$ vs the coefficient B_4 of the eigenfunctions expansion (cf. Eq. (2)) for the set of nucleon distribution amplitudes determined in Ref. [23], and various model amplitudes denoted by the acronyms of the corresponding authors. Solid and dashed lines are the results within the MCS at $Q^2 = 30 \text{ GeV}^2$ and $Q^2 = 10^3 \text{ GeV}^2$, respectively. The dashed-dotted line shows the original “fiducial orbit” in the form of an empirical fit. For a full discussion, see Ref. [23].

FIGURE 1

FIGURE 2

FIGURE 3

F I G U R E 4

FIGURE 5

FIGURE 6

FIGURE 7