Čerenkov radiation by neutrinos in a supernova core

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Abstract

Neutrinos with a magnetic dipole moment propagating in a medium with a velocity larger than the phase velocity of light emit photons by the Čerenkov process. The Čerenkov radiation is a helicity flip process via which a left-handed neutrino in a supernova core may change into a sterile right-handed one and free-stream out of the core. Assuming that the luminosity of the sterile right-handed neutrinos is less than $10^{38}$ ergs/sec gives an upper bound on the neutrino magnetic dipole moment $\mu_\nu < 0.5 \times 10^{-13} \mu_B$. This is two orders of magnitude more stringent than the previously established bounds on $\mu_\nu$ from considerations of supernova cooling rate by right-handed neutrinos.

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The observation of neutrinos [1] from the supernova SN1987A has provided a number of constraints on the properties of neutrinos [2-4]. Most of the mechanisms for the constraints on the mass and magnetic moment of the neutrinos depend upon the the helicity flip of a left-handed neutrino into a sterile right-handed one which can free-stream out of the core and hence deplete the energy of the supernova core within a timescale of $\sim 1$ sec. Since the observed time-scale of neutrino emission [1] is of the order of 1-10 secs, it is expected that the luminosity of the right-handed neutrinos is less than $10^{53}$ ergs/sec, which is the total neutrino luminosity of the supernova. The mechanism for helicity flip caused by a neutrino magnetic moment that have been considered so far are (i) helicity flip in an external magnetic field of the neutron star in the supernova core [2] and (ii) helicity flip by scattering with charged fermions \textit{i.e.} the processes $\nu_L e^- \rightarrow \nu_R e^-$, $\nu_L p \rightarrow \nu_L p$ [3]. The process (i) leads to an upper bound $\mu_\nu < 10^{-14} \mu_B$ ($\mu_B = \epsilon/2m_e$, the Bohr magneton), but is unreliable since it relies on a high magnetic field ($\sim 10^{14}$ Gauss) in a supernova core which has not been observed. The scattering process (ii) leads to an upper bound $\mu_\nu < (0.2 - 0.8) \times 10^{-11} \mu_B$ [3].

In this letter we propose a third mechanism for the neutrino helicity flip which occurs via a Čerenkov radiation process in the medium of the supernova core. In the supernova core the refractive index of photons is determined by the electric perimitivity of the $e^-$, $p^+$ plasma and the paramagnetic susceptibility of the degenerate neutron gas. We find that Čerenkov emission of a photon from a neutrino is allowed in the photon frequency range $\omega_p \lesssim \omega < 2E/\left(n+1\right)$, (where $\omega_p$ is the plasma frequency, $E$ is the initial neutrino energy and $n$ is the refractive index of the medium). Since the Čerenkov emission process is due to the magnetic dipole operator $\mu_\nu \sigma_{\mu\nu} k_\nu \epsilon_\mu$, it is a helicity flipping process $\nu_L \rightarrow \nu_R \gamma$. The helicity flipping is more efficient in the Čerenkov process because unlike the process (i) there is no dependence on external magnetic field and unlike (ii) it is a single vertex process, so the rate is larger than the scattering rate $\nu_L e^- \rightarrow \nu_R e^-$ by $\alpha_{em} e^{-\bar{\mu}_e/T}$, where the exponential factor is due to the Pauli blocking of the outgoing charged fermion. We compute the luminosity $Q_{\nu_R}$ of the right-handed neutrinos produced by the Čerenkov process. The constraint that
$Q_{\nu_R} < 10^{53}$ ergs/sec (the total observed luminosity) leads to the bound on the neutrino magnetic moment $\mu_\nu < 0.5 \times 10^{-13} \mu_B$. This is two order of magnitude improvement over the previously established bound [3] owing to the fact that the Čerenkov radiation is a single vertex process unlike the scattering processes considered in ref [3].

The amplitude for the Čerenkov radiation process $\nu_L(p) \rightarrow \nu_R(p') \gamma(k)$ is given by

$$\mathcal{M} = \frac{\mu_\nu}{n} \mathcal{A}(p', s') \sigma^{\mu\nu} k_\nu u(p, s) e_\mu(k, \lambda),$$

(1)

where $\mu_\nu$ is the magnetic dipole moment of neutrino and $n$ is the refractive index of the medium. The transition rate of the Čerenkov process is

$$\Gamma = \frac{1}{2E} \int \frac{d^3p'}{(2\pi)^3 2E' (2\pi)^3 2\omega} (2\pi)^4 \delta^{(4)}(p - p' - k) |\mathcal{M}|^2,$$

(2)

where $p = (E, p)$, $p' = (E', p')$ and $k = (\omega, k)$ are the four momenta of the incoming neutrino, outgoing neutrino and the emitted photon respectively. Using the identity

$$\int \frac{d^3p'}{2E'} = \int d^4p' \theta(E') \delta(p'^2 - m_\nu^2),$$

and integrating over the $\delta$ function in (2) we obtain

$$\Gamma = \frac{1}{16\pi} \int \frac{k^2 \, dk}{E^2 \omega^2 n} d(\cos \theta) \delta\left(\frac{2\omega E - k^2}{2|k||p|} - \cos \theta\right) |\mathcal{M}|^2,$$

(3)

where $\cos \theta$ is the angle between the emitted photon and the incoming neutrino.

In a medium with the refractive index $n(= |k|/\omega)$, the $\delta$ function in (3) constrains $\cos \theta$ to have the value

$$\cos \theta = \frac{1}{nv} [1 + \frac{(n^2 - 1)\omega}{2E}],$$

(4)

where $v = |p|/E$ is the particle velocity and $k^2 = -(n^2 - 1)\omega^2$. It is clear that the kinematically allowed region for the Čerenkov process is where $|\cos \theta| \leq 1$. This leads to an upper limit on the range of the allowed photon frequency

$$\omega < \frac{2E}{n + 1},$$

(5)
where we have taken $|p| = E(1 - m^2/E^2)^{1/2} \approx E$ since we are dealing with extremely relativistic neutrinos with $m^2/E^2 < 10^{-6}$. The refractive index of photons in a medium is $n^2 = \epsilon\mu$, where $\epsilon$ and $\mu$ are the electric permittivity and magnetic permeability of the medium. For the case of supernova the medium is a plasma consisting of degenerate electrons, protons and neutrons at temperature $T \approx 60$ MeV [5]. The permittivity $\epsilon$ is given by

$$\epsilon = (1 - \frac{\omega_p^2}{\omega^2}),$$

where $\omega_p$ is the plasma frequency that is determined by the chemical potential of the electrons $\tilde{\mu}_e \approx 280$ MeV and $\omega_p = (4\alpha/3\pi)^{1/2}\tilde{\mu}_e \approx 15$ MeV [6]. The neutrons do not contribute to $\epsilon$, but they do contribute towards $\mu$ due to the paramagnetic susceptibility of the neutron gas which can be treated as a degenerate Fermi gas of magnetic dipoles with para-magnetic susceptibility $\chi$ given by [7]

$$\mu = 1 + 4\pi\chi,$$

with $\chi$ equals to

$$\chi = \frac{(2m_n)^{3/2}\mu_n^2\tilde{\mu}_n^{1/2}}{2\pi^2},$$

where $m_n$, $\mu_n (\approx -1.91e/2m_p)$ and $\tilde{\mu}_n$ being the mass, magnetic moment and chemical potential of neutron respectively. In the supernova core the neutron density is $\rho_n \approx 4 \times 10^{14}$ gm/cm$^3$ and $\tilde{\mu}_n \approx 400$ MeV [5]. The paramagnetic susceptibility is $\chi = 0.084$ and the magnetic permeability $\mu = 1 + 4\pi\chi = 2.055$. Thus the refractive index of the medium becomes

$$n^2 = 1 + 4\pi\chi - \frac{\omega_p^2}{\omega^2}.$$  

For $\chi = 0.084$, we have $n = 1.433$ at $\omega >> \omega_p$. The refractive index is positive definite and the Čerenkov condition (5) is satisfied in the frequency range

$$0.70\omega_p \leq \omega \leq 0.82E,$$

which is the parameter space of the allowed Čerenkov radiation frequency.
Evaluating from (1) we have

$$|\mathcal{M}|^2 = \frac{\mu_\nu^2}{n^2}[-16(p.k)^2 + 12(p.k)k^2 + 16m^2_{\nu}k^2].$$

(10)

Substituting the above expression in (3) and performing the integral over $\delta$ function and using (4) for $\cos \theta$, we have the expression for transition rate for the Čerenkov process [8,9]

$$\Gamma = \frac{\mu_\nu^2}{16\pi E^2} \int_{\omega_1}^{\omega_2} d\omega [2(n^2 - 1)\omega^4 - 16m^2_{\nu}(n^2 - 1)\omega^2],$$

(11)

where the limits of the integral are from (9), $\omega_1 = 0.7\omega_p$ and $\omega_2 = 0.8E$. The Čerenkov process $\nu_L(p) \rightarrow \nu_R(p')\gamma(k)$ changes the $\nu_L$'s to sterile $\nu_R$'s which can free stream out of the supernova core. Neglecting the second term (since $m_{\nu} \sim 1$ eV and $\omega \sim 60$ MeV) in the expression for $\Gamma$, the luminosity of the sterile $\nu_R$'s is

$$Q_{\nu_R} = \frac{V \mu_\nu^2}{8\pi E^2} \int_0^\infty dE [f_\nu(E) - f_\pi(E)] E^2 \int_{\omega_1}^{\omega_2} d\omega (E - \omega)(n^2 - 1)^2\omega^4,$$

(12)

where $f_\nu(E) = [e^{(E-\mu_\nu)/T} + 1]^{-1}$ and $f_\pi(E) = [e^{(E+\mu_\nu)/T} + 1]^{-1}$ are the statistical distribution function of the $\nu_L$ and $\bar{\nu}_L$ in the supernova. Performing the integrals over $E$, the luminosity of right handed neutrinos is

$$Q_{\nu_R} = V \mu_\nu^2(n^2 - 1)^2(0.002)\left[\frac{\bar{\mu}_\nu^7}{T} + \pi^2 T^2 \mu_\nu^{-5} + \frac{7}{3} \pi^4 T^4 \mu_\nu^{-3}\right].$$

(13)

We take the volume $V \approx 4 \times 10^{18}$ cm$^3$, $\mu_\nu \approx 160$ MeV, $T = 60$ MeV [3,5] for the supernova core parameters within 1 second after collapse. The luminosity turns out to be

$$Q_{\nu_R} = 0.164 \times 10^{53} \mu_\nu^2 \ (GeV^4),$$

(14)

in terms of the magnetic moment $\mu_\nu$. Assuming that the entire energy of the core collapse is not carried out by the right handed sterile neutrinos, i.e. $Q_{\nu_R} < 10^{53}$ ergs/sec, we have from (13) the upper bound on the neutrino magnetic dipole moment

$$\mu_\nu < 0.5 \times 10^{-13} \mu_B.$$

This is two orders of magnitude better than the previously established [3] upper bound from the $\nu_R$ luminosity of supernova. The process for generating $\nu_R$ in the supernova core
considered in ref. [3] is via the helicity flip scattering $\nu_L e^{-} \rightarrow e^{-}\nu_R$ and $\nu_L p^{-} \rightarrow p^{-}\nu_R$ etc. This process has an extra electromagnetic vertex and a Pauli blocking factor for the outgoing charged fermion compared to the process that we have considered. That accounts for the more stringent bound we established compared to ref. [3].
REFERENCES


