ABSTRACT

During the dark current measurements on the CTF RF gun, done by R. Bossart et al [1], a slight detuning of the cavity was observed in the presence of an electron beam. At the same time, a non zero reflection coefficient was measured. In this note, we try to explain this phenomenon by beam loading. Computing electron motion in a simplified model we find a qualitatively correct prediction of this effect. But both detuning and residual reflection are a factor 10 smaller than observed.
**Equivalent circuit of the RF gun cavity**

![Equivalent circuit of the RF gun cavity](image)

**Figure 1.** Equivalent circuit of RF gun cavity without beam.

We represent the cavity of the RF gun for the CLIC Test Facility (see e.g. [2]) by the equivalent circuit sketched in Figure 1. The phasor $V_0$ will be proportional to the accelerating axial electric field $E_0$. With

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (1)$$

and

$$Q_0 = \frac{\sqrt{C/L}}{\omega_0} \quad (2)$$

the admittance $Y_0$ of the RF gun cavity as seen from the klystron port is given by

$$\frac{Y_0}{G_0} = 1 + jQ_0 \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \quad (3)$$

For small deviations from $\omega_0$, $\omega = \omega_0 + \Delta \omega$, this becomes

$$\frac{Y_0}{G_0} \approx 1 + jQ_0 \frac{2\Delta \omega}{\omega_0} \quad (4)$$

If the klystron is matched at $\omega_0$ ($G_K = G_0$), the input reflection coefficient $r_0$ is

$$r_0 = \frac{G_0 - Y_0}{G_0 + Y_0} \approx -jQ_0 \frac{\Delta \omega}{\omega_0} \quad (5)$$

It is sketched in the Smith diagram in Figure 2 versus $\omega$.

If $V_0$ is considered as peak value, the power delivered by the klystron and dissipated in the cavity is

$$P = \frac{V_0^2}{2} G_0 \quad (6)$$

**Influence of the electron beam**

In the simplified equivalent circuit of the RF gun cavity, the electron beam would appear as a parallel admittance as sketched in Figure 3.

![Influence of the electron beam](image)

**Figure 2.** Input reflection without beam loading. Smith diagram representation of the input reflection of the RF gun matched at $\omega_0$.

If we ignore the frequency dependence of $Y_L = G_L + jB_L$, we have:

$$\frac{Y}{G_0} = \left( 1 + \frac{G_L}{G_0} \right) \left( 1 + jQ_1 \frac{2(\omega - \omega_1)}{\omega_1} \right) \quad (7)$$

The beam loading will thus have the following effects:

1. The cavity Q will be decreased to

$$Q_1 = Q_0 \frac{\omega_1}{\omega_0} \frac{1 + G_L/G_0}{1 + G_L/G_0} \quad (8)$$

2. The resonant frequency will be detuned to $\omega_1$, where

$$\frac{\omega_1 - \omega_0}{\omega_0} = -\frac{B_L}{2G_0Q_0} \quad (9)$$
3. There is a remaining mismatch even at $\omega = \omega_1$ of

$$r_1(\omega_1) = \frac{-G_L}{2G_0 + G_L}.$$  \hspace{1cm} (10)

By means of (9) and (10), both $G_L$ and $B_L$ can be determined from the measurement of mismatch and detuning. In the following, we will follow a different approach and try to estimate these values.

The reflection coefficient

$$r = \frac{G_0 - Y}{G_0 + Y}$$  \hspace{1cm} (11)

is sketched in the Smith diagram in Figure 4.

![Smith diagram](image)

Figure 4. Input reflection with beam loading

At $\omega_0$, this reflection coefficient is approximately $-(G_L + jB_L)/(2G_0)$.

The beam conductance

The beam admittance $Y_L$ will have a positive real part, $G_L > 0$, if it extracts energy from the RF field. In addition it may have an imaginary part, $B_L$. In an ideal structure, a short bunch of electrons starting when the electric field at the cathode is maximum, would have to see this maximum field on its whole way through the gun. Just like a "surfer" it would ride on (or − for stability reasons − slightly before) the crest of the electromagnetic wave. But this would require a travelling wave with a phase velocity adapted to the changing speed of this electron. For such an "ideal" structure, $Y_L$ could be purely real.

The power absorbed by $G_L$ increases the kinetic energy of the electrons. It can be estimated from the observed peak energy $W$ and the total charge $q$ measured in the time interval $T$:

$$P_{\text{beam}} \approx \frac{V^2}{2} G_L = \frac{1}{2} \frac{q}{T} W e^{-e}.$$  \hspace{1cm} (12)

Here we have assumed that the average energy is by a factor 2 smaller than the observed peak energy. With the measured data, $T = 2\mu$s, $q = 50\, \text{nC}$, and $W = 4.5\, \text{MeV}$, one obtains $P_{\text{beam}} \approx 56\, \text{kW}$. With an input power of $P = 6\, \text{MW}$ from the klystron, we have

$$\frac{G_L}{G_0} \approx 1\%.$$  \hspace{1cm} (13)

With this ratio, the residual reflection coefficient is in the order of 0.5% which is significantly below the observed value.

The beam susceptance

In the following we will discuss why we expect $Y_L$ to have a negative (inductive) imaginary part in the case of the real, standing wave cavity. To this end we consider a coarse model of the cavity. In Figure 5 a) we have depicted the contour function of the real RF gun cavity, overlaid with arrows for the electric field from computer simulations [3]. In Figure 5 b) the field profile $f(z)$ is plotted which will be used here. The axial electric field is given by $E(z, t) = V/\rho(z) \sin(\omega t)$. We use $L_1 = 20\, \text{mm}$, $L_2 = 40\, \text{mm}$, and an intermediate region without field of 12 mm length.

![RF gun cavity model](image)

Figure 5. RF gun cavity model with regions of constant field amplitudes

Electrons will start during the positive half wave of the electric field at the cathode. We will assume that the current emitted from the cathode is a function only of this electric field, and that this function
is given by the modified Fowler-Nordheim characteristic,

\[ \frac{q(E)}{\Delta t} \propto E^{2.5} 10^{-E_1/E}, \quad (14) \]

where we have adapted the parameter \( E_1 \) to the measured value \((E_1 = 123 \text{ kV})\) and \( E = E_z(0, t_0) \) is the electric field at the cathode at the starting time \( t_0 \). The trajectory of an electron \( k \) is given by the system of ordinary differential equations

\[ \frac{d}{dt} p_k = eE_z(z_k, t), \]
\[ \frac{d}{dt} z_k = v_k \quad (15) \]

where

\[ v_k = \frac{p_k}{m_0 \sqrt{1 + \left( \frac{p_k}{m_0 c} \right)^2}} \quad (16) \]

is the instantaneous velocity of the electron \( k \). The initial conditions are \( z_k(t_0) = 0, \ p_k(t_0) = 0 \). The instantaneous power absorbed by electron \( k \) is given by

\[ P_k(t) = q(E_z(0, t_0)) E_z(z_k, t) v_k, \quad (17) \]

and the current as it appears as load to the klystron is

\[ I_k(t) = \frac{P_k(t)}{V} = \frac{q(V_{f_{\text{max}}}) \sin \omega t_0 f(z_k) v_k \sin \omega t}{\sqrt{1 + \left( \frac{p_k}{m_0 c} \right)^2}}. \quad (18) \]

with \( q(E) \) from (14) and \( z_k \) and \( v_k \) from (15) and (16). The currents of all electrons have to be added up. These equations have been integrated numerically for a number of electrons, the result is shown in Figure 6. We have used \( V_{f_{\text{max}}} = 100 \text{ MV/m} \).

In Figure 6 a) we have plotted the trajectories of electrons which leave the cathode during the positive half wave of the cathode voltage. \( \omega z/c \) is plotted versus \( \omega t \). In Figure 6 b) the corresponding velocities \((v/c)\) are plotted. Figure 6 c) finally shows the resulting current these electrons according to (18). For comparison, the voltage \((\propto \sin(\omega t))\) is also plotted.

The Fourier component of this current at \( \omega \) is lagging by \( 32^\circ \) behind the voltage. With \( B_c/G_t = \tan( -32^\circ) \approx -0.6 \) and (13) we have

\[ \frac{B_c}{G_0} \approx -0.6 \%. \quad (19) \]

If we assume \( G_0 \) of \( 10^4 \), this corresponds according to (9) to a detuning of \( 3 \cdot 10^{-7} \) or 1 kHz.

This value is again significantly smaller than the observed detuning of about 10 kHz.

The dark current results from field emission out of all metallic surfaces in the RF gun cavity, not only from the cathode region. Many of the emitted electrons will never exit from the cavity but either hit the cavity walls or move in closed orbits. The measured dark current accounts only for those electrons which exit the gun. Comparing the surface of all potentially emitting surfaces to the cross section of the output tube, one might well expect a factor of 10 between the measured output current and the contribution of the "unseen" electrons to the load admittance \( Y_L \).
References

