Fluctuations and clustering in relativistic heavy ion interactions

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Abstract

The method of scaled factorial moments is used to study the fluctuations in the nuclear fragmentations of ^{197}Au at 10.6A GeV by disentangling the peripheral from more central events. Intermittent behavior is found only in the midcentral processes. Clustering size distributions also exhibit power-law behavior. These observations indicate that the nuclear system might have gone through a phase transition.

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Nuclear systems under various amount of excitation in relativistic heavy ion interactions show a wide variety of different behavior. Of the different regions, one of them is of great interest: the multifragmentation region, where the excitation energy is near the binding energy. Within this region lies the possibility of seeing a phase transition and critical phenomena in the nuclear matter limit. In this region, the initial number of baryons are redistributed into clusters of various sizes and single nucleons. Studies of two-fragment correlation functions [1] indicate that multifragmentation time scales are short, $\tau \approx 100 \text{ fm}/c$, and that fragment emission already sets in during the early non-equilibrium

stages of the reaction with large fluctuations. The occurrence of fluctuations which are large near a critical point are the signatures of a phase transition at that point. The mechanism of how nucleons in a nuclear system at moderate energy are redistributed among clusters (multifragments of charge $Z \geq 3$) is quite complex and is still in debate. Therefore, subtle experimental studies are necessary to understand the nature of copious production of fragments in heavy ion interactions. A cluster is a group of correlated hadrons which are independent of the other hadrons produced in the same process. Previous work on multifragmenting systems at low energy $\approx 1 \text{ A GeV}$ [2-5], over all impact parameters, exhibited large scale fluctuations. Many methods have been developed to analyze the fluctuations and the correlations for various physical quantities. In particular one of the most powerful and promising possibilities seems to be the analysis of event-by-event data in terms of intermittency which is a statistical concept initially developed to study turbulent flows [6] and is now applied in many different fields. This technique was applied to the multifragmenting systems at low energy [2-5] which indicated that fragmentation processes are scale invariant. The analysis of scaled factorial moments reveals the properties

of fluctuations through the magnitudes of the moments and the slopes of the moments. It has been argued [7] that the intermittent signal in the low energy work [2-5] could have arisen from the mixing of events from all impact parameters. So in the present paper, we show (i) the comparison of general characteristics of intermediate-mass-fragments (IMFs) for ^{84}Kr , ^{131}Xe and ^{238}U beams at low energy $\approx 1\text{A GeV}$ with ^{197}Au beam at 10.6A GeV , (ii) the presence of intermittency in the IMFs of ^{197}Au beam at 10.6A GeV from BNL (Expt. No. 875) within the exclusive data of midcentral interactions only and no intermittent signal in peripheral collisions, (iii) the existence of power-law behavior in the clustering size distributions of these beams.

For parts (i) and (iii), we have used four beams: ^{238}U at 0.96A GeV (beam A), ^{197}Au at 10.6A GeV (beam B), ^{131}Xe at 1.22A GeV (beam C) and ^{84}Kr at 15.2A GeV (beam D) and for part (ii), we used only ^{197}Au beam at 10.6A GeV (beam B). The details of these beams were given earlier [3-5, 8]. The charges of the projectile fragments (PFs) were estimated by a combination of different methods, which included grain, gap and δ -density; for very heavy fragments the technique of relative track width [9] was used. The data were collected on an event-by-event basis, where fragmentations corresponding to various impact parameters were extracted proportionally through the number of released projectile protons (N_p) [3]; these protons were determined very carefully [8]. Monotonic relationship between the number of projectile protons (N_p) and the impact parameter (b) for ^{197}Au beam is shown in Fig. 1(a), where b_{max} is the maximum impact parameter as shown previously for ^{238}U beam [4]. For large nuclei such as ^{238}U and ^{197}Au , the intermediate mass fragments (IMF) were considered between $3 \leq Z \leq 30$ and for relatively smaller nuclei, ^{131}Xe and ^{84}Kr , IMF charges $3 \leq Z \leq 15$ were considered. From the ^{238}U beam all the fission events were removed [9] for the present analysis.

The correlation between the average number of IMFs normalized by their maximum values, and the released number of protons normalized by the charge of their respective beam for all the four ions (i.e. the distribution $\langle N_{IMF} \rangle / \langle N_{max} \rangle$ vs N_p/Z_{beam}) is shown in Fig. 1(b). All four beams have the general characteristics of rise and fall, passing through their maximum which was first observed by the ALADIN Collaboration [1]. The peak in all cases is approximately in the mid center. The underlying mechanism responsible for the magnitude of these IMFs is strongly correlated with the excitation energy of the projectile through the number N_p [8]. Intermittency, as discussed in a later section (ii) for ^{197}Au beam, is generally sought for systems exhibiting larger than Poisson fluctuations (for which $\sigma^2 / \langle N_{IMF} \rangle = 1$). All four projectiles are of heavy mass and hence they show a broad multiplicity distribution of their fragments. In Fig. 1(c) we show the distribution of the ratio of $\sigma^2 / \langle N_{IMF} \rangle$ as a function of the centrality (N_p/Z_{beam}) for all four beams. We find that the ratio of $\sigma^2 / \langle N_{IMF} \rangle$ varies from 2 to 0.5 over all parameters. Recently, Lee and Mekjian [10] have done work on charge spacing distribution for ^{238}U and ^{197}Au fragments for small spacings and have found that the data follow the Wigner distribution rather than Poisson distribution indicating that the process may have some chaotic mechanism.

The fluctuations that cause the break up of a highly excited system into many fragments have been studied very extensively in the last few years. It has been shown recently that the projectile-fragment size distribution also shows signs of intermittency [2-5] and the emulsion data have played an important part in this work. We may mention that the values of moments and the intermittency slopes depend upon the selection criteria for the events. Recently, it has been argued [7,11] that the scaled factorial moments (SFM) signal may be due to the mixing of fragments produced under different initial condi-

tions (i.e. different impact parameters). In order to resolve this problem, we separated the peripheral events from the central collisions in the ^{197}Au beam at 10.6A GeV by using the number of released protons (N_p) from the projectile [3-5,12]. Generally, the occurrence of intermittency corresponds to the existence of large nonstatistical fluctuations which have self-similarity over a broad range of scales. This signal can be deduced from the SFM F_q of order q which measures the properties of dynamical fluctuations without the bias of statistical fluctuations and are calculated [6,12] using

$$\langle F_q \rangle =$$

$$\frac{1}{\langle n_m \rangle^q} < \frac{1}{M} \sum_{m=1}^M n_m(n_m-1)\dots(n_m-q+1) \rangle \quad (1)$$

where, $\langle n_m \rangle = \frac{1}{M} \sum_{m=1}^M n_m$ for events with mean fragment multiplicity $\langle n_m \rangle$ in the fragment charge interval Δs , which is divided into M bins of size $\delta s = \Delta s/M$. The number of fragments in the m th bin ($m = 1, 2, \dots, M$) is n_m . For self-similar fluctuations at all scales δs , the factorial moments of the order q are given by $\langle F_q \rangle = (\Delta s/\delta s)^{\alpha_q}$. The exponent α_q is the slope characterizing a linear rise of $\ln \langle F_q \rangle$ with $-\ln \delta s$ for all bins of δs . The exponent α_q increases with increasing order q of the moment; however, for random uncorrelated particle production, $\langle F_q \rangle$ should be constant for all values of q . For non-flat fragments, multiplicity distributions varying within a finite bin of width δs introduce an extra M -dependent correction factor R_q which is given by [13] $R_q = \frac{1}{M} \sum_{m=1}^M M^q \langle n_m \rangle^q / \langle N_{fr} \rangle^q$, where $\langle n_m \rangle = \frac{1}{N_{events}} \sum_{i=1}^{N_{events}} n_{m,i}$ and $\langle N_{fr} \rangle$ is the average fragment multiplicity in the interval Δs . Thus, $\langle F_q \rangle / R_q = \langle F_q^c \rangle$ measures the properties of dynamical fluctuations.

Previously with a minimum-bias sample, we have shown a Campi scatter plot for IMFs in Fig.

4(b) of Ref. [8] which measures the correlation between second moment and the largest fragment of an event. The events which fall within the critical region of the scatter plot might be associated with a second order liquid-gas phase transition in nuclei. The critical region is the area where most of the multifragments might show the presence of intermittency. For peripheral events with $N_p \leq 20$, we show in Fig. 2(a) the distribution of the corrected $\langle F_q^c \rangle$ as a function of $-\ln \delta s$. The values of the slopes of α_q of the fitted lines for different orders of $q \leq 7$ are either negative or near zero, with no intermittent signal. On the other hand, midcentral events with $20 < N_p \leq 65$ [Fig. 1(a)], SFM $\langle F_q^c \rangle$ using events with $N_{fr} \geq 3$ with charges $Z \geq 3$ including protons and α -particles show a clear intermittency signal in Fig. 2(b). The values of α_q are $\alpha_2 = 0.0065 \pm 0.0009$, $\alpha_3 = 0.0089 \pm 0.0012$, $\alpha_4 = 0.0115 \pm 0.0015$, $\alpha_5 = 0.0142 \pm 0.0018$, $\alpha_6 = 0.0170 \pm 0.0021$, $\alpha_7 = 0.0200 \pm 0.0025$ and $\alpha_8 = 0.0231 \pm 0.0028$. All α_q are positive. For a system having only finite number of critical indices, one expects that the corresponding intermittency indices would depend linearly on the order of the moment. For a phase transition the suggested relation [13] $\alpha_q = \alpha_2 q/2$ works reasonably well with the present results, within their statistical errors.

Now we look at the dynamical mechanisms that effect the growth process of cluster formation as suggested in Ref. [15]. Each cluster which is leaving the boundary contributes to the cluster size distribution. Suppose that $n_i(Z)$ is the number of clusters of size Z , after all emerging clusters are collected for the event and N_i is the total number of clusters of all sizes in the i th event (i.e. $N_i = \int n_i(Z) dZ$), where $P(Z)$ is the average distribution for N_{ev} and is defined by

$$P(Z) = \frac{1}{N_{ev}} \sum_i^{N_{ev}} n_i(Z) / N_i. \quad (2)$$

In this process no thermal equilibrium is required

and no equation of state is assumed. In order to look at the distribution of cluster sizes, we used all four beams (A, B, C and D), where the charges of all the fragments were determined as discussed earlier. For the present analysis, we used charges $2 \leq Z \leq 30$ for beams A and B and $2 \leq Z \leq 15$ for beams C and D. The individual $\ln P(Z)$ distributions as a function of $\ln Z$ of beams A and B have approximately the same slopes, as shown in Fig. 3(a), with a linear relation having a slope $m = -1.544 \pm 0.116$. Similarly, in Fig. 3(b) we show the plot for beams C and D together, whose slopes are almost similar with $m = -1.803 \pm 0.223$. In Fig. 3(c) we give the combined $\ln P(Z)$ distribution for the four beams together, which is linear up to their finite sizes and is given by $P(Z) = -(1.641 \pm 0.095)Z - 1.4$ on a log-log plot. $P(Z)$ possesses a power-law behavior given by $P(Z) \propto Z^{-\tau}$, where $\tau \approx 1.6$ is the critical exponent. This power-law behavior, a common feature observed among all the beams, as shown in Fig. 3(c) alone, can not be considered as conclusive proof for a critical behavior that recalls a phase transition in a finite system. Self-similarity, as shown earlier for ^{197}Au beam in Fig. 2(b), is also a power law behavior of the function. Since scaling is the underlying mechanism of critical phenomena, which is revealed through intermittency analysis in ^{197}Au beam, as shown in Fig. 3(b), together with a power-law behavior observed in Fig. 3(c), it is reasonable to consider that a phase transition might have occurred at least in ^{197}Au beam in mid-central collisions. Any dynamical process which could explain the production of such large clusters would be of great help.

To emphasize the fluctuations rather than the average size Z , one can calculate the normalized moments $C_2 = \langle Z^2 \rangle / \langle Z \rangle^2$, where $\langle \dots \rangle$ denotes averaging over $P(Z)$. These moments are frequently used to compare multiplicity distributions (n_s) for different processes at different energies (for distributions with different

$\langle n \rangle$). Here one should not expect power-law behavior of C_q as a function of resolution in size as observed in intermittency in Fig. 2(b). In Fig. 4(a) is shown the normalized C_2 -moments for a minimum-bias sample of data for ^{238}U and ^{197}Au beams and in Fig. 4(b) for ^{131}Xe and ^{84}Kr beams. In all cases, the C_2 moments show linearity in the beginning up to $Z \approx 20$ for ^{238}U , $Z = 10$ for ^{197}Au , $Z = 10$ for ^{131}Xe and $Z = 8$ for ^{84}Kr and after that all cases show different fluctuations in C_2 values. The limit on the linear behavior of the C_2 function for a beam is determined by the fragment charge distribution; the larger the charge, the longer is the interval over which the C_2 moments are linear. The linear behaviors are shown in all cases until the finite size effects cut off the approximate linearity. The values of C_2 moments varied differently for different projectiles for the same charge Z parameter after certain Z -values. In general, the fluctuations observed are more important than expected from superposition of independent sources corresponding to elementary collisions. This also implies that the observed correlations are due to the collective effect (collectivity) which we have also observed in heavy fragments [16].

The source of fluctuations in the fragment size distribution which are largest near the critical point has also been discussed by Campi [17] who uses the percolation model. In Fig. 4(c), we plot for the ^{197}Au beam, the average distribution of the largest fragment produced per event as a function of n , the reduced multiplicity $n = m/Z_0$ (here m is the measured charge $-Z_{max}$ and this is normalized by the total charge Z_0). Fig. 4(d) represents the fluctuations (standard deviation, σ) of that quantity as function of n . This is the direct measurement of the critical fluctuations near the transition point. Campi has shown that the deviations (σ) peak at the critical n_c . The fluctuations in the fragment size distributions are largest near the critical point and this is also exhibited in Fig. 4(a).

We conclude that the observation of intermit-

tency in the exclusive data of multifragmentation of a ^{197}Au beam at 10.6A GeV in midcentral collisions, can be interpreted in a natural way as an evidence for self-similarity and scaling in the decompressed phase of evolution preceeding the fragmentation of a hot residue in relativistic heavy ion collisions. Thus the presence of large fluctuations as indicated by the intermittency analysis plus the power-law observed in the charge distribution of ^{197}Au beam indicate a self-similar behavior both for fluctuations and for averages. These combined features are reminiscent of a phase transition. The present observations on the ^{197}Au beam at high energy will most likely have important consequences for the theories of dynamics of heavy ion collisions.

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Figure Captions

Fig. 1 (a) Relationship between the impact parameter b/b_{max} and the proton multiplicity N_p . (b) Variation of $\sigma^2 / \langle N_{IMF} \rangle$ with N_p/Z_{beam} for ^{197}Au at 10.6A GeV,

^{238}U at 0.96A GeV, ^{84}Kr at 1.52A GeV and ^{131}Xe at 1.22A GeV. (c) A plot of $\langle N_{IMF} \rangle / \langle N_{max} \rangle$ vs. N_p/Z_{beam} for the above four beams.

Fig. 2 The variation of $\ln \langle F_q^c \rangle$ as a function of $-\ln \delta s$ for the ^{197}Au events with: (a) $N_p \leq 20$ and (b) $20 < N_p \leq 65$ and having $N_f \geq 3$ of $Z \geq 3$. The solid lines in these figures correspond to the best fits.

Fig. 3 $\ln P(Z)$ as a function of $\ln Z$: (a) for ^{197}Au at 10.6A GeV and ^{238}U at 0.96A GeV, (b) for ^{84}Kr at 1.52A GeV and ^{131}Xe at 1.22A GeV, and (c) for all these four beams together. Solid line shows the best linear relation.

Fig. 4 C_2 as a function of Charge, Z: (a) for ^{197}Au and ^{238}U beams, and (b) for ^{84}Kr and ^{131}Xe beams. For the ^{197}Au data: (c) a plot of $\langle Z_{max} \rangle / Z_{beam}$ vs n and (d) σ as a function of n .

Jain and Singh Fig. 1

Jain and Singh Fig. 2

Jain and Singh Fig.4

Jain and Singh Fig.3