SOLAR NEUTRINOS AND LEPTON MIXING
A. Yu. Smirnov
International Centre for Theoretical Physics, I-34100, Trieste, Italy and
Institute for Nuclear Research, RAS, 107 370 Moscow, Russia

ABSTRACT

With latest experimental data the solar neutrino problem enters new phase when crucial aspects of the problem can be formulated in an essentially (solar) model independent way. Original neutrino fluxes can be considered as free parameters to be found from the solar neutrino experiments. Resonance flavour conversion gives the best fit of all experimental results. Already existing data allow one to constraint both the neutrino parameters and the original neutrino fluxes. The reconciliation of the solution of the solar neutrino problem with other neutrino mass hints (atmospheric neutrino problem, hot dark matter etc.) may require the existence of new very light singlet fermion. Supersymmetry can provide a framework within which the desired properties of such a light fermion follow naturally. The existence of the fermion can be related to axion physics, mechanism of $\mu$-term generation etc..
1. Introduction.

There are three phases in understanding of the solar neutrino problem:
1. Theory without experiment. The problem was predicted by B. Pontecorvo: Even before the first Homestake results he suggested that neutrino oscillations can influence the solar neutrino fluxes, diminishing the detected signals.
2. Theory and Experiment. During more than 20 years we had one experiment and one model. Namely, Davis’s experiment and Bahcall’s predictions of neutrino fluxes in the Standard Solar Model (SSM). The problem was formulated as the smallness of the Homestake signal in comparison with Bahcall’s prediction.
3. Experiment without theory. Results (now rather precise) from Homestake\(^1\), Kamiokande\(^2\), SAGE\(^3\) and GALLEX\(^4\) experiments as well as calibration of Kamiokande and recent GALLEX experiment with \(^{51}\)Cr source allow one to formulate the problem in almost solar model independent way.

There is some hope that forthcoming experiments SuperKamiokande and SNO will resolve the problem (at least establish, finally, whether the astrophysics or neutrino properties are responsible for the observable deficit) without referring to the predictions of the specific solar models.

In this paper we summarize essential points of this third phase and consider some implications of the data to the lepton mixing.


In spite of serious progress in the solar modeling and very good agreement of SSM and helioseismological data, the predicted solar neutrino fluxes still have rather large uncertainties. Mainly, they are related to the nuclear cross-sections (first of all, for the reaction \(p + ^7\text{Be} \rightarrow ^8\text{B} + \gamma\)) and probably to some plasma effects which have not yet been properly taken into account. These uncertainties will hardly be fixed before new experiments on solar neutrinos start to operate. In this connection it is instructive to use as much as possible model independent approach to the problem, and try to resolve it using solar neutrino data only. Main points of the approach are the following\(^5\)–\(^18\).

1. Only general notion is used about the solar neutrinos: the composition, the energy spectra of components, but not the absolute values of fluxes. These absolute values are considered as free parameters to be found from the solar neutrino experiments. In particular, the boron neutrino flux can be written as

\[ \Phi_B = f_B \cdot \Phi_B^{SSM}, \]  

where \(f_B\) is free parameter, and \(\Phi_B^{SSM}\) is the flux in the reference SSM\(^{19}\). Similarly, parameters \(f_i\) (\(i = \text{Be, pp, NO}\)) for other important fluxes can be introduced.
2. One confronts the data from different experiments immediately.
3. The normalization of the solar neutrino flux is used which follows from the solar luminosity
at the condition of thermal equilibrium of the Sun.

In fact, present experimental situation makes the analysis of data to be very simple. There are two key points in this analysis.

**Kamiokande versus Homestake**\(^5\)\(^{\text{-}16}\). Boron neutrino flux measured by Kamiokande gives the contribution to Ar-production rate \(Q_{Ar}^B = 3.00 \pm 0.45\) SNU \(^1\) which exceeds the total signal observed by Homestake: \(Q_{Ar}^B = 2.55 \pm 0.25\) SNU. This means that the contributions of all other fluxes to \(Q_{Ar}\), and in particular, of the Beryllium neutrinos should be strongly suppressed.

**Gallium Experiment Results versus Solar Luminosity**\(^7\)\(^\text{-}20\),\(^17\). The luminosity of the Sun allows one to estimate the pp-neutrino flux and consequently its contribution to Ge-production rate: \(Q_{Ge}^{pp} \approx 71\) SNU. This value plus small (\(\sim 5\) SNU) contribution of boron neutrinos coincide within \(1\sigma\) with total signal observed by GALLEX and SAGE. Consequently, gallium results can be reproduced if the beryllium neutrino flux as well as all other fluxes of the intermediate energies are strongly suppressed.

Thus both these points indicate on strong suppression of the \(^7\)Be-neutrino line. Statistical analysis gives \(f_{Be} < 0.4\) \((2\sigma)\) (for more detail see\(^13\),\(^14\),\(^15\)).

It follows from the above consideration that the data fix uniquely values of fluxes which give the *best fit*\(^17\):

1. Boron neutrino flux should be \(\approx 0.4\Phi_{B}^{SSM}\).
2. Beryllium neutrino flux as well as other fluxes of the intermediate energies (pep, N, O) give negligible contributions to the signals.
3. There is little or no suppression of the pp-flux.

Moreover, to reproduce central values of signals one should suggest that there is an additional flux which contributes to the Kamiokande signal, \(\Delta\Phi_{B} \approx 0.09\Phi_{B}^{SSM}\), but does not contribute to the Ar-production rate. Any deviation from this picture gives a worse fit. Thus the energy dependence of the suppression factor \(P(E)\) can be represented as

\[
P(E) \sim \begin{cases} 
0.9 - 1 & E < 0.5 \text{ MeV} \\
0 & E \sim 0.7 - 1.5 \text{ MeV} \\
0.4 - 1 & E > 7 \text{ MeV} 
\end{cases}
\] (2)

Large uncertainty of the suppression in high energy region is related to the uncertainty in the original boron neutrino flux. Kamiokande admits a mild distortion of the recoil electron spectrum.

Evidently the astrophysics can not reproduce such a picture\(^5\),\(^6\),\(^8\),\(^15\). Typically one gets more strong suppression of the boron neutrino flux than the beryllium neutrino flux.

\(^1\)In this estimation it was suggested that the contribution to the Kamiokande follows from the electron neutrinos only.
3. Neutrino parameters and neutrino fluxes

There are several recent studies of the particle physics solutions of the solar neutrino problem for unfixed values of original fluxes\(^{16,17,18,21,22}\).

Long length vacuum oscillations can reasonably reproduce the desired suppression factor. For \(\Delta m^2 > 7 \cdot 10^{-11} \text{ eV}^2\) the pp-neutrino flux is in the region of averaged oscillations, where \(P = 1 - 0.5 \sin^2 2\theta\), the Beryllium neutrinos are in the fastly oscillating region of the \(P(E)\) (so that one expects an appreciable time variations of the Be-neutrino flux due to annual change of distance between the Sun and the Earth). Boron neutrinos are in the first (high energy) minimum of \(P(E)\). This allows one to reach the inequality \(P_{\text{pp}} > P_B > B_{\text{Be}}\) implied by (2). However, there is an obvious relation between maximal suppression of the Be-line and the suppression of pp-neutrinos: \(P_{\text{Be,min}} = 2P_{\text{pp}} - 1\), and due to this the best fit configuration (2) can not be realized.

With diminishing \(f_B\) the needed suppression of B-neutrino flux due to the oscillations becomes weaker. Therefore for fixed values of \(\Delta m^2\) the allowed regions of parameters shift to smaller \(\sin^2 2\theta\)\(^{21,22}\). In particular, for \(f_B = 0.7\), the region is at \(\sin^2 2\theta < 0.7\) thus satisfying the potential bound from SN87A\(^{23}\). For \(f_B \sim 0.4\) the mixing can be as small as \(\sin^2 2\theta < 0.5 - 0.6\). Moreover, for \(f_B = 0.5\) the allowed region appears at \(\Delta m^2 \sim 5 \cdot 10^{-12} \text{ eV}^2\) which corresponds to a position of the Be-neutrino line in the first high energy minimum of \(P\). Depending on neutrino parameters and \(f_B\), \(f_{\text{Be}}\) ... one can get a variety of distortions of the boron neutrino energy spectrum.

Being excluded at \(f_B = 1\), the oscillations into sterile neutrino are allowed for \(f_B < 0.7\)\(^{22}\).

Resonance flavour conversion can precisely reproduce the desired energy dependence of the suppression factor (2). In the region of small mixing angles:

\[
P_{\text{pp}} \sim 1, \quad P_{\text{Be}} \sim 0, \quad P_B \sim \exp(-E_{\text{na}}/E),
\]

where \(E_{\text{na}} \equiv \Delta m^2 l_n \sin^2 2\theta\). An additional contribution to Kamiokande \(\Delta f \approx 0.09\) follows from scattering of the converted \(\nu_\mu\) (\(\nu_\tau\)) on electrons due to the neutral currents. As the result: \(R_{\nu e} \sim f_B [P_B + 1/6(1 - P_B)]\). With diminishing \(f_B\) the desired suppression due to conversion relaxes, and therefore \(\sin^2 2\theta\) decreases according to (3)\(^{17,18}\). At \(\Delta m^2 = 6 \cdot 10^{-6} \text{ eV}^2\) the best fit of the data for flavour mixing corresponds to (7):

\[
\begin{array}{cccccc}
\hline
f_B & 0.4 & 0.5 & 0.75 & 1.0 & 1.5 & 2.0 \\
\sin^2 2\theta & 1.0 \cdot 10^{-3} & 1.8 \cdot 10^{-3} & 4.3 \cdot 10^{-3} & 6.2 \cdot 10^{-3} & 9 \cdot 10^{-3} & 10^{-2} \\
\hline
\end{array}
\]

(For \(f_B \sim 0.38\) the best fit is at \(\Delta m^2 = 4 \cdot 10^{-6} \text{ eV}^2\). The decrease of \(f_{\text{Be}}\) gives an additional shift of the allowed region to smaller values of \(\sin^2 2\theta\). A consistent description of the data has been found for\(^{17}\)

\[
f_B \sim 0.4 - 2.0.
\]

(see (4) and table in (5)). For unfixed values of the original fluxes, \(f_B\), \(f_{\text{Be}}\) ..., the allowed region of neutrino parameters is controlled immediately by Gallium data and by the “double
Namely, the mass squared difference

$$\Delta m^2 = (6 \pm 2) \cdot 10^{-6} \text{eV}^2,$$

is restricted essentially by results from Gallium experiments which imply that the adiabatic edge of the suppression pit is in between the end point of the pp-neutrino spectrum and the Be-line. This bound does not depend on mixing angle in a wide region of \(\theta\). (For sterile neutrinos the bound is approximately the same). For fixed \(\Delta m^2\) the mixing \(\sin^2 2\theta\) is determined by the “double ratio”

$$R_{H/K} \equiv \frac{R_{Ar}}{R_{\nu e}},$$

where \(R_{Ar} \equiv Q_{Ar}^{obs}/Q_{Ar}^{SSM}\) and \(R_{\nu e} \equiv \Phi_{B}^{obs}/\Phi_{B}^{SSM}\) are the suppressions of signals in Cl–Ar and Kamiokande experiments, respectively. Here \(Q_{Ar}^{SSM}\), \(\Phi_{B}^{SSM}\) are the predictions in the reference model\(^{19}\) and \(Q_{Ar}^{obs}\), \(\Phi_{B}^{obs}\) are the observable signals. The ratio \(R_{H/K}\) depends very weakly on the solar model. It has however different behaviour for the conversion into active and into sterile neutrinos. For \(\Delta m^2 = 6 \cdot 10^{-6} \text{ eV}^2\) we get

$$\sin^2 2\theta \quad 2 \cdot 10^{-3} \quad 5 \cdot 10^{-3} \quad 10^{-2} \quad 2 \cdot 10^{-2}$$

$$R_{H/K}^{a} \quad 0.75 \quad 0.64 \quad 0.56 \quad 0.21$$

$$R_{H/K}^{s} \quad 0.77 \quad 0.74 \quad 0.72 \quad 0.69.$$  

The experimental value is \(R_{H/K} = 0.65 \pm 0.11\). In the case of active neutrinos \(R_{H/K}\) drops quickly when \(\sin^2 2\theta\) becomes larger than \(10^{-2}\). The reason is that the Kamiokande signal is dominated by NC scattering of \(\nu_\mu\) and \(\nu_\tau\) and \(R_{\nu e} \to 1/6\), whereas \(R_{Ar}\) is strongly suppressed. Central value of \(R_{H/K}\) can be achieved at \(\sin^2 2\theta = 5 \cdot 10^{-6}\) and \(f_B \approx 1.1\).

In the case of conversion into sterile neutrinos there is no NC effect for \(\nu_s\) and the suppression of both Homestake and Kamiokande signals strengthen with \(\theta\) increase simultaneously. As the result one has weak dependence of \(R_{H/K}^{s}\) on mixing angle. However, for large \(\sin^2 2\theta_{es}\) the original flux of Boron neutrinos should be large (to compensate for a strong suppression effect). If we restrict \(\Phi_B \leq 1.5\Phi_B^{SSM}\), then the bound on the mixing angle becomes: \(\sin^2 2\theta_{es} < 1.5 \cdot 10^{-2}\).

For very small mixing solution: \(f_B \sim 0.5\), \(\sin^2 2\theta_{es} \sim 10^{-3}\), all the effects of conversion in the high energy part of the boron neutrino spectrum \((E > 5 – 6 \text{ MeV})\) become very weak. In particular, the distortion of the energy spectrum disappears, and the ratio \((CC/NC)^{exp}/(CC/NC)^{th}\) approaches 1. Thus studying just this part of spectrum it will be difficult to identify the solution (e.g., to distinguish the conversion and the astrophysical effects).

Recent calculations in SSM with diffusion of heavy elements give larger boron neutrino flux\(^{24}\), so that even with 25% decrease of nuclear cross-section and 2% decrease of central temperature of the Sun one still needs an appreciable conversion effect. This gives a hope that the problem can be resolved by SuperKamiokande/SNO experiments.

With increase of \(f_B\) the fit of the data in the large mixing domain becomes better\(^{17}\). Here the Kamiokande signal can be explained essentially by NC effect and the mixing can be relatively small. Be- neutrinos are sufficiently suppressed and suppression of the pp-neutrinos is rather weak. For \(f_B = 2\) the values \(\sin^2 2\theta = 0.2 – 0.3\) become allowed. The corresponding
mass squared difference is $\Delta m^2 = 6 \cdot 10^{-6} - 10^{-4} \text{eV}^2$.

4. Lepton mixing: pattern, implications

Let us consider possible implications of the solar neutrino data to the lepton mixing. The scale of masses

$$m_2 = (2 - 3) \cdot 10^{-3} \text{eV}$$  \hspace{1cm} (7)

needed for solar neutrinos can be obtained by the see-saw mechanism with the mass of the RH neutrino component in the intermediate range: $M \sim 10^{11} \text{GeV}$. The common observation is that $M_R$ can be related to the scale of the Peccei-Quinn symmetry breaking or to SUSY breaking in the hidden sector etc.. The desired mixing is consistent with the following relation

$$\theta_{e\mu} = \sqrt{\frac{m_e}{m_\mu}} - e^{i\phi} \theta_\nu,$$ \hspace{1cm} (8)

where $\theta_\nu$ comes from diagonalization of neutrino mass matrix. The relation (8) may follow from Fritzsch ansatz in the context of the see-saw mechanism. There are however two cautions. In many models the angle $\theta_\nu$ is very small, and from (8) one finds $\sin^2 2\theta_{e\mu} \approx 4(\frac{m_e}{m_\mu}) \approx 2 \cdot 10^{-2}$ which is too large (see table in (6)). For very small mixing solution, $\sin^2 2\theta_{e\mu} \sim 10^{-3}$, one needs strong cancellation of contributions in (8).

Is the solution of the solar neutrino problem compatible with explanations of other neutrino anomalies like deficit of the atmospheric $\nu_\mu$ flux, possible signal of the $\bar{\nu}_\mu - \bar{\nu}_\tau$ oscillations, existence of the hot component of dark matter? Here the key words are the “pattern” and the “scenarios” of neutrino masses and mixing. Let us outline two possibilities.

1. Standard scenario of neutrino masses and mixing.

(i). Neutrino masses are generated by the see-saw mechanism with masses of the RH components $M_R = 10^{11} - 10^{12} \text{GeV}$. This scale can originate from Grand Unification scale, $M_{GU}$, and the Planck scale, $M_{Pl}$, as $M_R \sim M_{GU}^2 / M_{Pl}$.

(ii) Second mass, $m_2$, is in the range (7), so that the resonance flavour conversion $\nu_e \rightarrow \nu_\mu$ solves the solar neutrino problem.

(iii) The third neutrino (at $m_3 \sim 50 \text{GeV}$ and $M = 10^{12} \text{GeV}$) has the mass about 5 eV. It composes the desired hot component of dark matter.

(iv) The decays of the RH neutrinos with mass $10^{12} \text{GeV}$ can produce the lepton asymmetry of the Universe which can be transformed in to the baryon asymmetry during the electroweak phase transition$^{25}$.

(v) Large Yukawa coupling of neutrino from the third generation, e.g. $Y_\nu \sim Y_{top}$, give appreciable renormalization effects in the region of momenta $M_R - M_{GU}$. The $b - \tau$ mass ratio increases by $(10 - 15)\%$ in the SUSY. In turn this disfavours the $b - \tau$ mass unification for low values of $\tan \beta$ $^{26,27}$.
(vi) Simplest schemes with quark-lepton symmetry lead to mixing angle for the $e$ and $\tau$ generations: $\theta_{e\tau} \sim (0.3 - 3)V_{ub}$ which is close the bound from the nucleosynthesis of heavy elements (r-processes) in the inner part of the supernova: $\sin^2 2\theta_{e\tau} < 10^{-5}$ ($m_3 > 2$ eV)$^{28}$.

(vii) For $\mu - \tau$ mixings one expects $^{29} \theta_{\mu\tau} \sim k V_{cb} \eta$, where $k = 1/3 - 3$ and $\eta \sim 1$ is the renormalization factor. For $m_3 > 3$ eV some part of expected region of mixing angles is already excluded by FNAL 531. Large part of the region can be studied by CHORUS and NOMAD. The rest (especially $m_3 < 2$ eV) will be covered by E 803.

(viii) The depth of $\bar{\nu}_\mu - \bar{\nu}_e$ oscillations with $\Delta m^2 \approx m_3$ turns out to be $4|U_{3\mu}|^2|U_{3e}|^2 \approx 4|\theta_{e\tau}|^2|\theta_{\mu\tau}|^2$. The existing experimental data give the bound on this depth: $< 10^{-3}$ $^{30}$ which is too small to explain the LSND result.

The standard scenario does not solve the atmospheric neutrino problem. One can sacrifice the HDM suggesting that some other particles are responsible for the structure formation in the Universe, or consider strongly degenerate neutrino spectrum which has a potential problem with neutrinoless double beta decay. In both cases no appreciable effects in KARMEN/LSND experiments are expected.

2. Neutrinos and light singlet fermion

More safe way to accommodate all the anomalies is to introduce one new neutrino state$^{31-37}$. As follows from LEP bound on the number of neutrino species this state should be sterile (singlet of standard group). Taking into account also strong bound on parameters of oscillations into sterile neutrino from Primordial Nucleosynthesis one can write the following “scenario” $^{31-35}$:

(i) Sterile neutrino has the mass $m_S \sim (2 - 3) \cdot 10^{-3}$ eV and mixes with $\nu_e$, so that the resonance conversion $\nu_e - \nu_s$ solves the solar neutrino problem;
(ii) The masses of $\nu_\mu$ and $\nu_\tau$ are in the range $2 - 3$ eV, they supply the desired hot component of the DM;
(iii) $\nu_\mu$ and $\nu_\tau$ form the pseudo Dirac neutrino with large (maximal) mixing and the oscillations $\nu_\mu - \nu_\tau$ explain the atmospheric neutrino problem;
(iv) $\nu_e$ is very light: $m_1 < 2 \cdot 10^{-3}$ eV. The $\bar{\nu}_\mu - \bar{\nu}_e$ mixing can be strong enough to explain the LSND result.
(v) Production of heavy elements in supernova via “r-processes” is problematic for this scenario.

What is the origin of sterile neutrino? Of course, the RH neutrino components are natural candidates. However in this case the see-saw mechanism does not operate. Another possibility is that singlet fermion $S$ exists beyond the standard see-saw structure$^{36}$. Its appearance is motivated by some reasons not related to the neutrino physics. And moreover, this scalar can be family blind. The lightness of $S$ is nontrivial since standard model symmetry does not protect $S$ from acquiring the mass $m_s \gg m_W$. 
We suggest\textsuperscript{36} (see also\textsuperscript{37}) that $S$ mixes with active neutrinos via interactions with RH neutrino components only. So that in the basis $(S, \nu_e, N)$ the mass matrix has the following form

$$M = \begin{pmatrix}
0 & 0 & m_{es} \\
0 & 0 & m_e \\
m_{es} & m_e & M_e
\end{pmatrix}.$$ \hspace{1cm} (9)

Diagonalization gives one massless and one light state with mass

$$m_1 \simeq -(m^2_e + m^2_{es})/M_e.$$ 

The $\nu_e$–$S$ mixing angle is determined by

$$\tan \theta_{es} = m_e/m_{es}.$$ 

This mechanism allows one to generate simultaneously the mass and mixing without introduction of very small mass scale. Taking for $m_e$ the typical Dirac mass of the first generation: $m_e \sim (1 - 5)$ MeV, and suggesting that $\nu_e \rightarrow S$ conversion explains the solar neutrino problem, we find

$$m_{es} = \frac{m_e}{\tan \theta_{es}} \simeq (0.02 - 0.3) \text{ GeV.}$$

How the scale $0.1 - 1$ GeV appears in singlet sector? One possibility is the supersymmetry endowed by some spontaneously broken global symmetry, e.g. $U(1)_G$ in the simplest case. Spontaneous violation of $U(1)_G$ results in goldstone boson, and in the supersymmetric limit corresponding superpartner (fermion) is massless. Supersymmetry breaking parametrized by soft breaking terms leads in general to the $S$-mass which can be as big as $O(m_3/2)$. That is the supersymmetry alone can not protect very small mass scales, and one needs some additional care to further suppress $m_S$.

One possibility is the based on $R$-symmetry ($G \equiv R$) spontaneously broken up to the R-parity\textsuperscript{36}. The R-parity conservation requires for the fermion $S$ to be a component of singlet superfield which has no VEV. This allows one to construct a simple model in which the properties (mass and mixing) of $S$ follow from the conservation of R-symmetry. $S$ is mixed with RH neutrinos by the interaction with additional singlet field $y$ which can acquire VEV radiatively after soft SUSY breaking. The model can naturally incorporate the spontaneous violation of Peccei-Quinn symmetry or/and lepton number. The fields involved can spontaneously generate the $\mu$-term. Approximate horizontal (family) $U(1)^h$ symmetry can provide simultaneous explanations for the predominant coupling of $S$ to the first generation (thus satisfying the Nucleosynthesis bound) and for the pseudo-Dirac structure of $\nu_\mu$–$\nu_\tau$ needed in solving the atmospheric neutrino and hot dark matter problem. Breaking of $U(1)^h$ can be arranged in such a way that the parameters of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations are in the region of sensitivity of KARMEN and LSND experiments.

Conclusion

After top quark discovery the neutrinos are the only known fermions with unknown masses. There is some hope that things are comming to a head and in 2 - 3 years we will know the answer.

REFERENCES

2. K. Inoe, these Proceedings.
3. S. Eliott, these Proceedings.
4. C. Tao, these Proceedings.
15. V. S. Berezinsky, these Proceedings.