Rotating magnetic quadrupole field traps for neutral atoms

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Abstract

New effective magnetic methods of trapping neutral spin-polarized atoms with much stronger confinement than previously known methods are proposed. The methods differ from the recently invented rotating field "top" trap by employing rotating fields substantially nonuniform and of much lower rotation frequencies. Also a new approach which provides a simple picture of trapping with rotating magnetic fields is presented.

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a. Introduction.

While trapping of atoms by laser light mixes atomic polarization states and imposes bounds on the density of the atom cloud, trapping by quasistatic (varying ad-

abatically) magnetic fields does not, and the usage and development of magnetic traps is in progress, e.g. Refs [1-5]. An important recent invention in this field is the "top" trap [5].

It combines spherical quadrupole static field trapping with a large-amplitude uniform field rotating about the symmetry axis of the spherical quadrupole field. The rotating field considerably enhances the confinement of the spherical quadrupole, prevents from leakage of the atoms from the trap center, and gives rise to the synchronous with the field rotation of atomic polarization which is also an advantage, e.g., of importance for controlling systematic effects of asymmetry in atomic polarization experiments. The present communication is about new and more effective rotating field methods, with rotating fields substantially nonuniform and of frequencies commensurate with the orbiting atoms, rather than the much faster field rotation used in the top trap. Also, a new approach to and a simple theory of rotating field traps is presented. That the oscillating component of magnetic force is large and influences the confinement critically, makes the analysis of confinement more complex than that of a static field trap. We find that the picture of trapping is essentially simplified employing the coordinate frame rotating with the field.

b. Main equations.

Let us consider trapping of neutral atoms by a magnetic field $B = B_s(\mathbf{r}) + b(t, \mathbf{r})$, where B_s is a static field of axial symmetry with respect to the vertical axis (axis z) and b is a field rotating about z at a frequency Ω much smaller than Ω_z , the frequency of Zeeman splitting of the trapped atoms in the field $B(\mathbf{r}(t), t)$. Like for the static traps, the confinement appears to be possible only for the "weak field seekers", i.e. for the atoms in select Zeeman levels with negative magnetic numbers conserved in the magnetic field at hand. The atomic magnetic energy $-\mu B$ reduces then to μB where $\mu = const > 0$ is the atomic (average) magnetic moment and $B = |B(\mathbf{r}(t), t)|$. The confinement in the rotating frame is governed by the time-independent potential

$$U(\mathbf{R}) = \mu B(\mathbf{R}) - \frac{1}{2} M \Omega^2 (X^2 + Y^2) + M g z \quad (1)$$

with M the atomic mass, g the gravity acceleration, $B(\mathbf{R}) = B(\mathbf{r}, t)$ and $\mathbf{R} = (X, Y, z)$ the rotating frame coordinates. The second term in the r.h.s. of (1) represents the centrifugal forces. Neglecting other influences, the equation of atomic motion takes the form

$$\ddot{\mathbf{R}} = -\frac{1}{M} \frac{\partial U}{\partial \mathbf{R}} - 2[\boldsymbol{\Omega} \times \dot{\mathbf{R}}]. \quad (2)$$

Here $\boldsymbol{\Omega} = (0, 0, \Omega)$ and $2M[\boldsymbol{\Omega} \times \dot{\mathbf{R}}]$ is the Coriolis force. The equilibrium point $\mathbf{R} = \mathbf{R}_o$ of Eq. (2), determined by $\partial U / \partial \mathbf{R} = 0$, corresponds to the synchronous rotation of trapped atoms about axis z at frequency Ω .

The balance condition between the gravitational and magnetic forces, $\partial U / \partial z = 0$, is the same as for the static field trap with $\mathbf{B}$ aligned vertically in the equilibrium state: $\mu \partial B / \partial z = -Mg$. But now $\partial B_z / \partial z$, where B_z is the vertical component of $\mathbf{B}(\mathbf{R}_o)$, is not fixed by the quantity Mg/μ and can be made much larger, enhancing in result the confinement when the atomic moment μ lies (rotates with the field) almost in horizontal plane. In such rotating field geometries, called below “r-geometries”, the transverse component of the rotating field is large compared with B_z while its variation in z is small, $|\partial B_z^2 / \partial z \ll \partial B_z^2 / \partial z|$. The top trap is of this class.

Taking $\mathbf{R} = 0$ for the center of equilibrium rotation and the axis X along $\mathbf{B}_\perp(\mathbf{R}_o)$, we obtain $\mathbf{R}_o = (X_o, 0, 0)$ with $X_o = (\mu/M\Omega^2) \partial B / \partial X$. Linearizing Eq. (2) near $\mathbf{R} = \mathbf{R}_o$ one arrives at the system of three coupled oscillators describing the confinement of small atomic clouds

$$\begin{aligned} \ddot{X} - \Omega^2(X - X_o) &= 2\Omega\dot{Y} + O_1, \\ \ddot{Y} - (\Omega^2 - 2\omega_\perp^2)Y &= -2\Omega\dot{X} + O_2, \\ \ddot{z} + \omega_z^2 z &= O_3. \end{aligned} \quad (3)$$

Here $\omega_\perp^2 = (\mu/M) \partial^2 B / \partial Y^2$ and $\omega_z^2 = (\mu/MB)(\partial B / \partial z)^2$. The $O_{1,2,3}$ are linear in $X - X_o, Y, z$ terms and their coefficients are small compared with ω_z^2 and $\omega_\perp^2$ in case of r-geometries. The coupling between the vertical and transverse oscillations is then a tiny effect while the coupling between X and Y oscillations is strong and should be taken into

account carefully. The repulsing springs of partial X and Y oscillators of Eq. (4) are different which results in a specific gyroscopic modulation of free oscillation, as elucidated below. The natural frequencies of transverse oscillation (neglecting $O_{1,2,3}$) are

$$\Omega_\pm^2 = \Omega^2 + \omega_\perp^2 \pm \sqrt{4\Omega^2\omega_\perp^2 + \omega_\perp^4}. \quad (4)$$

It follows from (4) that the regime of small oscillation is stable, provided $\Omega^2 > 2\omega_\perp^2$. In case of Ω large compared with $\omega_\perp$, the transverse oscillation reduces to a simple form which after returning to the fixed coordinate frame is given by the complex function $\rho(t) = x(t) + iy(t)$

$$\rho(t) = X_o e^{i\Omega t} + (\rho_+ e^{i\omega_+ t} + \rho_- e^{-i\omega_- t}) e^{i\delta t}. \quad (5)$$

Here $\rho_\pm$ are complex constants determined by initial conditions, $\omega = (\Omega_+ - \Omega_-)/2 \approx \omega_\perp$ and

$$\delta = \Omega - \frac{\Omega_+ + \Omega_-}{2} \approx \frac{\omega_\perp^4}{8\Omega^3}. \quad (6)$$

So, the normal transverse oscillations in the fixed coordinate frame are of different frequencies, $\omega + \delta$ and $\omega - \delta$. They represent rotation in opposite directions. This results in the “gyroscopic” modulation: the polarization of $\mathbf{r}(t)$ of frequency ω rotates slowly, at frequency δ , in the direction of the rotating magnetic field. The modulation can be 100 % depending on the conditions of loading and manifests itself when the time of observation is commensurate or larger than $1/\delta$. Since δ is relatively small the scale of transverse confinement springs is given by the quantity $\omega_\perp^2$.

Thus, evaluating ω_z , $\omega_\perp$ and X_o via the field derivatives from the algebraic equations cited above allows one to simply determine the scales of the picture described and investigate most appropriate trapping configurations. In case, e.g., of a spherical quadrupole static field and a rotating field of planar quadrupole geometry, we arrive at a structure linear in $\mathbf{R}$ for the $\mathbf{B}(\mathbf{R})$ field, with the components

$$\mathbf{B} = [b_o + (\alpha - q/2)X, -(\alpha + q/2)Y, B_o + qz]. \quad (7)$$

Here B_o and b_o are respectively the static and rotating field components at the center, q is the gradient field characterizing the spherical quadrupole static field and α the rotating quadrupole field. The latter can be formed by a pair of vertical bars with alternating currents of frequency Ω and the spherical quadrupole by a pair of round coils with the axis along z and of opposing direct currents. The balance condition between magnetic and gravitational forces imposes on the parameters the relation $qB_o/B = -Mg/2\mu$ (with $B = B(R_o)$) and B_o for the r-geometries should be made small compared with B . With this the parameters b_o , q and α can be rather arbitrary and controlled independently.

c. The top trap. Such trap can be viewed as a particular case of configuration (7) with

$\alpha = 0$. In this case

$$\omega_z^2 = \frac{1}{8} \omega_z^2 = \frac{\mu g^2}{8MB} \quad (8)$$

and

$$b = b_o - \frac{1}{2} q X_o = \frac{b_o \Omega^2}{\Omega^2 - 2\omega_z^2}. \quad (9)$$

Here $b = b_o - qX_o/2$ is the horizontal field at the equilibrium orbit. In case of r-geometries b is large compared with the vertical field and $B \approx b$. The top trap assumes fast rotation, $\Omega \gg \omega_z, \omega_L$, so b is close to b_o . For the atoms loaded into the center and with isotropic velocity distribution, i.e. equal trapping temperatures $T_i = T$ (where $T_i/2 = M < \bar{x}_i(0)^2 > /2$, $i = x, y, z$), the transverse and vertical sizes of the cloud are given by

$$<|\rho|^2> = 4 <z^2> = \frac{2}{\bar{x}_i(0)^2} \frac{\omega_z^2}{\omega_z^2},$$

provided T is large compared with $T_{min} = M\Omega^2 X_o^2$ but smaller than $T_{max} = \mu b/4$. For the loading with T small compared with T_{min} , the transverse cloud size varies with T insignif-

icantly and is close to $|\rho|_{min} = \Omega X_o/\omega_L$. Note that this minimal cloud size is larger than X_o by Ω/ω_L times. For $b_o = 10$ G, $q = 10^2$ G/cm and $\Omega/\omega = 10^2$ (the data of trap [5])

$|\rho|_{min} = 0.04$ mm. The bound T_{max} corresponds to the cloud size $|\rho| = |\rho|_{max} = 2b/q$. This is the radius of the ring boundary where the magnetic field periodically reduces to zero and,

and transverse directions as is evident from what follows.

Advantages appear when employing rotating fields substantially nonuniform both in vertical and horizontal directions. The difference is that now the transverse confinement spring is four times stronger. More imposed when the alignment is critical. The same bound is intrinsic for the top trap [6]. when it is small compared with 1, we obtain $1 - P \approx T/\mu b$. So, a bound on the latter is the atomic coordinates $R(t)$ is smaller than 1. Evaluating the level of misalignment, $1 - P$, exactly synchronously. As a result, the amplitude P of rotating polarization averaged over in different points the instant field is directed differently and the polarization is rotating not cloud is of concern. The atomic polarization follows adiabatically the field, $\mu/\mu = -B/B$, A restriction on the confinement appears when the atomic polarization alignment over the oscillation amplitude from that given by approximation (5) is smaller than 10 %.

decrease Ω considerably, up to $\Omega \sim 2\omega_L$. Note that at $\Omega = 2\omega_L$ the deviation of transverse trap Ω/ω_L should be taken large, to have $|\rho|_{min}$ small, for the RQ trap it is possible to springs increase by four times the same order. Also of importance is that while for the top vertical confinement spring increases by the same order of magnitude and the transversal provided $B_o/b_o \ll 1$ and $\Omega \ll \Omega_z$ which are rather weak restrictions. Correspondingly, the confinement there is no need to take b_o large and it can be diminished by orders of magnitude, free of the bounds on the cloud sizes intrinsic for the top trap. Moreover, to achieve stronger Then the radius of equilibrium rotation vanishes: $X_o = 0$. As a result, the trap becomes *d. The rotating quadrupole (RQ) trap.* We call so the configurations (7) with $\alpha = q/2$.

the rotating field coils should provide a uniform field rotating at a high frequency ($\sim$ kHz). several times larger than 10 G. But to increase b by orders of magnitude is difficult, since place. Only the field magnitude b enters the bound. It is possible to arrange for fields The trap is not effective for the atoms loaded with $T > T_{max}$ because their leakage takes

$$T_{max} = \mu b/4 \sim 10^{-4} \text{ K}.$$

as a result, the atoms occurring in this area leak from the trap due to spin-flips. Taking for example $\mu = \mu_B$, the Bohr magneton, and $b_o = 10$ G one obtains $|\rho|_{max} = 2$ mm and

e. The rotating double quadrupole (RDQ) trap. The idea of trap proposed here is to use two rotating quadrupole planar fields, one is the same plane quadrupole as in the RQ trap, the other is the quadrupole in the plane Y, z . The latter can be formed by horizontal current bars with alternating currents of frequency Ω . Thus, the arrangement is of three quadruples, one static and two rotating. The resultant field components (compare with (7))

$$\mathbf{B} = [b_0 + (\alpha - q/2)X, -(\alpha + \beta + q/2)Y, B_0 + (\beta + q)z] \quad (10)$$

where β is the field gradient of the planar, in the Y, z plane, quadrupole field rotating around z . The picture of atomic motion in the RDQ trap remains the same as in RQ trap but with the modified transverse and vertical confinement springs:

$$\omega_1^2 = \frac{1}{2}\omega_z^2 = \frac{\mu(\beta + q)^2}{MB} \quad (11)$$

With $\alpha = q/2$ all the advantages of the RQ trap discussed above hold, while for the RDQ trap the confinement springs can be made larger, provided $\alpha\beta > 0$. With β of same magnitude as q , one obtains the transverse and vertical confinements respectively sixteen and eight times larger than that of the top trap with the same b_0 . With this the equilibrium radius of orbiting atoms equals zero and a slow rotating field, $\Omega \sim 2\omega_1$, can be used as in the case of RQ trap. This provides additional advantages since employing a rotating field frequency of a few tenths of a Hz, rather than a few kHz, makes it significantly easier to arrange large currents in the magnetic coils.

Note that the magnitude of b in the top trap can be increased by employing a slow rotating field, resonant to the orbiting atoms, as is evident from (9). An important advantage of resonant orbit trapping is in better polarization alignment of atom cloud and in facilitating the separation of particles of different ground states. The separation (via e.g. evaporative cooling) becomes very sensitive to the factor μ/M which enters the resonance detuning. The expense is in enlarging the radius of orbital motion. This idea was proposed in [6] where one can find more details. Employing RQ or RDQ trapping configurations with $\alpha \neq q/2$ allows one to use the idea of resonant orbit trapping more effectively, because the orbit resonance can be significantly shifted from the frequency ω_1 figuring in (3).

Thus, the methods of magnetic trapping proposed eliminate the bounds on trapping temperatures intrinsic for the top trap and can provide the confinement springs many times larger. They also are more effective in using the resonant orbit trapping for evaporative cooling of neutral atoms and for separation of particles with different ground states. The possibilities opening up in this field are wide and can be dealt in a simple way with the approach presented.

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