State-independence in atom interferometry, and insensitivity to acceleration and rotation

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Abstract

We demonstrate that for atom interferometers with lasers serving as beam splitters state independence (including Doppler-freeness) of the phase shift is obtained if the paths of the two interfering beams are closed and have obtained the same momentum transfer. This is also the case when the center-of-mass motion of the atoms is subject to a constant force field (acceleration). The simplest interferometer geometry is deduced for which the phase shift does not depend on acceleration. Its sensitivity to rotation is discussed. The geometry of a state independent interferometer which is practically insensitive to both acceleration and rotation is worked out. As an application its relevance for the detection of the influence of space-time curvature on the atomic motion is studied.

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1 Introduction

Atomic interferometry has turned out to be a powerful tool to measure with high accuracy gravitational and inertial effects, atomic properties, fundamental constants, and to perform tests of quantum mechanics [1]. Furthermore, atom interferometer may serve as frequency standards. To reach this high sensitivity in experiments where laser beams are used to split and recombine an atomic beam it is, among other things, necessary to get rid of Doppler-induced effects which can wash out the interference pattern. One instructive example is an array of the Ramsey type [2, 3, 4] which can also be used as an atomic interferometer [5]. Another one, which is more suited for our purposes, is the atomic fountain [6] shown in Fig. 1, for which one can use an effective two-level picture to describe the momentum transfer from the laser to the atoms. In this picture copropagating laser pulses are used as beam splitters (see Fig. 1). If the flight times are chosen properly ($T_1 = T_2$ in Fig. 1) the additional phase shift induced by the Doppler effect is exactly the same for the pair (I,II) of partial beams in Fig. 1 (see below).

When performing precision measurements with atom interferometers not only the Doppler effect must be eliminated but the influence of all types of external potentials must be eliminated, too. The earth's gravitational field and the rotation of the earth are naturally present in the laboratory and produce large phase shifts if no special care is taken. It is therefore an important task to eliminate experimentally these influences. The aim of this paper is to describe how the influences of a homogeneous force field and the rotation of the interferometer can theoretically be controlled. This will lead to the proposition of Doppler-free interferometer arrays for which these influences are highly suppressed. To give an idea of the effectiveness of the suppression we compare the remaining influence of the earth's acceleration and rotation with the influence of the space-time curvature of the earth.

In Ref. [7] we studied the influence of a general class of potentials on an atom interferometer of Ramsey type. The measurability of the influence of space-time curvature was discussed in detail in Ref. [8] and including other interferometer types in Ref. [9]. In the latter we gave a general treatment of the influence of a class of external perturbations, which may be physically realized by gravity and inertia, acting on the atoms of an atomic interferometer. In the limit of small interaction times an exact expression for the resulting shift of the interference pattern is given for any number of laser beam splitters with traveling or standing waves having arbitrary wave vectors.

In Ref. [9] the influence of the perturbing potential in the laser zones has been neglected. We closed this gap in our paper [10] where the time evolution of a two-level atom which is simultaneously exposed to the field of a running laser wave and a homogeneous gravitational field is studied. The result of the coupled dynamics of internal transitions and center-of-mass motion is worked out exactly (in the rotating wave approximation) and can be directly applied to the laser zones of atom interferometers. For a different approach to the same problem see Ref. [11]

The following calculations are based on our results of Ref. [9]. Because these results are derived for an arbitrary array of lasers the respective relations are an ideal

Figure 1: The splitting of an atomic beam in the atomic fountain. Dashed lines correspond to excited atoms. The atoms are initially deexcited and travel freely for times T_i between the lasers with wave vectors $\vec{k}_i$. Beam I and II interfere in a state independent way (which includes Doppler-freeness). They fulfill the conditions of equal momentum transfer and closed geometry.

starting point to specify laser arrays which satisfy certain conditions, this means to answer questions of the type: what is the smallest number of lasers necessary to build up an interferometer with Doppler-free interference which is insensitive to acceleration, and how have the respective laser beams to be oriented. We turn to this and related questions below.

In section 2 we repeat the general result of Ref. [9]. In section 3 we discuss conditions for state-independence including Doppler-freeness and work out in section 4 an interferometer which is insensitive to acceleration ("figure 8" interferometer). The influence of rotation on this interferometer is studied in section 5. To suppress also this influence we show in section 6 that a more complicated array ("figure $8\frac{1}{2}$ " interferometer) is necessary. Remaining influences are discussed in section 7.

2 Phase shift for arbitrary interferometer configurations

We consider as in Ref. [9] two-level atoms propagating in the time intervals between the laser influences according to the Hamiltonian

$$H = \frac{\vec{p}^2}{2M} + H_{Atom} + H_p. \quad (1)$$

H_p represents the influence of external potentials which effect only the center-of-mass motion. The initial state at $t = 0$ is $|\psi_0\rangle$. At times t_i the matter state interacts

with a running or standing laser beam with wave vector $\vec{k}_i$. The total number of these interactions is N . The interaction time with a laser beam is assumed to be small as compared with the flight time between the lasers so that the influence of the Hamiltonian H_p in a laser zone may be neglected (but compare section 7). That the lasers act at fixed times t_i is characteristic for our approach. This may be realized in using laser pulses or, approximately and if $|\psi_0\rangle$ has a nearly definite momentum, by narrow laser zones fixed in space. The atomic beam is thereby separated into partial beams by the momentum transfer connected with atomic excitation or deexcitation processes induced by the laser waves. The *history* of each partial beam can be characterized by a sequence of numbers $\{n_i\}$. For instance, $n_1 = 1$ denotes excitation at the first laser with momentum transfer $+\hbar\vec{k}_1$, and $n_2 = -1$ denotes deexcitation at the second laser with momentum transfer $-\hbar\vec{k}_2$. Correspondingly for larger positive or negative numbers n_i . For $n_i = 0$ the state of the atoms belonging to this partial beam is not altered by the i th laser wave.

In Ref. [9] that part of the phase shift between the two partial beams $|\psi_I\rangle$ with history $\{n_i^I\}$ and $|\psi_{II}\rangle$ with history $\{n_i^{II}\}$ was worked out, which is caused by the center-of-mass motion under the influence of H_p of the form

$$H = -M\vec{a} \cdot \vec{x} + \frac{Mc^2}{2} R_{0l0m} x_l x_m - \vec{\omega} \cdot (\vec{x} \times \vec{p}). \quad (2)$$

Summation convention for $l, m, \dots = 1, 2, 3$ is assumed throughout the paper. H_p describes the influence of linear and quadratic external potentials or any angular momentum coupling. Here M , $\vec{x}$, and $\vec{p}$ are the mass, the position operator, and the momentum operator of the atom. $\vec{\omega}$ is the rotation of the total interferometer array. The first two terms of Eq. (2) may be regarded as the first terms of a Taylor expansion of some scalar potential $V(\vec{x})$. We will call $\vec{a}$ of the term linear in $\vec{x}$ the *acceleration* and R_{0l0m} of the next Taylor term the *curvature* because a typical realization to which we will refer below as an example is the influence of gravity. In this case $\vec{a}$ denotes the gravitational acceleration related to the homogeneous part of the gravitational field. R_{0l0m} are the components of the Riemannian tensor representing the space-time curvature. To a very good approximation they can be replaced by $\partial_l \partial_m V(\vec{x}) / (Mc^2)$ where $V(\vec{x})$ is the Newtonian potential. For details see Ref. [8]. As compared to Ref. [9] we have changed the sign of the acceleration. Of course all considerations below apply as well for other scalar potentials $V(\vec{x})$.

We discuss the interference of two partial beams denoted by I and II . It was shown in Ref. [9] that the part of the interference term $\langle \psi_{II} | \psi_I \rangle$ which depends on the center-of-mass motion can be written as

$$\langle \psi_{II} | \psi_I \rangle = \langle \psi_0 | U_{II}^\dagger U_I | \psi_0 \rangle \quad (3)$$

where the unitary operator U_I related to beam I is given by Eq. 25 of Ref. [9],

$$U_I = \exp[i \sum_{i=1}^N n_i^I r_i] \exp[\frac{i}{2} \sum_{i=1}^N \sum_{j=i}^N n_i^I n_j^I (\vec{\mu}_i \cdot \vec{v}_j - \vec{v}_i \cdot \vec{\mu}_j)], \quad (4)$$

and the corresponding operator U_{II} is given by the same expression with n_i^I replaced by n_i^{II} . The operators

$$r_i = \vec{\mu}_i \cdot \vec{x} + \vec{v}_i \cdot \vec{p} + \vec{\rho}_i \cdot \vec{a} \quad (5)$$

are linear in $\vec{x}$, $\vec{p}$, and $\vec{a}$ with c-number coefficients. Their vector components are given by

$$\begin{aligned} (\vec{\mu}_i)_l &= (\vec{k}_i)_l - t_i(\vec{k}_i \times \vec{\omega})_l + \frac{1}{2}t_i^2((\vec{k}_i \times \vec{\omega}) \times \vec{\omega})_l - \frac{1}{2}t_i^2c^2R_{0l0m}(\vec{k}_i)_m \\ (\vec{v}_i)_l &= \frac{1}{M}\{t_i(\vec{k}_i)_l - t_i^2(\vec{k}_i \times \vec{\omega})_l + \frac{1}{2}t_i^3((\vec{k}_i \times \vec{\omega}) \times \vec{\omega})_l - \frac{1}{6}t_i^3c^2R_{0l0m}(\vec{k}_i)_m\} \\ (\vec{\rho}_i)_l &= -\frac{1}{2}t_i^2(\vec{k}_i)_l + \frac{1}{3}t_i^3(\vec{k}_i \times \vec{\omega})_l - \frac{1}{8}t_i^4((\vec{k}_i \times \vec{\omega}) \times \vec{\omega})_l + \frac{1}{24}t_i^4c^2R_{0l0m}(\vec{k}_i)_m \end{aligned} \quad (6)$$

where we have neglected terms of order $(\vec{\omega})^3$, $\vec{\omega}R_{0l0m}$, and $(R_{0l0m})^2$. Throughout the paper summations over $i, j, \dots = 1, \dots, N$ are always explicitly written out. Note that in Eq. (4) operators appear only in the first two terms of r_i . All the other terms are c-numbers. Note also that $\vec{a}$ appears only in Eq. (5) and not in $\vec{\mu}_i$, $\vec{v}_i$, and $\vec{k}_i$.

3 State-independence of the phase shift

In the first step of our specification of interferometer arrays with particular properties we assume $\vec{\omega}$ and R_{0l0m} to be vanishing but allow the presence of an acceleration $\vec{a}$. We want to derive conditions under which the interference fringes are not washed out. This is the case when the phase shift is independent of the initial state $|\psi_0\rangle$ (*state independence*). This includes Doppler-freeness. Our second aim is to demonstrate the influence of the acceleration on the phase shift. For these purposes it is therefore sufficient when working out Eq. (3) to omit all terms which do neither depend on $\vec{a}$ nor on $|\psi_0\rangle$. We obtain

$$\langle \psi_I | \psi_{II} \rangle = \exp \left\{ \frac{i}{2} \vec{a} \cdot \sum_{i=1}^N \Delta n_i \vec{k}_i t_i^2 \right\} \langle \psi_0 | \exp \left\{ i \vec{x} \cdot \sum_{i=1}^N \Delta n_i \vec{k}_i \right\} \exp \left\{ \frac{i}{M} \vec{p} \cdot \sum_{i=1}^N \Delta n_i \vec{k}_i t_i \right\} | \psi_0 \rangle. \quad (7)$$

with $\Delta n_i = n_i^{II} - n_i^I$. No approximation has been made beside the limit of short interaction times with the laser beams. Note that the state dependent part, which goes back to the $\vec{x}$ and $\vec{p}$ terms in Eq. (5), is independent of $\vec{a}$. Because of the structure of r_i this is also the case for non-vanishing $\vec{\omega}$ and R_{0l0m} .

Each term in Eq. (7) allows a proper physical interpretation. The first term describes the influence of a homogeneous force field. That it is linear in $\vec{a}$ and that it is independent of $|\psi_0\rangle$ can be explained in referring to the equivalent situation of a freely propagating state in an accelerated interferometer array. It has been worked out in detail in Ref. [9]. For a homogeneous gravitational field this argument amounts to the equivalence principle. Important is again that the lasers act at both partial beams simultaneously at well defined times t_i (laser pulses). The simple $\vec{a}$ -dependence described above is not obtained exactly if the beam splitters act instead at certain planes in space as is the case in neutron interferometry and for atom interferometers

with gratings as beam splitters. The simple $\vec{a}$ -dependence is an advantage of atom interferometry.

We turn now to the state dependent influences. The second term in Eq. (7) can be written in the form $\exp\{i\vec{q} \cdot \vec{x}\}$ with

$$\vec{q} = \sum_{i=1}^N n_i^{II} \vec{k}_i - \sum_{i=1}^N n_i^I \vec{k}_i. \quad (8)$$

The operator $\exp\{i\vec{q} \cdot \vec{x}\}$ when applied to a wave function $|\psi\rangle$ amounts to a shift of the momentum by $\hbar\vec{q}$. Accordingly the second term describes the difference between the total momentum transfers by the lasers to partial beam I and II , respectively, after the last laser field has been passed. A necessary condition for any interference is that the wavefunctions $|\psi_I\rangle$ and $|\psi_{II}\rangle$ overlap in momentum space. If the initial state $|\psi_0\rangle$ is a relatively narrow wavepacket in momentum space the best overlap is reached when both partial beams have the same total momentum transfer. We therefore demand in addition to Eq. (10) the *condition of equal momentum transfer*

$$\vec{q} = \sum_{i=1}^N \Delta n_i \vec{k}_i = 0 \quad (9)$$

as the first condition necessary to guarantee state independence.

The third term in Eq. (7) describes a displacement in space of the atoms caused by the momentum transfer because the operator $\exp\{i\vec{p} \cdot \vec{y}/\hbar\}$ stands for a shift in position space with amount $\vec{y}$. This term reflects the influence of the Doppler effect which induces a shift of the i th lasers frequency ω_i according to $\omega_i' = \omega_i - \vec{k}_i \cdot \vec{v}$ where $\vec{v} = \vec{p}/M$ is the velocity of the atoms. This substitution can be read off if one reintroduces the laser phases $\exp\{i\omega_i t_i\}$ which have been omitted in the equations above. Since the Doppler term varies with the velocity of the atoms an interference pattern is washed out if the momentum distribution of the atoms is not very narrow. The condition of *Doppler-free interference* is fulfilled if the Doppler term in Eq. (7) vanishes, i.e., if

$$\sum_{i=1}^N \Delta n_i \vec{k}_i t_i = 0 \quad (10)$$

holds.

Only if the two conditions (10) and (9) are both fulfilled the demanded state independence of the phase shift is obtained. Multiplying Eq. (9) with t_N and subtracting Eq. (10) from the result the condition of state independence amounts to the demand

$$\Delta s^{II} - \Delta s^I = \sum_{i=1}^N n_i^{II} (t_N - t_i) \frac{\hbar \vec{k}_i}{M} - \sum_{i=1}^N n_i^I (t_N - t_i) \frac{\hbar \vec{k}_i}{M} = 0. \quad (11)$$

$t_N - t_i$ is the time which still passes after the action of the laser i . Note that either two of the conditions (10), (9), and (11) have the third as consequence. Important is that for vanishing $\vec{\omega}$ and R_{0l0m} these conditions guarantee state independence regardless if an influence of acceleration $\vec{a}$ is present or not.

A particular interferometer array is determined by the laser wave vectors $\vec{k}_i$ and the times t_i when they influence the atomic state. Two particular partial beams are fixed by the histories $\{n_i^I\}$ and $\{n_i^{II}\}$ describing the momentum transfer. Given an array of two partial beams, how can one read off exact state independence at a glance? This can be done by referring to the well known diagrams of the type of Fig. 1 which are usually drawn in this context. After introducing time and space axes to fix a reference frame the diagram may be interpreted as representing the space-time trajectories of *freely moving classical* point particles (no influence of acceleration) which suffer the specific momentum transfers at times t_i . By construction the reference frame is thereby the one in which the particles are at rest at time t_1 . Of course it is not assumed that the wavevectors $\vec{k}_i$ are orthogonal to the particles momentum or restricted any way else. In general the $\vec{k}_i$ will not all lie in one plane as in Fig. 1. Referring now to these classical trajectories the condition (9) of equal momentum transfer means that the two classical trajectories in question leave the array after the last laser influence at the time t_N with the same momentum vector (space-time direction). In Eq. (11) the quantities $\Delta\vec{s}^{III}$ have now the meaning of the total classical displacement in space of the respective beam due to the momentum transfers. Condition (11) has therefore the very intuitive interpretation that the corresponding classical trajectories have to come together after the last laser beam (*closed geometry*). Interference is only possible if the atoms are in the same internal state. To visualize this, too, the classical trajectories corresponding to the excited are drawn as dashed lines. Our result then is: equal final momentum and closed geometry guarantee state independence (row 1 of the table). As pointed out above: If acceleration $\vec{a}$ is present one may draw nevertheless the straight trajectories for the free motion between the laser influences to control the conditions because these are independent of $\vec{a}$. A discussion of recoil diagrams can also be found in the paper [12] of Friedberg and Hartmann where the closure condition is derived on the basis of a billiard-ball model. As compared with this model, which allows the calculation of the complete interference pattern, our aim is to discuss directly the influence of the center-of-mass motion on the phase shift only.

4 An acceleration-insensitive interferometer

In all earth-bound experiments the homogeneous gravitational field of the earth and the related acceleration $\vec{a}$ are always present. For the existing atom beam interferometers the influence of $\vec{a}$ produces a large phase shift. For the atomic fountain this phase shift is of the order of 10^7 [6]. For many precision measurements in which phase shifts are observed which do not go back to $\vec{a}$ this is an unwanted influence. Eq. (7) shows that it can be eliminated in principle in choosing $\vec{a} \cdot \vec{k}_i = 0$ for all i . But this can in practise not be realized because a small deviation of this orientation of the laser beams leads already to a considerable effect. For illustration we mention as a particular high precision measurement for which the problem appears the detection of the influence of space-time curvature on quantum matter. This would be of great interest since it would be the first demonstration that the geometry of space-time

also affects the motion of quantum particles. The influence of a rotation [3] or the homogeneous gravitational field of the earth [6] on atoms is now well established, but these two effects arise from the choice of a particular reference frame and have nothing to do with the geometry of space-time. Taking the earth as a source of gravity it was shown in Ref. [8] that one of the major problems in the measurement of the curvature-induced phase shift is the phase shift caused by the earth's acceleration which can be several orders of magnitude larger than the curvature part of the phase shift. Since both shifts can be varied only simultaneously (by turning the interferometer) this prevents a detection of the curvature effect. To get rid of this problem also in experiments where the curvature is caused by laboratory sized objects [8] it would be advantageous to use an interferometer which is insensitive to any homogeneous force field including the earth's acceleration regardless of its orientation in space.

In this section we discuss acceleration-insensitivity under the condition that $\vec{\omega}$ and R_{olom} are vanishing. In this case it is possible to eliminate the influence of $\vec{a}$ exactly (for very small interaction times with the lasers and ignoring relativistic effects). The respective interferometer set up will then serve as a good starting point for the study of the additional exclusion of the influence of rotation $\vec{\omega}$ in the next sections

Eq. (7) can be used to derive uniquely the simplest interferometer geometry which is exactly insensitive to acceleration. We demand that the $\vec{a}$ -dependent part of the phase shift in Eq. (7) vanishes for any choice of $\vec{a}$. This results in the condition

$$\sum_{i=1}^N \Delta n_i \vec{k}_i t_i^2 = 0. \quad (12)$$

Together with Eqs. (10) and (9) for state independence we have here obtained conditions for the number N of laser beams, their wavevectors $\vec{k}_i$, the flight times t_i , and the histories $\{n_i^I\}$ and $\{n_i^{II}\}$ of the two partial beams. These three conditions represent a hierarchy of equations containing increasing powers of the times t_i . To be an efficient beam splitter each laser must be nearly in resonance with the atomic transition. We therefore also assume that the absolute values of all wavevectors are equal,

$$|\vec{k}_i| = |\vec{k}_j| \quad \forall i, j, \quad (13)$$

so that only their directions have to be determined.

It is not difficult to work out under which conditions Eqs. (10), (9), and (12) can be simultaneously fulfilled. We sketch the proof. There is no nontrivial solution for $N = 1, 2, 3$. For $N = 4$ we can assume $\Delta n_i \neq 0 \quad \forall i$ since for partial beams with $\Delta n_i = 0$ for one laser beam this beam would not be necessary to fulfill any of the three conditions. If we restrict to running laser waves then $n_i^{II/I}$ can only take the values -1, 0, and +1 and there remain only five different pairs of partial beams for which all Δn_i are nonzero. It is straightforward but long to study all five pairs. We will only sketch the calculations for the pair with

$$\Delta n_i = \{1, -2, 2, -1\}. \quad (14)$$

which is the only one which can fulfill all three conditions. One may use Eqs. (9)

Figure 2: The splitting of an atomic beam by four appropriate laser pulses. For beam I and II, which build a "figure 8" geometry, a pure acceleration (homogeneous force field) causes no phase shift.

and (10) to eliminate $\vec{k}_4$ and $\vec{k}_3$ from Eq. (12). It remains

$$\vec{k}_1(t_4 - t_1)(t_3 - t_1) = 2\vec{k}_2(t_4 - t_2)(t_3 - t_2). \quad (15)$$

The time differences are positive so that $\vec{k}_1$ and $\vec{k}_2$ must be parallel and therefore equal. Inserting this into Eq. (9) one finds that all laser beams must be parallel

$$\vec{k}_1 = \vec{k}_2 = \vec{k}_3 = \vec{k}_4. \quad (16)$$

We then obtain from Eqs. (10) and (12) for the flight times $T_i := t_{i+1} - t_i$ between the lasers

$$T_1 - T_2 + T_3 = 0, \quad -2T_1^2 + 2T_0(T_2 - T_1 - T_3) + 2(T_1 + T_2)^2 - (T_1 + T_2 + T_3)^2 = 0 \quad (17)$$

with the nontrivial solution

$$T_1 = T_3 =: T, \quad T_2 = 2T. \quad (18)$$

Because of Eq. (14) the two partial beams are characterized by the histories

$$\{n_i^I\} = \{0, 1, -1, 1\}, \quad \{n_i^{II}\} = \{1, -1, 1, 0\}. \quad (19)$$

They form a "figure 8" (see Fig. 2). We have noted this in row 2 of the table. The last numbers n_4^I are to some extent arbitrary and reflect the fact that only excited atoms are detected behind the interferometer as it is usually the case. If one wishes to detect atoms in the ground state they have to be replaced by $n_4^I = 0$ and $n_4^{II} = -1$.

We should mention that there is a problem with the proposal made here. Using the criterion of the proceeding section we can read off from Fig. 2 that there are two more pairs of partial beams which interfere state-independent and are in the

excited state after the influence of the fourth laser beam. They will therefore in general contribute to the fringe pattern. But their phase shift does depend on the acceleration. To circumvent this problem one has to use a sequence of four laser pulses which picks out only the partial beams I and II. This is the case for the sequence $\pi/2, \pi, \pi, \pi/2$. To produce efficient π and $\pi/2$ pulses it is necessary to use an atomic beam with a narrow momentum spread what rules out the use of thermal atoms, for instance.

The idea to construct an interferometer which is insensitive to acceleration or rotation is not new. For neutron interferometry Anandan [13] proposed a geometry for the measurement of the Lense-Thirring effect, and Clauser [14] also proposed to measure the influence of space-time curvature in this way by using gratings as beam splitters for atoms. Both proposals include a "figure 8" geometry which here is shown to be the simplest possibility.

5 State independence including Doppler-freeness in the presence of rotation

We showed above that the "figure 8" is the simplest array within a class of interferometers which fulfill the conditions (9), (10), and (12). These conditions amount exactly to the physically desired properties of state independence and acceleration independence if rotation $\vec{\omega}$ and curvature R_{0l0m} vanish exactly. But to assume that $\vec{\omega}$ vanishes exactly is unrealistic. The question therefore arises to what extent the intended two properties are still realized for the class of interferometers if a nonvanishing $\vec{\omega}$ is allowed. We assume thereby $R_{0l0m} = 0$ and include in the Eqs. (3) to (6) $\vec{\omega}$ only up to first order.

Do we have to impose additional demands to guarantee state independence (including Doppler-freeness) up to first order in $\vec{\omega}$? The surprising answer is: no! Because of the validity of Eqs. (10), (9), and (12) we can infer that

$$\sum_i n_i^{II} \vec{\mu}_i = \sum_i n_i^I \vec{\mu}_i, \quad \sum_i n_i^{II} \vec{v}_i = \sum_i n_i^I \vec{v}_i. \quad (20)$$

This implies in turn the equality of the q-number parts $\sum n_i^{II} r_i$ and $\sum n_i^I r_i$ of the operators U_{II} and U_I (see Eqs. (4) and (5)). Because U_I and U_{II} are unitary operators we therefore have shown that the operator $U_{II}^\dagger U_I$ appearing in the interference term (3) is a pure c-number. Hence the interference term is state independent. In particular the presence of rotation does not induce a washing out of the fringe pattern of a "figure 8" interferometer.

Note, however, that because of the presence of a rotation a new acceleration dependence of the phase shift arises. Exploiting the equality of the operator parts of U_I and U_{II} it is not difficult to show that for the case under consideration the interference term (3) reduces to the phase factor

$$\langle \psi_I | \psi_{II} \rangle = \exp\left\{i \sum_i \Delta n_i \vec{\rho}_i \cdot \vec{a}\right\} \exp\left\{\frac{i}{2} \sum_i \sum_{j \geq i} (n_i^{II} n_j^{II} - n_i^I n_j^I) (\vec{\mu}_i \cdot \vec{v}_j - \vec{v}_i \cdot \vec{\mu}_j)\right\} \quad (21)$$

with $\vec{\rho}_i$ of Eq. (6). By using

$$n_i^{II} n_j^{II} - n_i^I n_j^I = \Delta n_i n_j^{II} + n_i^I \Delta n_j \quad (22)$$

and the resummation formula

$$\sum_{i=1}^N \sum_{j=i}^N \dots = \sum_{j=1}^N \sum_{i=1}^j \dots \quad (23)$$

one can show that the second exponential in Eq. (21) vanishes. Applying Eq. (12) the complete phase shift is then given by

$$\langle \psi_I | \psi_{II} \rangle = \exp \left\{ \frac{i}{3} (\vec{\omega} \times \vec{a}) \cdot \sum_{i=1}^N \Delta n_i \vec{k}_i t_i^3 \right\} \quad (24)$$

(compare row 3 of the table). Despite the reappearance of the acceleration in the phase shift its influence is substantially reduced as compared to the general case even if the $\vec{\omega} \times \vec{a}$ term does not vanish. The reason is that the pure accelerational phase factor is proportional to $\vec{a} t_i^2$ whereas the new term is proportional to $\vec{\omega} \times \vec{a} t_i^3$. The magnitudes of both factors are different by a factor of $|\vec{\omega}| t_i$ which is a very small quantity for typical interferometers ($t_i < 1$ s) resting on the earth ($|\vec{\omega}| = 2\pi/\text{day}$). It is desirable to reduce by a special interferometer design this combined rotation and acceleration dependence further.

6 A Doppler-free interferometer which is practically insensitive to both acceleration and rotation

Although the disturbing effect of the acceleration is relatively small for the "figure 8" interferometer it still remains a big problem for the detection of the influence of the second order Taylor terms R_{0l0m} of a scalar potential if this is very small. This is obviously the case if R_{0l0m} represents the space-time curvature of the earth or some laboratory-sized object. We will give the respective data below. Our aim is therefore to propose an interferometer set up for which the term (24), which is of first order in $\vec{\omega}$, vanishes. Such an interferometer may be regarded as being state independent and practically insensitive to both acceleration and rotation.

Accordingly, we demand in addition to Eqs. (10), (9), and (12) the new condition

$$\sum_{i=1}^N \Delta n_i \vec{k}_i t_i^3 = 0 \quad (25)$$

which extends our hierarchy of conditions to one containing third powers of t_i . If all four conditions are fulfilled we can adopt the reasoning of the previous section, but now including all terms of the vectors $\vec{\mu}_i$ and $\vec{v}_i$ of Eq. (6). Again Eq. (20) is valid,

Figure 3: Atom beams with a "figure 8 $\frac{1}{2}$ " geometry are practically insensitive to both acceleration and rotation. They may be used to measure the influence of space-time curvature.

and in the same manner as before we can infer that the total phase shift including R_{0l0m} and terms of second order in $\vec{\omega}$ is independent of the initial state and given by

$$\begin{aligned} \langle \psi_I | \psi_{II} \rangle &= \exp \left\{ i \sum \vec{\rho}_i \Delta n_i \cdot \vec{a} \right\} \\ &= \exp \left\{ \frac{i}{8} \left[-(\vec{\omega} \times (\vec{\omega} \times \vec{a}))_m + \frac{c^2}{3} a_l R_{0l0m} \right] \sum_{i=1}^N \Delta n_i t_i^4 (\vec{k}_i)_m \right\}. \end{aligned} \quad (26)$$

Inserting for $\vec{a}$, $\vec{\omega}$, and R_{0l0m} the data of the earth, that is roughly $|\vec{a}| = 10 \text{ m s}^{-2}$, $|\vec{\omega}| = 2\pi/\text{day}$, and $|R_{0l0m}| = 10^{-23} \text{ m}^{-2}$, it is obvious that the quadratic term in $\vec{\omega}$ is in general much smaller than the curvature part of the phase shift. All interferometers of this type are therefore an excellent tool for the detection of space-time curvature.

It remains the question by which interferometer array we can fulfill all four conditions (10), (9), (12), and (25). It is clear that we need more than $N = 4$ laser beams since the "figure 8" geometry was the unique solution of the first three condition with four lasers. The simplest interferometer fulfilling all conditions must therefore include at least $N = 5$ beam splitters. Fig. 3 shows the "figure 8 $\frac{1}{2}$ " interferometer which is of this type (row 4 of the table). Here one half of the "8" is added to the "figure 8" array of Fig. 2. All laser beams are parallel (all $\vec{k}_i$ agree), and the partial beams are characterized by the histories $\{n_i^I\} = \{0, 1, -1, 1, 0\}$ and $\{n_i^{II}\} = \{1, -1, 1, -1, 1\}$. For this device the curvature dependent phase shift is given by

$$\Delta \phi_R = \frac{c^2}{6} a_l R_{0l0m} k_m (17 + 12\sqrt{2}) T^4. \quad (27)$$

For the experimental data of Kasevich and Chu [6] ($k = 2.2 \cdot 10^7 \text{ m}^{-1}$) this phase shift is about 0.1 rad if the total flight time $T(4 + 2\sqrt{2})$ of the atoms is 0.5 s, and about 0.4 rad if the total flight time is 0.7 s. It should therefore be measurable.

Interferometer set up:		Insensitivity obtained:		
No.	Specification	state dependence	$\vec{a}$ -dependence	$\vec{\omega}$ -dependence
1	closed geometry, equal momentum transfer, $\vec{\omega} = 0, R_{010m} = 0$	exactly vanishing		
2	"figure 8", $\vec{\omega} = 0, R_{010m} = 0$	exactly vanishing	exactly vanishing	
3	"figure 8", $\vec{\omega} \neq 0, R_{010m} = 0$	of order $\omega^2 k v t^3$	of order $\omega k t^3$	of order $\omega k t^3$
4	"figure 8 $\frac{1}{2}$ ", $\vec{\omega} \neq 0, R_{010m} \neq 0$	of order $\omega^3 k v t^4$ and $\omega R_{010m} c^2 k v t^4$	of order $\omega^2 k t^4$ and $\omega c^2 R_{010m} k t^4$	of order $\omega^2 a k t^4$ and $\omega R_{010m} c^2 k v t^4$

Table 1: Properties of various interferometer geometries. v is the initial velocity of the atoms and t is the total flight time. k is the reciprocal wavelength of the laser beams. $\vec{\omega}$ is the rotation of the array. $\vec{a}$ and R_{010m} represent the first terms of a Taylor expansion of a scalar potential influencing the center-of-mass motion of the atoms (compare Eq. (2)).

7 Remaining influences

The model used above is based on certain simplifying assumptions. If these are not made additional influences appear: In the "figure 8" and "figure 8 $\frac{1}{2}$ " geometries an accelerational phase shift is present even in the absence of rotation and for vanishing R_{010m} . One reason for this to occur are relativistic corrections to the motion of the atoms. For the atomic fountain [6] Lämmerzahl [15] showed that this affects the phase shift only to a very small amount. A more important source of accelerational influence is the time evolution of the atoms in the laser beams under the influence of the homogeneous gravitational field which has been neglected here. This evolution was worked out exactly in Ref. [10]. For relatively short interaction times τ of the atoms with the laser field the corrections due to acceleration were shown to be proportional to $\vec{k} \cdot \vec{a} \tau^2$. Adopting the data of Ref. [6] to specify the interferometer array we have an interaction time of about 20 μ s which results in $\vec{k} \cdot \vec{a} \tau^2 \approx 0.08$. This crude estimate which could easily be made precise on the basis of the results in Ref. [10] shows that the corresponding accelerational phase shift may become of comparable magnitude as $\Delta\phi_R \approx 0.4$. A more complete discussion of this topic will be done by Lämmerzahl and Bordé extending Ref. [11]. It should be noted that from the point of view of practical realizations it is likely that the main source of error may be a misalignment of the lasers as was discussed in Ref. [8].

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