THE RELATION OF CONSTITUENT QUARK MODELS TO QCD:
WHY SEVERAL SIMPLE MODELS WORK "SO WELL"

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SUMMARY. We discuss the relationship between exact QCD and constituent quark models (non relativistic, bag or others), to clarify why different models work reasonably in many cases. For this we use the general parametrization method (G.Morpurgo, Phys. Rev. D40, 2997(1989)), now expressed in terms of the standard current quark fields (m_u, m_d a few MeV; $m_s \approx 150$ MeV-at the usual mass renormalization point $q=1$ GeV). The method provides for several quantities the most general exact form of the spin-flavor structure derivable from the QCD Lagrangian. We can thus determine for many important quantities (masses of lowest baryons and mesons, baryon magnetic moments, semileptonic decays ...), from a fit to the data, the coefficients of the parametrization, the same that a direct QCD calculation, if feasible, would give. It turns out that only a few coefficients are relatively important. Because different models, each with its few free parameters, can produce these terms by some choice of parameters, one can see why models so different as non relativistic or quasi-chiral, work "well". Finally, expressing dimensionally the coefficients in the general parametrization in terms of current quark masses and Λ , we find that the m_s expansion of broken $SU_3 \times SU_3$ is just an expansion in $\Delta m / (\xi \Lambda) \approx m_s / (\xi \Lambda) \approx 0.3$. The ξ 's determined from different data result rather close (from 2.3 to 3.7). The effective light quark masses in constituent models result of order $(\xi \Lambda)$. No conclusion above depends on whether the chiral limit $m_u, m_d, m_s \rightarrow 0$ is mathematically sound or not.

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I. INTRODUCTION

The connection between the quasi chiral approach to QCD and the non relativistic quark model [1] (NRQM) of light hadrons (called also naive model) has been rather mysterious since many years. The NRQM not only works qualitatively in the classification of light hadrons, but it also leads to fairly good *quantitative* predictions. How can this happen and be compatible with a description so dissimilar as the chiral one? Can both descriptions be derived from QCD? If so, which is the relationship between the two, and that between current (quasi-chiral) quarks and constituent ones; and, first, what are constituent quarks? Incidentally all these questions apply not only to the NRQM but to any constituent or potential model, such as the MIT bag [3,2b], of "relativistic" type with Dirac four spinors.

To exemplify, consider the De Rujula, Georgi and Glashow (DGG) treatment [4] which was the first attempt to connect the NRQM to the QCD Hamiltonian starting the "QCD inspired" treatments. DGG write the QCD Hamiltonian and calculate, in the semirelativistic Fermi-Breit approximation, the one gluon exchange QCD potential between a quark and an antiquark or between two quarks. By examining the effect of the hyperfine interaction $\vec{g}_i \cdot \vec{g}_k$ on the masses of the lowest hadron states, DGG derive a value for ΔM . From their Eqs. (5) and (11) they obtain for $\Delta M/M_\lambda$ values around 0.35 and conclude: "The value of M_p/M_λ given by (5) or (11) does not coincide with the value obtained from the pseudoscalar meson masses via current algebra. Ours are effective masses of quarks bound in hadrons, not the masses appearing in the phenomenological Lagrangians describing the breaking of $SU(3) \times SU(3)$ ". This statement is correct, but cryptic. It leaves obscure the reason why, having set the task to calculate the hadron masses in terms of parameters of the QCD Hamiltonian (including masses), DGG must conclude that the masses in their final formulas are something different; moreover this "effective quark mass" appears abruptly in their treatment, without having been defined. Of course in QCD the quark masses are running; yet it is not clear from DGG how the quark masses M_i, M_k in their term $(M_i M_k)^{-1} (\vec{g}_i \cdot \vec{g}_k)$ of the hadron masses are related to the masses m in the Lagrangian of QCD.

Recently we have shown [5,6,7] that a description of constituent type can be derived exactly from a relativistic field theory of quark and

gluons, such as QCD. It appeared that any constituent model is nothing but a convenient parametrization of certain physical quantities (e.g. hadron masses, magnetic moments....) in the spin-flavor space. It was shown [5] that the general structure of this parametrization can be derived exactly (thus relativistically, though non covariantly) from QCD, using general properties of QCD, namely the flavor structure of the Lagrangian and the fact that gluons are flavorless and neutral. However we left open the relationship between current and constituent quarks, in particular their masses; we shall fill this gap here.

We divide the presentation in two parts, starting with pure QCD, and concluding with models. *In the first part* (Sects. I to VII) we express the parameters of the general parametrization in terms of the QCD masses of current quarks, and show that: a) Expanding the parameters in powers of $\Delta m = m_s - m$, the scale of this expansion ($\approx 3\Lambda$) extracted from the baryon and meson masses, is found to coincide with the standard scale of the m_s expansion in broken $SU_3 \times SU_3$. b) One is led to a natural definition of the effective mass of a light "constituent quark", the scale of $\approx 3\Lambda$ (more precisely from 2.3Λ to 3.7Λ) just mentioned.

In the second part (Sects. VIII-XI) the general parametrization is shown to allow checking any proposed constituent quark model in a way more useful than a direct comparison of the model predictions to masses, magnetic moments etc.; applying the analysis to several models, we clarify why many are "so good".

Two points of notation and language: a) Above we spoke of "current" quarks and "quasi-chiral" quarks. The two are synonymous and refer to the light (u, d, s) quark fields in the Lagrangian of QCD renormalized at a $q \geq 1 \text{ GeV}$. b) Except when noted, capital M will be used for the effective mass of constituent quarks. Instead $m(q)$ will be the running quark masses of QCD at the renormalization point q; for the standard $q = 1 \text{ GeV}$, we omit writing q and call such masses simply m_u or m_d or m_s (m_u and m_d of a few MeV and $m_s \approx 120-180 \text{ MeV}$).

II. THE GENERAL PARAMETRIZATION OF BARYON MASSES IN TERMS OF CURRENT QUARKS
Call H_{QCD} the exact Hamiltonian of QCD. Its strong part is:

$$H_{\text{QCD}} = H_c + \int d^3x [m(\bar{u}u + \bar{d}d + \bar{s}s) + \Delta m \bar{s}s] \equiv H_c + \int d^3x (m \bar{\Psi}\Psi + \Delta m \bar{\Psi} P^S \Psi) \quad (1)$$

where, for simplicity, we neglected intrinsic isospin breaking, setting:

$$m_u = m_d \equiv m \quad \Delta m \equiv m_s - m \quad (2)$$

and H_C is the chiral invariant part of the Hamiltonian. On the right in (1) Ψ is the quark field:

$$\Psi(x) = \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix} \quad (3)$$

and P^S is the projector on the strange quark field $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \equiv (1 - \lambda_8)/3$:

$$P^S s = s \quad P^S u = 0 \quad P^S d = 0 \quad (4)$$

As to flavor, it is broken only by the Δm term. Recall that M_K^2/M_π^2 calculated using H_{QCD} (1) gives [8] $m_s/m \approx 25$ (or, depending on the corrections [9], $(m_s/m) = 8 \div 25$). This same QCD Hamiltonian leads to the equally time honored value (from 45 MeV to ≈ 60 MeV) of the πN σ -term [10,11].

Now consider the general parametrization. In Refs. [5-7] we selected the renormalization point of the running quark masses in the region of low q 's, so as to have H_{QCD} expressed in terms of renormalized quark fields with mass values in a range typical of those usually assigned to constituent quarks. But, in fact, the parametrization in [5-7] is independent of the choice of the renormalization point of the quark masses. Because here we intend to relate constituent and current quarks, we now think to the QCD Hamiltonian expressed in terms of the standard quark fields with standard masses [8,10], a few MeV for m_u, m_d , corresponding to the conventional renormalization point at $q \approx 1$ GeV. The coefficients of the parametrization are also thought as expressed in terms of the standard masses. Accordingly we adopt the standard u, d, s , for quark fields instead of $\mathcal{P}, \mathcal{N}, \lambda$ of Ref. [5]. We stress that adopting this standard choice of renormalization point does not alter the deduction [5] of the general parametrization. We recall briefly below such treatment.

We start with the masses of the lowest 8 and 10 baryons:

$$M_i = \langle \Psi_i | H_{QCD} | \Psi_i \rangle \quad (5)$$

Here H_{QCD} is the QCD Hamiltonian (1) and $|\Psi_i\rangle$ is the exact eigenstate of the i -th 8 and 10 baryon at rest:

$$H_{\text{QCD}}|\Psi_i\rangle = M_i|\Psi_i\rangle \quad (6)$$

To parametrize a property of the lowest baryons, one imagines to construct the exact baryon eigenstates $|\Psi_i\rangle$ applying a unitary transformation V to a set of simple 3-quark states $|\phi_i\rangle$. (Important: In [5-7] we called these states $|\phi_i\rangle$ "model states"; now, to avoid any possibility of confusion with constituent quark models, we will refer to $|\phi_i\rangle$'s as "auxiliary states"):

$$|\Psi_i\rangle = V|\phi_i\rangle \quad (7)$$

The auxiliary states $|\phi_i\rangle$ and V are defined in Ref. [5] where it was shown how, in principle, V can be constructed (see also Sect. IV).

As shown in [5] it is convenient to select the wave function ϕ_i of the auxiliary states as products of a space (or momentum) factor $X_{L=0}(\underline{r}_1, \underline{r}_2, \underline{r}_3)$ with orbital angular momentum $L=0$ and a symmetrical spin-flavor factor W_i constructed in terms of the spin flavor variables of three quarks:

$$\phi_i = X_{L=0}(\underline{r}_1, \underline{r}_2, \underline{r}_3) \cdot W_i \quad (8)$$

W_i accounts for all the angular momentum of the state i , and, therefore has, necessarily, the SU_6 spin flavor structure. For instance for the proton P (S means symmetrization over 1,2,3) and for Δ^{++} it is:

$$W_P(\uparrow) = (18)^{-1/2} S[\alpha_1(\alpha_2\beta_3 - \alpha_3\beta_2)u_1u_2d_3] \quad (9)$$

$$W_{\Delta^{++}}(\uparrow) = \alpha_1\alpha_2\alpha_3u_1u_2u_3 \quad (10)$$

We underline that all physical results (e.g. the baryon masses to be considered now) are obviously independent of the choice of the auxiliary states $|\phi_i\rangle$. They only depend on H_{QCD} .

Let us recall the general parametrization [5,6d,6f] of the masses M_i of the 8 and 10 baryons. As shown in Ref. [5] it is (i specifies the baryon):

$$M_i = \langle \Psi_i | H_{\text{QCD}} | \Psi_i \rangle = \langle \phi_i | V^\dagger H_{\text{QCD}} V | \phi_i \rangle = \langle W_i | \text{"Parametrized mass"} | W_i \rangle \quad (11)$$

where the last form is what we call the "general parametrization" (W_i are the spin flavor functions defined in (8); the fact that the space variables

have disappeared from the last form of (11) is due to the factorizability (8) of ϕ_i). From ref. [5,6f] it is (compare also Appendix I):

$$\begin{aligned}
 \text{"Parametrized mass"} = & M_0 + B \sum_i P_i^S + C \sum_{i>k} (\underline{g}_i \cdot \underline{g}_k) + D \sum_{i>k} (\underline{g}_i \cdot \underline{g}_k) (P_i^S + P_k^S) + \\
 + E \sum_{\substack{i \neq k \neq j \\ (i>k)}} (\underline{g}_i \cdot \underline{g}_k) P_j^S + & + a \sum_{i>k} P_i^S P_k^S + b \sum_{i>k} (\underline{g}_i \cdot \underline{g}_k) P_i^S P_k^S + c \sum_{\substack{i \neq k \neq j \\ (i>k)}} (\underline{g}_i \cdot \underline{g}_k) (P_i^S + P_k^S) P_j^S \\
 & + d P_1^S P_2^S P_3^S \quad (12)
 \end{aligned}$$

In (12) the $\underline{g}_i$'s are the Pauli matrices; the projectors P_i^S 's on the strange quark were defined above in (4).

A few comments on (12): Because the different masses of the lowest octet and decuplet baryons are 8 (barring e.m. and isospin corrections), Eq.(12), containing 9 parameters (A,B,C,D,E,a,b,c,d), is certainly true, no matter what is the underlying theory. Nevertheless Eq.(12) can be regarded as an exact deduction from QCD in the following sense: We could not write the parametrization (12) if the exact states $|\Psi_i\rangle$ were not related, as in (7), to a set $|\phi_i\rangle$ of three quark-no gluon states. This is the feature of QCD that enters. This being clear, the exact parametrization (12) is not trivial: The values of the 8 parameters obtained fitting the masses (in the analysis only (a+b) intervenes) decrease strongly moving to terms with increasing number of indices (that is [6f] with increasing number of gluons exchanged and/or flavor breaking P_k^S factors).

Note that, in deriving (12) from QCD, $\Delta m \bar{\Psi} P^S \Psi$ in the Lagrangian is treated exactly; Eq.(12) is always true, in particular no matter how large is Δm in the QCD Lagrangian.

In [5,6f] we gave A,B,C,D,E,(a+b),c,d. Here we reanalyzed the data (Appendix II) to determine A,B,...,d after subtraction of electromagnetic and intrinsic isospin effects, using (for wide resonances) both the pole [12] and the conventional masses [13]. The pole values of parameters in (12) are given below in (13), omitting errors, if unimportant. The parameters from conventional baryon masses are similar (Appendix II-A10), but the small ones not identical. The pole parameters (in MeV) are:

$$\begin{aligned}
 A=1076 \quad , \quad B=192 \quad , \\
 C=45.6 \quad , \quad D=-13.8 \pm 0.3 \quad , \quad (a+b)=-16 \pm 1.4, \quad (13) \\
 E=5.1 \pm 0.3 \quad , \quad c=-1.1 \pm 0.7 \quad , \quad d=4 \pm 3
 \end{aligned}$$

The hierarchy of these numbers is evident. The values (13) decrease so strongly that, omitting c and d, the following mass formula results [6d]:

$$\frac{1}{2} (P + \Xi^0) + T = \frac{1}{4} (3\Lambda + 2\Sigma^+ - \Sigma^0) \quad (14)$$

The symbols stay for masses and T is the following combination of decuplet masses:

$$T = \Xi^{*-} - \frac{1}{2} (\Omega + \Sigma^{*-}) \quad (15)$$

The combinations of masses in (14) are independent of electromagnetic effects, to zero order in flavor breaking. This is the reason of the charge combinations in (14,15). The data satisfy Eq.(14) as follows:

$$\text{l.h.s.} = 1132.6 \pm 1.2 \quad \text{r.h.s.} = 1132.6 \pm 0.1 \quad (16)$$

an impressive agreement confirming the smallness of the terms neglected in (12). One more remark: A QCD calculation, if feasible, would express each (A, B, ..., c, d) in (12) in terms of the quantities in the QCD Lagrangian, the running quark masses-normalized at any q that we like to select-and the dimensional (mass) parameter Λ of QCD [$\alpha_s(q^2) = 4\pi(\beta_0 \ln q^2 / \Lambda^2)^{-1}$ for $q \gg 1$ GeV]; for instance (recall that we set $m_u = m_d \equiv m$):

$$A \equiv \Lambda \hat{A}(m(q)/\Lambda, m_s(q)/\Lambda) \quad (17)$$

where $\hat{A}$ is some function. Similarly for B, C, D, E, a, b, c, d. To simplify the notation we set in what follows $\Lambda=1$, reinstalling Λ when appropriate and suppress the upper $\hat{}$ on the r.h.s. of A etc.

Note finally that, while the derivation of the general parametrization needs only general properties of the Lagrangian of QCD, the asymptotic freedom typical of QCD enters when we introduce $\Lambda \approx 150-200$ MeV in (17), as will be essential in what follows.

III. PARAMETRIZATION OF OCTET BARYON MAGNETIC MOMENTS AND MESON MASSES

For later use, we display also the parametrizations of magnetic moments of octet baryons [5] and the masses of lowest meson nonets [6b].

Baryon magnetic moments

Introduce the magnetic moment operator in the rest frame:

$$\underline{M} = (1/2) \int d^3 \underline{r} [\underline{r} \times \underline{j}(\underline{r})] \quad (18)$$

where $\underline{j}(\underline{r})$ is the space part of the electromagnetic current at $t=0$:

$$j_\mu(x) = ie \left[\frac{2}{3} \bar{u}(x) \gamma_\mu u(x) - \frac{1}{3} \bar{d}(x) \gamma_\mu d(x) - \frac{1}{3} \bar{s}(x) \gamma_\mu s(x) \right] \equiv \frac{ie}{2} [\bar{\Psi}(x) (\lambda_3 + \frac{1}{3} \lambda_8) \gamma_\mu \Psi(x)] \equiv ie [\bar{\Psi}(x) Q \gamma_\mu \Psi(x)] \quad (19)$$

The charge Q in (19) is, in terms of the λ 's or of the projectors P^u, P^d, P^s :

$$Q = \frac{1}{2} (\lambda_3 + \frac{1}{3} \lambda_8) = \frac{2}{3} P^u - \frac{1}{3} P^d - \frac{1}{3} P^s \quad (20)$$

The formula that now replaces (11) for the masses is:

$$M_i = \langle \Psi_i | M_Z | \Psi_i \rangle = \langle \phi_i | V^\dagger M_Z V | \phi_i \rangle = \langle W_i | \text{"Parametrized magn.moment"} | W_i \rangle \quad (21)$$

Here we give the general parametrization of magnetic moments keeping only terms linear in P^S . For baryon masses Eq.(12) was exact to all orders in P^S ; the same is true for Eq.(28) below for meson masses. For the magnetic moments we might easily write the parametrization to all orders in P^S , but then one would have too many parameters to make it useful. The terms neglected, bilinear or cubic in P^S , are expected to be at most 5% of the dominant term [6]. Keeping only terms linear in P^S the parametrization of the magnetic moments of the baryon octet has eight terms [5,14]:

$$\text{"Parametrized magn.moment"} = \sum_{\nu=0}^7 g_\nu (\underline{G}_\nu)_Z \quad (22)$$

with:

$$\begin{aligned} \underline{G}_0 &= \text{Tr}[Q P^S] \cdot \sum_i \underline{\sigma}_i & \underline{G}_1 &= \sum_i Q_i \underline{\sigma}_i & \underline{G}_2 &= \sum_i Q_i P_i^S \underline{\sigma}_i \\ \underline{G}_3 &= \sum_{i \neq k} Q_i \underline{\sigma}_k & \underline{G}_4 &= \sum_{i \neq k} Q_i P_i^S \underline{\sigma}_k & \underline{G}_5 &= \sum_{i \neq k} Q_i P_k^S \underline{\sigma}_k, \\ \underline{G}_6 &= \sum_{i \neq k} Q_i \underline{\sigma}_i P_k^S & \underline{G}_7 &= \sum_{i \neq k \neq j} Q_i P_k^S \underline{\sigma}_j \end{aligned} \quad (23)$$

As remarked in [7a] the coefficient g_0 of $\underline{G}_0$ is expected, due to general arguments, $\approx 10^2$ times smaller than g_1 and, therefore, negligible. Omitting $\underline{G}_0$ the data determine the other 7 coefficients $g_1, g_2, \dots, g_7$. Fitting the observed moments gives (in proton magnetons):

$$(24)$$

$$g_1 = 2.79, g_2 = -0.94, g_3 = -0.076, g_4 = 0.41, g_5 = 0.097, g_6 = -0.134, g_7 = 0.155$$

showing that the first two terms are appreciably larger than the remaining ones; thus one understands why the "naive" NRQM (in which only g_1 and g_2 are kept) gives a fair description of the magnetic moments. Indeed neglecting,

besides g_0 , all coefficients from g_3 to g_7 we remain with the "naive" [1a] additive form of the "Parametrized magnetic moment" operators, namely:

$$g_1 \sum_i [1 + (g_2/g_1) P_i^S] Q_i \underline{g}_i \quad (25)$$

With the above values of g_1 and g_2 :

$$g_1 = 2.79 \quad g_2/g_1 = -0.34 \quad (26)$$

(25) gives a fit (Ref.[5], fig.1) correct to about 15% of all octet magnetic moments. Note: Keeping g_0 and neglecting g_7 , which is also expected [6f] small because is a two gluon exchange, flavor breaking term, the values of g_1 and g_2 in (22) would remain essentially the same listed in (24).

Meson masses

The general parametrization of meson masses proceeds as for baryons. Here we give the parametrization only for the lowest pseudoscalar and vector mesons with isospin $I \neq 0$ (that is π, K, ρ, K^*). The $I=0$ mesons ($\eta, \eta', \omega, \phi$) are treated in [6b]. Eq.(11) now becomes

$$M_i = \langle \psi_i | H_{\text{QCD}} | \psi_i \rangle = \langle \phi_i | V^\dagger H_{\text{QCD}} V | \phi_i \rangle = \langle w_i(1,2) | \text{"Parametrized mass"} | w_i(1,2) \rangle \quad (27)$$

where:

$$\text{"Parametrized mass"} = A + B(P_1^S + P_2^S) + C \underline{g}_1 \cdot \underline{g}_2 + D \underline{g}_1 \cdot \underline{g}_2 (P_1^S + P_2^S) \quad (28)$$

Similar formulas ([5,6b]) could be written for any power of the masses.

Once more the parametrization (28) is exact, to all orders in P^S . Again this formula looks trivial: four masses and four parameters. But two aspects of (28) are not trivial, as for baryons: 1) Its structure is typical of a NRQM description; yet (28) follows exactly from QCD. 2) The coefficients decrease in magnitude from A to D (see Eq.(31)).

In the last form of (27) the w_i 's are the spin-flavor functions

$$w_i(1,2) = \chi_i(1,2) \cdot f_i(1,2) \quad (29)$$

for the auxiliary states $|\phi_i\rangle$ of a quark (1)-antiquark (2) corresponding to each mesons π, K, ρ, K^* ; in (29) χ_i 's are obviously a singlet spin function for π and K , a triplet for ρ and K^* ; f_i 's are the flavor functions, e.g. $u_1 \bar{s}_2$ for a K^+ or K^{*+} . In (28) A, B, C, D are four real parameters. The other symbols are

obvious. Of course, see (17), A, B, C, D are Λ times functions of $m(q)/\Lambda, m_s(q)/\Lambda$, that could be determined if we were able to calculate with QCD.

Recalling that $\underline{g}_1 \cdot \underline{g}_2 = -3$ for $J=0$ and $\underline{g}_1 \cdot \underline{g}_2 = +1$ for $J=1$, the meson masses (indicated with the meson symbols) are:

$$\begin{aligned} \pi &= A-3C \quad (=138) & K &= A-3C+B-3D \quad (=495) \\ \rho &= A+C \quad (=770) & K^* &= A+C+B+D \quad (=894) \end{aligned} \quad (30)$$

Therefore (in MeV):

$$A=612, B=182, C=158, D=-58 \quad (31)$$

We conclude with the following remark. The pion mass is:

$$\pi = A-3C = 138 \quad (32)$$

with $A=612$ and $C=158$. The idea that in the perfect chiral limit (mass zero of u, d quarks) the pion would be massless, that is:

$$A-3C=0 \quad (33)$$

is almost universally held. Then the pion is looked as a quasi-Goldstone boson, getting its mass from explicit breaking of chiral symmetry due to the small u and d quark masses. This description, extended to all mesons of the lowest pseudoscalar octet, makes them all quasi-Goldstone bosons. We will not discuss this standard chiral picture (which accounts for, but is not strictly required by, the great classical successes of current algebra plus PCAC). We must add, nevertheless, that the pion mass on the r.h.s. of $A-3C=138$ is not so small on the scale of the parameters A, B, C, D . Due to the 3 in front of C , a percentually minor change of C (possibly produced in QCD by a comparatively small change of α_s) might equally well lead to $A-3C \approx 0$ or to $A-3C$, say 350, making the pion mass comparable to the others in the octet.

IV. THE DEPENDENCE OF THE TRANSFORMATION V ON THE RENORMALIZATION POINT OF THE RUNNING QUARK MASSES IN THE LAGRANGIAN

We digress briefly to examine more closely the question of the choice of the mass renormalization point in constructing the parametrization. We stated that the parametrization (thus V in Eqs. (7), (11), (21) or (27)) can be introduced, in principle, for any choice of the renormalization point q for the quark masses in the QCD Hamiltonian; also for $q \approx 1$ GeV, and, therefore, m_u and m_d a few MeV. As already stated, it is important to have this clear because in Ref. [5-7] we were thinking to the QCD Hamiltonian expressed in

terms of quark masses renormalized at a low value of q , so as to have u and d masses in the range—a few hundred MeV—usually assigned to constituent quarks. That choice is possible but unnecessary. *Here we have adopted the conventional choice.*

To see where the renormalization point enters in V , express, therefore, from now on, the QCD Hamiltonian in terms of quark fields with masses $m(q)$ defined at some definite freely chosen renormalization four momentum q . Decompose the QCD Hamiltonian as:

$$H_{\text{QCD}} = H_a + H_b \quad (34)$$

where H_b is the quark-gluon interaction plus the flavor breaking mass term and H_a is all the rest; thus H_a is flavor invariant (all quarks in it with mass $m(q)$). Introduce a complete set of states $|\nu(q)\rangle$ of H_a :

$$H_a |\nu(q)\rangle = E_a(q) |\nu(q)\rangle \quad (35)$$

where q indicates the selected renormalization point. To be definite we refer below to baryons; for mesons everything goes similarly.

Write the auxiliary states $|\phi_i\rangle$, introduced in (11), as:

$$|\phi_i\rangle = \sum_{\mathbf{p}, r} c_{r_1 r_2 r_3}^i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) a_{\mathbf{p}_1 r_1}^\dagger a_{\mathbf{p}_2 r_2}^\dagger a_{\mathbf{p}_3 r_3}^\dagger |0\rangle \quad (36)$$

where $\sum_{\mathbf{p}, r}$ stays for $\sum_{\mathbf{p}_1 \mathbf{p}_2 \mathbf{p}_3, r_1 r_2 r_3}$. In (36) the $a_{\mathbf{p}r}^\dagger$ are creation operators of quarks of momentum $\mathbf{p}$ and spin-flavor-color index r (we omit color except when necessary); $|0\rangle$ is the vacuum state, in a Fock space of quarks and gluons. In constructing the auxiliary states the masses of quarks u, d , and also s are taken equal. For simplicity—this is not necessary—we identify this value with the (common) mass $m(q)$ of u, d at the renormalization point q . In (36) $c_{r_1 r_2 r_3}^i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$ is the momentum space function of the auxiliary state of the three quarks; being in the rest frame, $c_{r_1 r_2 r_3}^i(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$ contains a factor $\delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3)$. The quark spin states are taken as four spinors with the upper components $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and two zeros in the lower components. A transformation of Foldy-Wouthuysen type is part of V . The auxiliary states $|\phi_i\rangle$ (36) can be seen [5] as the lowest (degenerate)

eigenstates of some auxiliary Hamiltonian $\mathcal{H}$, defined in the Fock sector of three quarks, no antiquark, no gluon; $\mathcal{H}$ is useful only to show how V can be constructed by the adiabatic procedure (appendix to Ref.[5]). As already stated, no physical result, in particular baryon or meson masses or magnetic moments, depends on the choice of the auxiliary state, that is of $\mathcal{H}$.

We now characterize V . The transformation V is simply a correspondence between a certain set of auxiliary states $|\phi_i\rangle$ and the exact states $|\psi_i\rangle$ of interest. To characterize V expand the exact state $|\psi_i\rangle$ in the complete set of states $|\nu(q)\rangle$ of Eq.(35):

$$|\psi_i\rangle = \sum_{\nu(q)} |\nu(q)\rangle \langle \nu(q) | V | \phi_i \rangle \quad (37)$$

In Eq.(37) the sum (that is the expansion of the exact state in terms of Fock quark-gluon states) extends to all possible eigenstates $\nu(q)$ of H_a .

Clearly Eq.(37) defines V through:

$$\langle \nu(q) | V | \phi_i \rangle \equiv \langle \nu(q) | \psi_i \rangle \quad (38)$$

for any $\nu(q)$; thus V can be defined selecting freely the renormalization point q of the running quark mass, as long as, for any q , the states $\nu(q)$ are a complete set. Less formally Eq.(37) means that the exact state $|\psi_i\rangle$ has an extremely complicated structure in Fock space. Schematically:

$$|\psi_i\rangle = |qqq\rangle + |qqq\bar{q}\bar{q}\bar{q}\rangle + |qqq, \text{Gluons}\rangle + \dots \quad (39)$$

where the ellipsis are states (in the $\underline{p}=0$ frame) with any number n of quarks, $n-3$ antiquarks and any number of gluons, provided only that the conserved quantum numbers of the Fock states on the r.h.s. of (39) (color, charge, baryonic number, strangeness, parity, angular momentum) are the same as those of $|\psi_i\rangle$ ($|\phi_i\rangle$ has of course the same quantum numbers as $|\psi_i\rangle$).

V. THE EXPANSION OF THE MESON MASS PARAMETERS IN TERMS OF $\Delta m_S - m_S$

We now analyze the mass parametrization of the lowest hadrons. For simplicity consider first the mesons. The result of a full QCD calculation of the "Parametrized mass" (28), showing the most general dependence of the "Parametrized mass" from the quark masses in the QCD Lagrangian, can be written as:

$$\text{"Parametrized mass"} = \phi(m, \Delta m | m + \Delta m P_1^S, m + \Delta m P_2^S) + (\underline{g}_1 \cdot \underline{g}_2) F(m, \Delta m | m + \Delta m P_1^S, m + \Delta m P_2^S) \quad (40)$$

Here the two functions ϕ and F of m and Δm multiplying the spin independent and spin dependent part are assumed to result from a QCD calculation of $V^\dagger H_{\text{QCD}} V$ after contraction of all creation and destruction operators and integration on the space (or momentum) variables. That is, we think to ϕ and F calculated, from first principles, in QCD.

The functions ϕ and F depend on the masses in the QCD Lagrangian, m and Δm , in two different ways, as illustrated in fig.1: a) A first dependence comes from the external lines and carries the indices of the quarks in the auxiliary state; the QCD Lagrangian shows that this dependence entails Δm multiplied by the projectors P_i^S . If, doing the QCD calculation, we keep all P_i^S (without exploiting $(P_i^S)^n = P_i^S$), the dependence of ϕ and F on the P_i^S 's is uniquely determined. In Eq.(40) this dependence on $\Delta m P_i^S$ appears in the arguments of ϕ and F on the right of the bar $|$. In fact it is slightly more convenient, as we did, to insert as arguments on the right of the $|$ in (40) $(m + \Delta m P_i^S)$ instead of $\Delta m P_i^S$. b) The second dependence of ϕ and F on m and Δm comes from internal quark loops in the "blob" of fig.1. This dependence is noted in the arguments on the left of the vertical bar in ϕ and F . It goes without saying that, though the numerical values of the quark running masses at a given q are definite, we imply, speaking of the m dependence of ϕ and F , that QCD makes sense also in a range of values of these masses (as QED can be expressed in terms of the electron mass, though the latter is 0.51 MeV).

Eq.(40) can be written slightly more compactly as:

$$A + B(P_1^S + P_2^S) + [C + D(P_1^S + P_2^S)] (\underline{g}_1 \cdot \underline{g}_2) \equiv \phi(m, \Delta m | m_1, m_2) + F(m, \Delta m | m_1, m_2) (\underline{g}_1 \cdot \underline{g}_2) \quad (41)$$

where we set:

$$m_1 = m + \Delta m P_1^S, \quad m_2 = m + \Delta m P_2^S \quad (42)$$

and $\phi(m, \Delta m | m_1, m_2)$ and $F(m, \Delta m | m_1, m_2)$ are symmetric in 1,2. Exploiting now

$$(P_i^S)^n = P_i^S \quad (i=1,2) \quad (43)$$

and recalling

$$m_s = m + \Delta m \quad (44)$$

it is:

$$\phi(m, \Delta m | m_1, m_2) = \phi(m, \Delta m | m, m) + [\phi(m, \Delta m | m_s, m) - \phi(m, \Delta m | m, m)] (P_1^S + P_2^S) \quad (45)$$

$$F(m, \Delta m | m_1, m_2) = F(m, \Delta m | m, m) + [F(m, \Delta m | m_s, m) - F(m, \Delta m | m, m)] (P_1^S + P_2^S)$$

so that:

$$A = \phi(m, \Delta m | m, m) \quad B = \phi(m, \Delta m | m_s, m) - \phi(m, \Delta m | m, m) \quad (46)$$

$$C = F(m, \Delta m | m, m) \quad D = F(m, \Delta m | m_s, m) - F(m, \Delta m | m, m)$$

Reinstalling Λ (see end of Sect. II), it is, more explicitly

$$\phi(m, \Delta m | m, m) \equiv \Lambda \phi(m/\Lambda, \Delta m/\Lambda | m/\Lambda, m/\Lambda) \quad (47)$$

$$F(m, \Delta m | m, m) \equiv \Lambda F(m/\Lambda, \Delta m/\Lambda | m/\Lambda, m/\Lambda)$$

with similar expressions for all other quantities. Again, with some exceptions, we set $\Lambda=1$ in what follows.

The ratios B/A and D/C are

$$\frac{B}{A} = \frac{\phi(m, \Delta m | m_s, m) - \phi(m, \Delta m | m, m)}{\phi(m, \Delta m | m, m)} \quad \frac{D}{C} = \frac{F(m, \Delta m | m_s, m) - F(m, \Delta m | m, m)}{F(m, \Delta m | m, m)} \quad (48)$$

From now on, to simplify the notation, we omit the arguments on the left of the bar $|$ in all functions; we keep memory of them by the notation:

$$\phi(m, \Delta m | m, m) \equiv \phi(|m, m) \quad (49)$$

using a similar $(|$ symbol for all the intervening functions. It is important to recognize that if a ratio as those in Eq. (48) above is expanded in Δm , no contributions to first order in Δm arise from the Δm dependence of the functions ϕ and F in the arguments on the left of the $|$. In other words for the first order terms in the above mentioned expansion one can forget the Δm dependence of ϕ and F on the left of the bar and consider only the Δm dependence from $m_s = m + \Delta m$ in ϕ and F .

We now expand $F(m, \Delta m | m_s, m)$ and $F(m, \Delta m | m, m)$ as well as $\phi(m, \Delta m | m_s, m)$, $\phi(m, \Delta m | m, m)$ in series of Δm , assuming the expansion possible at m . Reinstalling here Λ , it is:

$$\phi(|(m+\Delta m)/\Lambda, m/\Lambda) / \phi(m/\Lambda, m/\Lambda) = 1 + (\Delta m / \beta_o \Lambda) + \gamma_o (\Delta m / \beta_o \Lambda)^2 + \dots \quad (50)$$

$$F(|(m+\Delta m)/\Lambda, m/\Lambda) / F(m/\Lambda, m/\Lambda) = 1 + (\Delta m / \beta_h \Lambda) + \gamma_h (\Delta m / \beta_h \Lambda)^2 + \dots$$

where β, γ are some coefficients (the index h refers to the hyperfine terms, the index o to the spin independent ones).

Recalling $\Delta m \gg m$ and thus $\Delta m \approx m_s$ it is

$$B/A = (m_s/\beta_o \Lambda) + \gamma_o (m_s/\beta_o \Lambda)^2 + \dots; \quad D/C = (m_s/\beta_h \Lambda) + \gamma_h (m_s/\beta_h \Lambda)^2 + \dots \quad (51)$$

If the series on the r.h.s. of (51) converges fast enough, we have (compare the experimental values of A, B, C and D in (31)):

$$|m_s/\beta_o \Lambda| \approx |B/A| = 0.30 \quad |m_s/\beta_h \Lambda| \approx |D/C| = 0.37 \quad (52)$$

Note that B/A and D/C (and therefore β_o and β_h) have opposite signs.

In fact, assuming that the second term γ in the expansions (51) is of order (first term)² with an unknown sign, one should write, instead of (52):

$$|m_s/\beta_o \Lambda| = 0.30 \pm 0.09 \quad |m_s/\beta_h \Lambda| = 0.37 \pm 0.13 \quad (53)$$

This scale in m_s is compatible with that assumed in chiral perturbation theory [11], where the expansion parameter governing Kaon physics is taken to be $(M(K)/S)^2$ ($M(K)$ =Kaon mass and S a mass between that of the ρ and of a scalar meson ≈ 1 GeV -thus $(M(K)/S)^2$ between 0.25 and 0.40). Note that the actual values of the coefficients β depend, as Δm or m_s , on the chosen renormalization point q and we refer here to the standard point $q=1$ GeV.

It is of some interest to note that, in determining these expansion parameters $|m_s/\beta \Lambda|$, no assumption is made on the existence of the chiral limit of $SU_3 \times SU_3$, that is on the behavior of $\phi(|m, m)$ or $F(|m, m)$ near $m=0$. Non analyticity at $m \rightarrow 0$ might imply that expanding from m up to m_s is not possible. *But even then we can proceed exactly as above only expanding in Δm near m_s and moving down to m .* With trivial changes the same expansion holds. (More generally none of the above or the following conclusions depends on whether the exact chiral limit $m_u, m_d, m_s \rightarrow 0$ is mathematically sound or not).

The scale of the expansion in m_s is derived here simply from the B/A ratio +0.30 or D/C ratio -0.37, typical flavor breaking effects, e.g. $D/C = [F(|m_s/\Lambda, m/\Lambda)/F(|m/\Lambda, m/\Lambda)] - 1 = -0.37$. Indeed, long ago (before [8]), instead of the m_s expansion scale, one used to speak of a flavor breaking expansion. Below we may use occasionally this language; because $m_s \cong \Delta m$, the two are equivalent; the difference, of course, with respect to old times is that now the expansion is not in $\Delta m/m$ (as it was originally) but in $\Delta m/(\beta \Lambda)$.

As to the convergence of the expansion (51), we now will see that it is supported by the data in the analogous case of baryon masses.

VI. THE EXPANSION OF BARYON MASS PARAMETERS IN TERMS OF $\Delta m \equiv m_s - m \cong m_s$

In the baryon masses (Eq.(12)) consider first the hyperfine terms, with coefficients C, D, E, b, c . To $d = 3d_h + d_o$ contribute the hyperfine term ($3d_h$) and the spin independent one (d_o). Experimentally it is impossible to determine the magnitude of each. As to b , the data determine only $a+b$.

As for mesons (Sect.V) we write the coefficients of hyperfine terms as:

$$\sum_{i>k} \left[C + D(P_i^S + P_k^S) + E \sum_{j \neq k, i} P_j^S + b P_i^S P_k^S + c \sum_{j \neq k, i} (P_i^S + P_k^S) P_j^S + d_h \sum_{j \neq k, i} P_k^S P_i^S P_j^S \right] (\mathbf{g}_i \cdot \mathbf{g}_k) =$$

$$= \sum_{\substack{i \neq k \neq j \\ (i>k)}} F(m, \Delta m | m_i, m_k; m_j) (\mathbf{g}_i \cdot \mathbf{g}_k) \quad (54)$$

where the notation $x, y; z$ in $F(m, \Delta m | x, y; z)$ recalls that such function (derivable in principle from QCD) is symmetric in x, y , but not necessarily in z . As for mesons, we omit the arguments on the left of the bar, setting:

$$F(m, \Delta m | m_i, m_k; m_j) \equiv F(|m_i, m_k; m_j) \quad (55)$$

Again set, as in (42):

$$m_i = m + \Delta m P_i^S \quad (56)$$

and use the property (43). The function $F(|x, y; z)$ on the r.h.s. of (55) is determined for values of x, y and z that can be either m or $m_s = m + \Delta m$. To simplify the formulas we write s for m_s . Thus

$$\begin{aligned} C &= F(|m, m; m) & D &= F(|s, m; m) - F(|m, m; m) & E &= F(|m, m; s) - F(|m, m; m) \\ b &= F(|m, m; m) - 2F(|s, m; m) + F(|s, s; m) \\ c &= F(|m, m; m) - F(|s, m; m) + F(|s, m; s) - F(|m, m; s) \\ d_h &= F(|s, s; s) - F(|m, m; m) + F(|m, m; s) - F(|s, s; m) + 2\{F(|s, m; m) - F(|s, m; s)\} \end{aligned} \quad (57)$$

We also consider the spin independent part of the parametrization. It is:

$$M_o + B \sum_i P_i^S + a \sum_{i>k} P_i^S P_k^S + d_o P_1^S P_2^S P_3^S = \phi(|m_1, m_2, m_3) \quad (58)$$

In (58) ϕ is, as F , a function of $m, \Delta m$ and of the three indexed masses (or better of m_1/Λ , etc) now symmetric in 1, 2, 3 (thus the notation $x, y; z$ in F is replaced by x, y, z). It is:

$$\begin{aligned}
M_0 &= \phi(|m, m, m) & B &= \phi(|s, m, m) - \phi(|m, m, m) & a &= \phi(|m, m, m) - 2\phi(|s, m, m) + \phi(|s, s, m) \\
d_0 &= \phi(|s, s, s) - 3\phi(|s, s, m) + 3\phi(|s, m, m) - \phi(|m, m, m) & & & & (59)
\end{aligned}$$

Proceeding as for mesons, consider first the hyperfine terms and the ratio $D/C \cong -0.3$. Because it is

$$\frac{D}{C} = \frac{F(|m_s, m; m) - F(|m, m; m)}{F(|m, m; m)} \quad (60)$$

we can again expand F in powers of $\Delta m/\Lambda$ at m . The expansion is similar to (50), but of course not identical, since the function F of three variables in (54) differs from the F in (41). We thus have:

$$D/C = (m_s/\beta'_h \Lambda) + \gamma'_h (m_s/\beta'_h \Lambda)^2 + \dots \quad (61)$$

with β'_h, γ'_h replacing β_h and γ_h in (50). Because D/C is now -0.3 , instead of -0.37 for mesons, (53) is replaced by:

$$|m_s/\beta'_h \Lambda| \approx 0.30 \pm 0.09 \quad (62)$$

(having again, arbitrarily, estimated the uncertainty as the square of the first term in the expansion) so that the flavor breaking scale, or, if one prefers, the $SU_3 \times SU_3$ breaking scale, $(\beta'_h \Lambda)$ for baryons is near to that $(\beta_h \Lambda)$ for mesons. Note that the signs of β_h for mesons and baryons are the same. While for mesons the convergence of the expansion (50) was assumed, here the availability of more coefficients (and their strong decrease—see (13)) allows a check. We show first in general that the experimental hierarchy of the coefficients, together with the Eqs. (57) and (59) expressing the coefficients in terms of ϕ and F , strongly indicates convergence in $(\Delta m/\Lambda)$; next we analyze in more detail the situation.

General argument: Expanding in series of Δm the expressions of the coefficients given in (57) and (59), it appears immediately that the expansion starts with a term of order Δm for D, E , of order $(\Delta m)^2$ for a, b, c and of order $(\Delta m)^3$ for d_0, d_h . Of course to see this we do not need (57)(59). Just look to the number of P^S that multiply each coefficient, since in the Lagrangian only the product $\Delta m \cdot P^S$ intervenes. Thus:

$$D \text{ and } E = O(\Delta m), \quad a \text{ and } b \text{ and } c = O(\Delta m^2), \quad d_0 \text{ and } d_h = O(\Delta m^3) \quad (63)$$

Here by $O(\Delta m^3)$ we mean, for instance, that the first non vanishing term in an

expansion in Δm (or better $\Delta m/\Lambda$) is of third order. This general result alone, together with the experimental values of the coefficients, suggests that terms associated to higher powers of $(\Delta m/\Lambda)$ are indeed smaller.

The power in $\Delta m/\Lambda$ is however not the only reason for the striking decrease in the coefficients. For a more detailed analysis, consider first $E/C \approx 0.11$. As noted in Ref. [6f], where the hierarchy of the coefficients was discussed in detail, the term associated to E has just one P^S and therefore is of the same order in $\Delta m/\Lambda$ as D . However E multiplies a three index term whereas D multiplies a two index term. Three-index terms should arise, in a QCD calculation, from diagrams exchanging at least one more gluon than diagrams giving the main contribution to terms with two indices [15]. Because hyperfine terms represent chromo-magnetic interactions of two dipoles, they should be, intrinsically, short range. We take $|E/D| \approx 0.37$ [16] as an estimate of the reduction due to this additional gluon (hard, on the average) and refer to [6f] and Appendix II for some additional detail.

With the reduction factor for "one gluon more" 0.37 and the flavor scale $\Delta m/(\beta'_h \Lambda) = 0.3$, the order of magnitude of $|c|$ is expected $(0.37 \times 0.3) |D| = 0.11 |D| \approx 1.5$. It is (Appendix II) $c = -1.1 \pm 0.7$. We get an estimate for $|d_h|$ multiplying $|c|$ by 0.3; it is $|d_h| = +0.3 \pm 0.2$

Consider now the parameters of the spin independent terms: M_0, B, a and d_0 . From B and M_0 we have

$$\frac{B}{M_0} = \frac{\phi(|m_s, m, m) - \phi(|m, m, m)}{\phi(|m, m, m)} = (m_s/\beta'_0 \Lambda) + \gamma'_h (m_s/\beta'_0 \Lambda)^2 + \dots \approx 0.18 \quad (64)$$

from which we estimate:

$$(m_s/\beta'_0 \Lambda) = 0.18 \pm 0.03 \quad (65)$$

The order of magnitude is comparable to that of $|m_s/\beta'_h \Lambda|$ (though it can differ by as much as 2). The sign of (65) is opposite to that from the hyperfine terms, as for mesons.

Coming to a , the data determine only $(a+b) = -16 \pm 1.4$. Taking, as order of magnitude, $|b/D| \approx |D/C|$, we have $|b| \approx 4$. If $b > 0$ it is $a \approx -20$; for $b < 0$, it is $a \approx -12$. Therefore $|a/B| \approx 0.06 \pm 0.1$, which implies again a large reduction factor of a with respect to the additive term B . The different physical meaning of a and B does not however allow to relate this reduction factor to that of

E/D. With a similar depression factor, $|d_o|$ is expected $\approx 0.1|a| \approx 2$ leading to $|d| = |d_o + 3d_h| \approx 2 \pm 1$ (experiment: $d = 4 \pm 3$). In conclusion the m_s/Λ expansion scale for spin independent terms is near to that of hyperfine terms, though we don't have an equivalent for the "hard" gluon chromo-magnetic argument (except the usual hand waving one that soft gluons produce confinement and, after this is taken into account, the remaining are hard on the average).

For later use (Sect. VII) we add a remark: Consider the hyperfine terms for baryons. Assume that in (54) $F(|m_i, m_k; m_j\rangle)$ is approximately factorizable: $F(|m_i, m_k; m_j\rangle) \approx f(m_i)f(m_k)\varphi(m_j)$. Put $\rho = [f(m_s) - f(m)]/f(m)$, $\omega = [\varphi(m_s) - \varphi(m)]/\varphi(m)$. Eqs. (57) lead to:

$$D/C = \rho \approx -0.30 \quad E/C = \omega \approx 0.1 \quad b/C = +\rho^2 \approx +0.09 \quad c/C = \rho\omega \approx -0.03 \quad d_h/C = +\rho^2\omega \approx 0.01 \quad (66)$$

These agree with the above orders of magnitude. Incidentally, factorizability fixes the signs of b, c and d_h .

VII. THE MAGNETIC MOMENTS OF OCTET BARYONS

The general parametrization of the baryon octet moments was given in Sect. III. Differently from baryon and meson masses (where formulas (12) and (28) of the parametrization are exact), we limited for magnetic moments to terms linear in P^S . Recall that g_1 and g_2 in the general parametrization, provide alone a fit to 15%. With only g_1, g_2 the parametrization reduces to :

$$g_1 \sum_i [1 + (g_2/g_1)P_i^S] Q_i \underline{g}_i \quad (25)$$

and the physical meaning of

$$-g_2/g_1 = 0.34 \pm 0.01 \quad (67)$$

is obvious: $(1 + (g_2/g_1)) = 0.66$ is the ratio of strange to non strange quark magnetic moments.

The "Parametrized magnetic moment" of the baryon octet is written compactly similarly to the "Parametrized baryon masses" (Sect. V) with the help of three functions χ, η, ξ of $(m/\Lambda), (\Delta m/\Lambda)$. Again these functions, in principle, are derivable from QCD. It is:

$$\begin{aligned}
\text{"Parametrized magnetic moment"} = & (\text{TrQP}^S) \sum_{\substack{i \neq k \neq j \\ (i > k)}} \chi(m, \Delta m | m_i; m_k, m_j) \underline{g}_i + \\
& + \sum_{\substack{i \neq k \neq j \\ (i > k)}} \eta(m, \Delta m | m_i; m_k, m_j) Q_i \underline{g}_i + \sum_{\substack{i \neq k \neq j \\ (i > k)}} \xi(m, \Delta m | m_i, m_k, m_j) Q_k \underline{g}_i \quad (68)
\end{aligned}$$

with $m_i = m + \Delta m P_i^S$ and the functions χ and η symmetric in k, j . The TrQP^S term in (68) is of order $\Delta m/\Lambda$, and is related to g_0 in the general parametrization of Sect. III ($g_0 = \chi(m, \Delta m | m, m, m)$). The other terms produce the spin flavor structures $Q_i \underline{g}_i$ and $Q_k \underline{g}_i$ multiplied by unity or by products of P_j^S with up to three factors. Extracting from (68) all terms either with no P^S or linear in P^S we reobtain of course the terms listed in the general parametrization of Sect. III. (Some terms bilinear in P^S are unduly absent in the list of Ref. [5], Eqs. 37-39. This is of no consequence because we never used for magnetic moments terms bilinear or cubic in P^S).

Here we discuss the terms extracted from η in (68), the only ones of interest for Eq. (25) (in this comparison χ and ξ do not intervene). Again we shorten $\eta(m, \Delta m | m_1; m_2, m_3)$ in $\eta(|m_1; m_2, m_3)$. It is, identically:

$$\begin{aligned}
\eta(|m_i; m_k, m_j) = & \eta(|m; m, m) + [\eta(|s; m, m) - \eta(|m; m, m)] P_i^S + [\eta(|m; s, m) - \eta(|m; m, m)] (P_k^S + P_j^S) + \\
& + (\text{terms bilinear and cubic in } P^S). \quad (69)
\end{aligned}$$

Thus, comparing with (25),

$$g_2/g_1 = \frac{\eta(|s; m, m) - \eta(|m; m, m)}{\eta(|m; m, m)} = (\Delta m/\beta''\Lambda) + \gamma''(\Delta m/\beta''\Lambda)^2 + \dots \quad (70)$$

Identifying (as in Sects. V, VI) Δm with m_s :

$$g_2/g_1 \cong (m_s/\beta''\Lambda) + \gamma''(m_s/\beta''\Lambda)^2 + \dots \quad (71)$$

where β'' and γ'' are coefficients in the expansion. Assuming, as previously, that the second term in the expansion has the order of magnitude of the square of the first, it is:

$$(\Delta m/\beta''\Lambda) = -0.34 \pm 0.11 \quad (72)$$

A classical remark (to be inserted more properly in the ensuing sections, where we will consider constituent quarks) is this: Approximating magnetic moments with (25), and *defining effective masses of quarks* as inversely proportional to magnetic moments, the ratio between the effective masses of

the strange and nonstrange quarks, so defined, is $(1+g_2/g_1)^{-1} \approx 1.5 \pm 0.25$.

The expansion (71) also implies an order of magnitude for the effective mass of a quark $\beta''\Lambda$. With the conventional choice $\Delta m = 150$ MeV and $\Lambda \approx 200$ MeV, this is ≈ 450 MeV.

Coming back to QCD it is remarkable that β, β' and β'' in the expansions for the hyperfine parts of meson and baryon masses (53)(62) and baryon magnetic moments (72) are so close. Why it is so? Only a full QCD calculation can explain this, but a guess may help to relate the β' of the hyperfine mass term in baryons to the β'' in the Δm expansion of the magnetic moments (71). Assume that $F(|m_i, m_k; m_j\rangle)$ governing the hyperfine part of baryon masses is approximately factorizable as mentioned at the end of Sect. VI (it is so in some models—see Sect. IX—and is anyway true to first order in $\Delta m/\Lambda$); factorizability means $F(|m_i, m_k; m_j\rangle) = f(m_i)f(m_k)\varphi(m_j)$. Then D/C (60) is $[f(m_s) - f(m)]/f(m)$. If also $\eta(|m_i; m_k, m_j\rangle)$ is factorizable, that is $\eta(|m_i; m_k, m_j\rangle) = t(m_i)r(m_k)r(m_j)$, it is $g_2/g_1 = [t(m_s) - t(m)]/t(m)$. Then $(1/\beta')$ is the first order coefficient in the expansion of the quark-gluon chromomagnetic vertex in $\Delta m/\Lambda$, normalized to the vertex at $m_s = m$. As to $(1/\beta'')$, this is the same for the electromagnetic vertex. The similarity in structure of electromagnetic and chromomagnetic interactions suggests that to first order in m_s it is $[t(m_s) - t(m)]/t(m) = [f(m_s) - f(m)]/f(m)$, that is $\beta' = \beta''$; note the equality in sign.

So far we used only QCD (no assumption on models!). The quarks in play were the standard current (quasi chiral) quarks. From now on we shall deal, instead, with models, discussing how the parametrization provides a convenient way to test models. Before this we comment briefly on the notion of constituent quarks, which sometimes is the source of some confusion.

VIII. THE TWO MEANINGS OF "CONSTITUENT QUARK"

At present "constituent quark" has two meanings, both familiar, but rather different. We recall them only to avoid ambiguities in what follows.

In the first (less common) acception a constituent light quark is the QCD field having chosen a low q (near Λ) as the renormalization point for the mass [17, 18].

$$m_{\text{constituent}} = m(\text{at } q \text{ near } \Lambda) \quad (73)$$

This definition was implied in Refs.[5-6],when deriving the general parametrization. But, as noted, the derivation of the parametrization is independent of the renormalization point and can proceed using as quark fields the standard current (quasi-chiral) fields; thus the q in $\nu(q)$ in (37) can be as high as we like.

Constituent quarks defined by something like (73) would be related to the QCD Lagrangian. But it is hard to turn this definition, for light quarks, into something useful. At low q 's perturbative QCD fails. With (73) the fact that $(\Delta m/m)$ differ for current and constituents, in spite of scale invariance, might be due to this failure and/or to Politzer's [17] q^{-2} term.

The second meaning of "constituent quark" is the most common. It dates back to the NRQM [1]; its continuing use is due to the above difficulty of reaching a really useful operational definition of first type. In this second acception constituent quarks are defined with reference to specific models. Their (effective) masses are just some among the many parameters in a calculation with the selected model. *From now on constituent quarks will have this second meaning; we will use, as mentioned, capital M for the effective mass of a constituent quark.*

IX. MODELS AND QCD.

The proliferation of models of hadron structure in the past 20 years has brought a lack of predictive power: Too many models, all "so good"! Thus it is interesting to see why models work and record certain properties that a model should have to agree with some general consequences of QCD. The properties to be considered below are minimal properties: A model should fulfill them, but the model is not necessarily perfect if they are fulfilled (a "perfect" model coincides with the true theory, say QCD!).

Another question with models, raised in the introduction, is the relation of the effective masses of constituent quarks and the masses of current (quasi-chiral) quarks in the QCD Lagrangian. We will examine also this.

Consider, to exemplify, a model of baryon structure (the same applies to mesons). It should at least reproduce the masses of the lowest baryons and their magnetic moments. Of course it should reproduce much more, as already stated. But below we concentrate on these, because these alone are sufficient to substantiate our point.

We will examine four classes of models: Any non relativistic quark model, the semirelativistic QCD inspired one gluon exchange DGG model, the MIT bag model and the cloudy bag model. They are all characterized, at least at some stage in the calculation, by Hamiltonians with three quarks (for baryons). The cloudy bag model, that couples these quarks to pions, will be treated in the next section.

1. Non relativistic quark model

Consider a non relativistic quark model. Call H_{NR} a typical Hamiltonian for it, expressed in terms of the space, spin and flavor coordinates of the three quarks (any quark variable has an index $j=1,2,3$). H_{NR} may be quite general; flavor has to be broken only by λ_8^j matrices, or, if electromagnetism is included, also by λ_3^j matrices. The eigenstates of H_{NR} in general will be mixtures of various orbital angular momenta; in other words its lowest exact eigenfunctions $\psi_i(NR)$ (i refers to a baryon in the octet or decuplet) may have configuration mixing. Yet, for the lowest baryons (octet plus decuplet) we may write:

$$\psi_i(NR) = V_{NR} \phi_i \quad (74)$$

where ϕ_i is an auxiliary wave function having the product form:

$$\phi_i = \chi_{L=0}(\xi_1, \xi_2, \xi_3) w_i \quad (8)$$

In (74), of course, V_{NR} is a transformation producing, from the $L=0$ function ϕ , the exact configurationally mixed function ψ . This is certainly a much simpler transformation than the V introduced for QCD in Sect. II to construct the exact state $|\psi_i\rangle$ from the auxiliary state $|\phi_i\rangle$. There the V transformation had the gigantic task of dressing the $3q$ state with all sorts of $q\bar{q}$ pairs and gluons, plus producing configuration mixing, plus transforming two component spinors into 4 component ones. In the present case the V_{NR} transformation has just the task of producing configuration mixing. But formally Eq. (11) can be rewritten also in this case (in writing it we suppress NR in $\psi(NR)$). Thus:

$$M_i = \langle \psi_i | H_{NR} | \psi_i \rangle = \langle \phi_i | V_{NR}^\dagger H_{NR} V_{NR} | \phi_i \rangle = \langle w_i | ("Parametrized mass")_{NR} | w_i \rangle \quad (75)$$

Because Eq.(12) for the "Parametrized mass" in (11) follows only from the flavor dependence and invariance properties of the QCD Hamiltonian, with the factorizable choice of ϕ_i , the same expression (12) is true here. *Therefore a NRQM Hamiltonian gives a description of the masses of the lowest baryons identical to that of QCD, provided only that it has the number of parameters necessary to produce all terms in (12).* Of course since, as we saw, many terms in (12) are small (in particular those with three quark indices), even simple NR Hamiltonians may lead to good results. A similar argument holds for magnetic moments, where, in writing a NRQM Hamiltonian, one has to pay attention to gauge invariance. Also in this case the fact that the additive terms of the general QCD expression (25) already reproduce to 15% the magnetic moments, makes not so miraculous that simple NR Hamiltonians with few parameters give a good account of the magnetic moments. What is of interest in this case, as already remarked and so far unexplained [6f], is that in the general QCD parametrization (23) the coefficient g_3 is so small (-0.076). It is this smallness that produces the classical ratio $\cong -3/2$ of the magnetic moments of proton and neutron (which fact [19] greatly contributed, in 1965, to the birth of the quark model).

Finally, though in this paper we did not treat this problem, the analysis [6a] of semileptonic decays of lowest baryons leads to similar conclusions. Of course many more properties should be considered (just think of excited hadronic states). Still the conclusion is: Simple NR models work because the number of important terms in the QCD general parametrization is relatively small.

2. The QCD inspired model of De Rujula, Georgi and Glashow.

In their treatment DGG first calculate the one gluon exchange QCD potential v between two quarks in the Fermi-Breit approximation. Their three body Hamiltonian is $H_{DGG} = H_0 + v$, with H_0 flavor and spin independent. Treating v as a first order perturbation, the DGG baryon masses are:

(76)

$$M_i = \langle \psi_i | H_{DGG} | \psi_i \rangle \cong \langle \phi_i | H_{DGG} | \phi_i \rangle = \langle w_i | ("Parametrized mass")_{DGG} | w_i \rangle$$

In (76) H_{DGG} is the full DGG Hamiltonian, ψ_i the exact and ϕ_i the zero order

eigenfunctions (with the effect of v in H_{DGG} neglected); the last expression in (76) arises from the third after integration on the space variables. Because H_0 is flavor and spin independent, the unperturbed zero order wave functions ϕ_i are flavor independent for all lowest octet and decuplet states and they are factorizable $\phi_i = X_{L=0}(r_1, r_2, r_3) W_i$ as in (8). In this treatment the baryon masses have automatically the form (12) predicted by the QCD general parametrization, except for the absence of terms with three different quark indices; these would be there if DGG had included the exchange of two or more gluons. Thus for lowest baryons, the "parametrized mass" again agrees with the general QCD parametrization (12) although it does not contain all the parameters in (12)). This being clear, we pass to the question raised in the introduction, namely, what is the meaning of the "effective masses" of quarks in the hyperfine term of DGG.

In DGG [4] the hyperfine contribution to the baryon masses from quarks 1 and 2 (one must then sum over all pairs of quarks) is:

$$K_{\text{DGG}} = \frac{4\pi}{9} \alpha_s \frac{1}{M_1 M_2} \langle X_{L=0} | \delta^3(\underline{r}_{12}) | X_{L=0} \rangle (\underline{S}_1 \cdot \underline{S}_2) \equiv \frac{\tau}{M_1 M_2} (\underline{S}_1 \cdot \underline{S}_2) \quad (77)$$

where the last expression is just a definition of τ :

$$\tau \equiv (4\pi/9) \alpha_s \langle X_{L=0} | \delta^3(\underline{r}_{12}) | X_{L=0} \rangle \quad (78)$$

In (77) M_1 and M_2 are what DGG call the effective masses of quarks 1,2; they are not defined except that by (77). In the Fermi-Breit treatment of a He like atom containing, say, an electron and a muon, M_1 and M_2 would be the masses of the electron and muon. Here, on the other hand, they are not the masses of the current (quasi-chiral) quarks that appear in the original Lagrangian of QCD from which DGG move. They have the dimensions of a mass and differ for a strange and non strange quark, but this is all. The question is: How M_1, M_2 are related to quantities in the QCD Lagrangian? To simplify the answer assume, following DGG, that $\langle X_{L=0} | \delta^3(\underline{r}_{12}) | X_{L=0} \rangle$ is independent of M_1 and M_2 (this, essentially, corresponds to $X_{L=0}$ uniform inside a sphere). Thus the only dependence of K_{DGG} on M_1 and M_2 is in $(M_1 M_2)^{-1}$; note that this is factorizable.

Now go back to the exact general parametrization of baryon masses in QCD (Eq.(54)) and compare it with the DGG formula. If we wish that the DGG

result (77) approximates QCD, $F(|m_i, m_k; m_j)$ in (54) must be factorizable, that is $F(|m_i, m_k; m_j) = f(m_i)f(m_k)\varphi(m_j)$. We introduced factorizability at the end of Sect. VII, having in mind also the present application. To compare with the DGG one gluon exchange treatment one must put $\varphi(m_j)=1$ in $f(m_i)f(m_k)\varphi(m_j)$. Then comparing with (77) it is:

$$\frac{\tau}{M_1 M_2} = \frac{1}{f(m_1)f(m_2)} \equiv \frac{\Lambda^3}{\Lambda^2 f(m_1/\Lambda)f(m_2/\Lambda)} \quad (79)$$

In the last form we reinstalled the Λ 's to make dimensions explicit. Clearly the DGG "effective masses" M_i in terms of QCD masses m_i and Λ are:

$$(\tau/\Lambda^3)^{-1/2} M_i = \Lambda f(m_i/\Lambda) \quad (80)$$

This shows, as expected, that Λ is the QCD scale giving the effective mass scale of constituent quarks.

Eq.(80) shows clearly how the relationship of current and constituent quark masses depends on the model used to introduce the latter. We obtained (80) assuming that the integral in τ is independent of the M 's. Otherwise, the relation of the M 's to the QCD m 's is affected.

Similarly consider $(\Delta M/M_\lambda)_{DGG}$ and its relationship with the QCD masses. Comparing (77) and the general parametrization (12), $(\Delta M/M_\lambda)_{DGG} = -D/C = 0.3$. From (60) $D/C = [F(|m_s, m; m) - F(|m, m; m)]/F(|m, m; m)$ and, in the factorized approximation, $D/C = [f(m_s) - f(m)]/f(m)$. Thus, reinstalling Λ ,

$$0.3 = -(D/C) = (\Delta M/M_\lambda)_{DGG} = -[f(m_s/\Lambda) - f(m/\Lambda)]/f(m/\Lambda) \approx -\Delta m/(\beta' \Lambda) \quad (81)$$

having used in the last step the expansion (61). Eq.(81) shows that there is no contradiction between $M_\lambda/M \approx 1.4$ and m_s/m equal to 25 or 10 (or ∞ !); of course m_s/m is fundamental and M_λ/M model dependent; Eq.(81) displays the conceptual relationship between them. Again the model dependence enters in (81) because to get $(M_1 M_2)^{-1}$ (and therefore $-(D/C) = (\Delta M/M_\lambda)_{DGG}$), one must assume, as noted in [20], that the integral in τ is mass independent. This assumption, nearly true in a potential well, is not such for a harmonic oscillator, or, worse, for a Coulomb potential. In such cases we would not have approximate factorizability, as can be seen easily in the analogous simpler case of mesons; $\Delta M/M_\lambda$ might be quite different.

3. The MIT bag model

We now turn briefly to the MIT bag model, where the quarks are relativistic (Dirac equation in a bag with four component spinors; the quark masses in the model are taken very small or zero for u and d and, say, 100 MeV for s). We limit to consider here the case of the baryon masses and specifically the hyperfine contribution. Note that in spite of the fully relativistic nature of the four component spinors from which one starts, the hyperfine term appears of course at the end [21] in the Pauli form (82), in agreement with the general parametrization. Indeed the hyperfine contribution to the baryon mass from quarks 1 and 2 (one must sum over the three pairs of quarks) is the expectation value of:

$$K_{\text{bag}} = 8\alpha_s (\mathbf{g}_1 \cdot \mathbf{g}_2) \frac{\mu_1 \mu_2}{R^3} I_{1,2} \quad (82)$$

on the W_i spin-flavor states. In (82) R is the radius of the bag, μ_1 and μ_2 are the chromomagnetic moments and $I_{1,2}$ is an expression depending, as the chromomagnetic moments, on the radius R . (In principle also R in (82) might depend on the flavors of 1 and 2; in that case it should be R_{12}). The dependence on the quark masses m_i inserted in the model stays in μ_i and $I_{1,2}$. To a good approximation in (82) (and also in the baryon magnetic moments) intervene the effective masses of the quarks $M_i = (m_i^2 + x^2/R^2)^{1/2}$ where x/R is the quark momentum in the bag ($x=2.04$ in the limit $mR \rightarrow 0$).

The main point of interest is the following: The hyperfine term (82) is contained in the general parametrization. However in the simple version of the model treated so far, all terms with three indices, that in general also appear in the parametrization, are absent. Since these terms are relatively small, the situation is, in this respect, the same as the DGG treatment with one gluon exchange, in spite of the fact that the two models differ considerably.

Essentially the same conclusion is true for a variety of relativistic or semi-relativistic quark models. Any of them can be successful (but not superior to others, in spite, often, of complicated calculations) provided that it reproduces the spin flavor structure of the general parametrization and provided that it contains a number of parameters producing the dominant

coefficients of it. Of course one might object that different models will reveal differences in the calculation of hadron properties other than those considered here (think e.g. to the spectrum of excited states). This is certainly true but is of interest only if the models do not add for this purpose too many additional parameters.

X. EXCHANGE CURRENTS AND THE CLOUDY BAG MODEL

In the cloudy bag model we discuss only one point, the exchange pion current contribution to the baryon magnetic moments. The question is: Does the general QCD parametrization contain terms that can be interpreted as due to pion exchange currents? At first sight the question looks intriguing for the following reason: The Hamiltonian of the cloudy bag model contains, due to the coupling of pions to quarks, the Gell Mann flavor matrices λ_1, λ_2 and λ_3 . Thus one expects that the result of any calculation, say that of baryon magnetic moments, is expressed through λ_1, λ_2 or (in SU_2) τ_x, τ_y ; indeed [22] the magnetic moments of proton and neutron receive, due to pion exchange, a contribution with the spin flavor structure:

$$\sum_{i \neq k} (\mathbf{T}_i \times \mathbf{T}_k)_z (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_k) \quad (83)$$

On the other hand the QCD Lagrangian (including electromagnetism) contains only the flavor matrices λ_8 and λ_3 . They commute and form a closed algebra. Performing a pure QCD calculation, where virtual pions are $q\bar{q}$ aggregates, one expects that λ_1 and λ_2 (that is τ_x, τ_y), cannot enter in the final result, in contrast with (83). Indeed, to derive the flavor structure of the general parametrization, we used the fact that operating with λ_3 and λ_8 (a closed algebra) one cannot produce other flavor matrices, which thus cannot appear in the final expression. How can this problem be solved?

Below we will show that, *in apparent contradiction with the argument given above*, a term as (83) can arise from a QCD calculation; thus the cloudy bag model (and its pion exchange current) is compatible with QCD and we were incorrect in questioning this compatibility in Ref. [7a]. However we will also show that, *in agreement with the previous argument*, the term (83) can be identically rewritten as a sum of the spin flavor structures $\underline{G}_1, \underline{G}_3$

in (22) not containing at all τ_x, τ_y (recall: $\underline{G}_1 = \sum_i Q_i \underline{\sigma}_i, \underline{G}_3 = \sum_{i \neq k} Q_i \underline{\sigma}_k$). Thus nothing changes in the general parametrization and the term (83) is not a univocal signature of pion exchange or of the cloudy bag model.

The proof is simple. Consider the Majorana space exchange operator P_x^{ik} exchanging the space coordinates of quarks i, k . In a QCD calculation of $V^\dagger \underline{M} V$ such operators may intervene; that is $V^\dagger \underline{M} V$ for the baryon magnetic moments may contain space exchange terms. Then one can proceed in two fully equivalent ways. First way: Because the auxiliary function ϕ (Eq.(8)) is factorized as the product of a space factor $X_{L=0}$ times a spin-flavor factor W_i , we let P_x act on $X_{L=0}$ and integrate on $\underline{x}$. This is just the procedure adopted in deriving the general parametrization (22); the presence of P_x does not alter the result (22) (this will emerge clearly from Appendix I).

Second way: Operating, for simplicity, in SU_2 , use now the symmetry of the whole wave function, and write $P_x^{ik} = (1 + \underline{\sigma}_i \cdot \underline{\sigma}_k)(1 + \underline{\tau}_i \cdot \underline{\tau}_k)/4$ (a similar argument holds in SU_3). Consider then a term of the form $\sum_{i \neq k} Q_i \underline{\sigma}_i P_x^{ik}$, the existence of which is possible in QCD. Rewrite it as:

$$\sum_{i \neq k} Q_i \underline{\sigma}_i P_x^{ik} = \sum_{i \neq k} Q_i \underline{\sigma}_i (1 + \underline{\sigma}_i \cdot \underline{\sigma}_k)(1 + \underline{\tau}_i \cdot \underline{\tau}_k)/4 \quad (84)$$

Setting (in SU_2) $Q_i = (1/2)\tau_{zi} + (1/6)$, using the identities:

$$\tau_{zi}(\underline{\tau}_i \cdot \underline{\tau}_k) = \tau_{zk} - i(\underline{\tau}_i \times \underline{\tau}_k)_z \quad \sigma_{zi}(\underline{\sigma}_i \cdot \underline{\sigma}_k) = \sigma_{zk} - i(\underline{\sigma}_i \times \underline{\sigma}_k)_z \quad (85)$$

and limiting to terms that can contribute to the expectation value on the real functions W_i of the baryon octet and decuplet, one obtains from (84):

$$4 \sum_{i \neq k} Q_i \sigma_{iz} P_x^{ik} = \sum_{i \neq k} \left[(Q_i + Q_k) + \frac{1}{6}[(\underline{\tau}_i \cdot \underline{\tau}_k) - 1] \right] (\underline{\sigma}_i + \underline{\sigma}_k)_z - (1/2)(\underline{\tau}_i \times \underline{\tau}_k)_z (\underline{\sigma}_i \times \underline{\sigma}_k)_z \quad (86)$$

Because $\sum_{i \neq k} [(\underline{\tau}_i \cdot \underline{\tau}_k) - 1](\underline{\sigma}_i + \underline{\sigma}_k)_z$ gives zero when operating on the P, N or Δ states, and because $P_x^{ik} X_{L=0} = X_{L=0}$ we remain with the identity, *valid only for the non strange baryons of octet and decuplet*:

$$\sum_{i \neq k} (\underline{\tau}_i \times \underline{\tau}_k)_z (\underline{\sigma}_i \times \underline{\sigma}_k)_z = -8 \sum_i Q_i \sigma_{iz} + 4 \sum_{i \neq k} Q_i \sigma_{kz} \quad (87)$$

Thus the exchange term of the cloudy bag model is already contained in the

terms with coefficients g_1 and g_3 in the general parametrization (22). It may contribute (more or less) to g_1 and g_3 but it seems impossible to disentangle it from all other contributions to them. In other words there is no specificity of the cloudy bag model in this respect, unless one does not add other assumptions that may be interesting, but hard to prove.

XI. CONCLUSIONS

Several points were already listed in the summary, to which we refer. Here we wish to add or underline the following:

1) Using the general parametrization starting from the standard QCD Lagrangian with quasi-chiral (or current) light quarks, we have shown that for each model the effective "constituent quark" masses M are related to the quark masses m of the QCD Lagrangian and to Λ_{QCD} ; the existence of a Λ around 200 MeV intervenes in an essential way. In all sensible models the effective masses of light quarks are $\xi\Lambda$ with ξ a number between 2.3 and 3.7. There is in principle no contradiction in having $m_s/m \approx (8 \text{ to } 25)$ and $M_\Lambda/M \approx 1.4$.

2) The circumstance that different models may give, with a convenient choice of few parameters in each of them, results in agreement with the data is due to the fact that the general parametrization derived from QCD contains usually [5,6f] only few important terms. An appropriate selection of parameters in each model considered can produce these few important terms. This can be regarded, if one so wishes, as an extension of the so called "Cheshire Cat" principle, introduced originally [23] to assess the equivalence of descriptions of hadronic phenomenology in terms of bag models with different radii.

3) The important issue of predictivity of models remains the same as ever. Indeed assuming that QCD is "the theory", that is a full QCD calculation would reproduce all details of hadron physics, the extrapolation of any given model to new phenomena, beyond those where it has been tested, and which were used to fix the parameters in the model, cannot be expected to be hundred per cent exact. This applies to any model whatsoever. In this situation the "best" model is to some extent a matter of taste and, to a large extent, a matter of simplicity. To exemplify with a recent important

achievement [24] consider the measurement of the magnetic moment of the Ω^- , an experiment of extraordinary precision: $\mu(\Omega^-) = (-2.024 \pm 0.056) \mu_N$. At the end of their paper the authors state that this measurement disagrees with the static quark model value of $-1.84 \mu_N$ and express the hope that the result will provide a stringent test for future models of baryon structure. On the reality of this hope our point of view differs from that of the authors of [24]. It seems already remarkable that the NRQM *in its simplest form* predicts for $\mu(\Omega^-) = -1.84 \mu_N$, confirming the dominance of few terms stated above. We saw, in fact, that the general parametrization for the magnetic moments of the octet baryons has a large variety of terms (Eqs. 22-24) and the same is true for the decuplet baryons [7a]. Exploiting this variety it would be trivial to add some terms to the "naive" ones and obtain the measured $\mu(\Omega^-)$ value, even if the latter were known with a precision still higher than the extraordinary one given in [24]. But this would not too be fruitful. Yes, the model so constructed would produce the measured $\mu(\Omega^-)$, but it would necessarily still be approximate for some other quantity; unless the "model" and the true theory were the same thing.

APPENDIX I-OUTLINE OF THE DERIVATION OF THE GENERAL PARAMETRIZATION

We outline the calculation of the expectation value of the field operator $V^\dagger H_{\text{QCD}} V$ in the baryon 3-quark auxiliary state $|\phi_i\rangle$; i.e. we outline the derivation of the fourth form of (11), expressing the masses as expectation values of a spin flavor three-body operator (Eq. 12) on the spin-flavor functions W_i . In the third form of Eq. (11) the only part of $V^\dagger H_{\text{QCD}} V$ that contributes is its projection in the $|3q, \text{no gluon}\rangle$ Fock sector:

$$\tilde{H} = \sum_{3q} \sum_{3q'} |3q\rangle \langle 3q| V^\dagger H_{\text{QCD}} V |3q'\rangle \langle 3q'| \quad (\text{A1})$$

where the sums in (A1) are on all possible 3-quark, no gluon Fock states. After normal ordering of all creation and destruction operators in $\tilde{H}$ and their contraction with those arising from $\langle \phi_i|$ and $|\phi_i\rangle$ (see Eq. (36)), the operator $\tilde{H}$ becomes a function only of the spin-flavor-space variables of the three quarks in $|\phi_i\rangle$; thus parametrizing $\tilde{H}$ amounts to construct the most

general scalar operator of the $\mathfrak{g}_i$'s, λ_i 's and $\mathfrak{r}_i$'s of the three quarks ($i=1,2,3$) including in it, of course, only those terms, that have a non vanishing expectation value in ϕ_i . Note that (Sect. IV) we took the quarks in the auxiliary states $|\phi_i\rangle$ identical to those in the QCD Lagrangian at the chosen renormalization point in the no flavor breaking limit. Due to this the contraction of the creation and destruction operators in $V^\dagger H_{\text{QCD}} V$ with those in the auxiliary states $|\phi_i\rangle$ (Eq. (36)) is straightforward. After this contraction, the projection $\tilde{H}$ of the field operator $V^\dagger H_{\text{QCD}} V$ in the 3-body sector becomes, as stated above, a scalar (i.e. rotation invariant) function of the space $\mathfrak{r}_i$, spin $\mathfrak{g}_i$, flavor f_i and color operators of the three quarks (we suppress the color variables when possible). One has to write the most general expression of such operator. We call it $\tilde{H}'$ [we use a different symbol because $\tilde{H}$ (Eq. A1) operates in Fock space, whereas $\tilde{H}'$, obtained after contraction of the field operators, is just a three body quantum mechanical operator]. The number of independent scalar operators in the spin-flavor space of three quarks is finite; we use for them, in general, the symbol $Y_\mu(\mathfrak{g}, f)$ where μ specifies the operator to which we refer. Thus *the most general operator of the space and spin-flavor variables is necessarily*

$$\tilde{H}' = \sum_{\mu} R_{\mu}(\mathfrak{r}, \mathfrak{r}') Y_{\mu}(\mathfrak{g}, f) \quad (\text{A2})$$

where $R_{\mu}(\mathfrak{r}, \mathfrak{r}')$ are operators (not necessarily local) acting in the coordinate space of the 3 quarks. In (A2) $\mathfrak{r}$ means:

$$\mathfrak{r} \equiv (\mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3)$$

To calculate a physical quantity such as a mass (as we are doing) one must form the expectation of (A2) on ϕ_i . Now a most important point: The auxiliary ϕ_i is arbitrary, provided that it has the correct quantum numbers of the state $|\psi_i\rangle$ under consideration (in this case an octet or decuplet baryon). With this proviso one can choose ϕ_i freely. For instance, the 3-quark part of the correct $|\psi_i\rangle$ (first addend of the r.h.s. of (39)) certainly has configuration mixing; still one can select an auxiliary wave function ϕ_i

without configuration mixing. It is the task of the transformation V to produce configuration mixing, and, of course, the whole complexity and variety of Fock states present on the right hand side of (39). Thus we select the auxiliary wave function ϕ_i as simple as possible. An important feature in this choice is factorizability. That is we select ϕ_i as in (8), the product of a space part $X_{L=0}(\underline{r}_1, \underline{r}_2, \underline{r}_3)$ with orbital angular momentum $L=0$ and a spin-flavor (color) factor $W_i(1,2,3)$ carrying the whole J (see (9,10)). The factorization of ϕ_i implies for the expectation value (11) (that is the mass M_i) the following structure:

$$M_i = \sum_{\mu} \langle X_{L=0}(\underline{r}) | R_{\mu}(\underline{r}, \underline{r}') | \phi_{L=0}(\underline{r}') \rangle \langle W_i | Y_{\mu}(\underline{g}, f) | W_i \rangle \quad (A3)$$

that we also rewrite as:

$$\sum_{\mu} g_{\mu} \langle W_i | Y_{\mu}(\underline{g}, f) | W_i \rangle \quad (A4)$$

with:

$$g_{\mu} = \langle X_{L=0}(\underline{r}) | R_{\mu}(\underline{r}, \underline{r}') | X_{L=0}(\underline{r}') \rangle \quad (A5)$$

Because the space part of the model wave function has, by construction, $L=0$, the operators $R_{\mu}(\underline{r}, \underline{r}')$ in (A2) must be rotation invariant. Because $\tilde{H}$ is a scalar, the $Y_{\mu}(\underline{g}, f)$ that enter in the parametrization of the masses M_i must be scalar operators, to be constructed only in terms of the spins $\underline{g}_i$ of the three quarks; hence the parametrized masses are written as:

$$\text{"Parametrized mass"} = \sum_{\mu} g_{\mu} Y_{\mu}(\underline{g}, f) \quad (A6)$$

and the masses M_i are:

$$M_i = \langle \phi_i | \tilde{H} | \phi_i \rangle = \sum_{\mu} g_{\mu} \langle W_i | Y_{\mu}(\underline{g}, f) | W_i \rangle \equiv \langle W_i | \text{"Parametrized mass"} | W_i \rangle \quad (A7)$$

the result already given in Eq. (11).

To deduce the general parametrization (12) of the masses is now sufficient to list all scalars $Y_{\mu}(\underline{g}, f)$ formed with the spins and flavors of the three quarks. Because the W_i 's are symmetric in spin-flavor, the only intervening spin flavor structures are precisely those in Eq. (12). This is due to these points:

- 1) The only possible scalars constructed with the three spin Pauli $\underline{g}_i$'s

are 1 and $(\mathbf{g}_i \cdot \mathbf{g}_k)$. The scalar $(\mathbf{g}_1 \times \mathbf{g}_2) \cdot \mathbf{g}_3$ (times any hermitian real flavor operator) has vanishing expectation value on any real spin flavor state of three particles, as the W_i 's are.

2) The only flavor operator in the strong Lagrangian is P^S . Thus only P_i^S , $P_i^S P_k^S$, $P_1^S P_2^S P_3^S$ are possible flavor operators for three quarks; $(P_i^S)^n$ with any (integer) n reproduce P_i^S . Structures like $\text{Tr}(P_i^S)$ are a number.

A similar procedure leads to Eq.(28) for the parametrized masses of the mesons (with $I \neq 0$). For the baryon magnetic moments (22,23), the $Y_\nu(\mathbf{g}, f)$'s in (A2) must then be axial vectors under rotations; keeping only terms linear in P^S one then obtains (22,23)[5].

APPENDIX II-THE COEFFICIENTS IN THE BARYON PARAMETRIZATION

In [5,6d,f] we determined the coefficients of the parametrization (12) from the baryon masses. For $\Delta, \Sigma(1385) \equiv \Sigma^*$, $\Xi(1530) \equiv \Xi^*$ we used for this in [6] the conventional masses (resonance peaks) as given in [13]. One of us [12] noted that it might be preferable to use the "pole" masses. For the "large" coefficients A, B, C , the differences between the values of the coefficients $A, B, C, D, E, (a+b), c, d$ in (12) derived using the conventional or the pole masses are irrelevant or of little interest. For the smaller coefficients $D, E, (a+b), c, d$, the two determinations may differ significantly. Below we will list the coefficients obtained from the conventional and pole fits. Because the general parametrization (and therefore its coefficients) refers to the strong interaction only (the masses in (12) are the eigenvalues of H_{QCD} , without the electromagnetic interaction) it is necessary, especially for the smaller coefficients, to extract from the experimental masses the strong part. That is, to determine the coefficients of the parametrization (12), one must construct and use combinations of baryon masses independent of the e.m. and isospin breaking ($m_u \neq m_d$), at least to first order. We did this already [6d] when writing Eq.(14). Below we write these combinations of baryon masses. To determine the "large" coefficients M_0, B and C the precision stated above is unnecessary and we simply averaged on Coulomb and isospin effects:

$$M_0 = (\bar{N} + \bar{\Delta})/2, \quad B = \Lambda - \bar{N} + 3E, \quad C = (\bar{\Delta} - \bar{N})/6 \quad (\text{A8})$$

where $\bar{N}=(n+p)/2$ and $\bar{\Delta}=(\Delta^{++}+\Delta^++\Delta^0+\Delta^-)/4$ and E is the coefficient in (12) given by below in Eq.(A10). As to the coefficients D,E,(a+b),c,d, they are determined from the following Eqs.(A9) which are Coulomb and isospin independent to first order

$$\begin{aligned}
 D &= (1/6)[(\Sigma^{*-}-\Delta^-)+(\Sigma^+-p)] \\
 E &= (1/6)(\Sigma^{*-}-\Delta^-)+(1/12)(\Sigma^0-3\Lambda+2n) \\
 c &= (1/3)[(\Xi^{*-}-\Xi^-)-(\Sigma^{*-}-\Sigma^-)] -2E \\
 a+b &= \Xi^- - \Sigma^- + (1/2)(\Sigma^0-3\Lambda+2n)+2c \\
 d &= \Omega-\Delta^{++}-3(\Xi^{*0}-\Sigma^{*+})
 \end{aligned} \tag{A9}$$

where in the above formulas Δ^- stays for:

$$\Delta^- = \Delta^{++}+3(n-p)$$

From Eqs.(A9), using the conventional and pole values of the masses-also the latter are found in Ref.[13] (we recall, e.g., $\Delta^{++}(\text{conv})=1231\pm 1$, $\Delta^{++}(\text{pole})=1210.5\pm 1$) we get for the coefficients (in MeV):

(A10)							
M_0	B	C	D	E	(a+b)	c	d
pole: 1076	192	45.6 $\pm$ 0.3	-13.8 $\pm$ 0.3	5.1 $\pm$ 0.3	-16 $\pm$ 1.4	-1.1 $\pm$ 0.7	4 $\pm$ 3
conv: 1086	184	49.2 $\pm$ 0.3	-16.4 $\pm$ 0.2	2.5 $\pm$ 0.2	-7.5 $\pm$ 0.8	+3.1 $\pm$ 0.4	-5.7 $\pm$ 2

The pole values (first line) were already listed in (13). Note the appreciable difference in the smaller coefficients (E to d) according to the two determinations; although, for reasons on which we do not come back here [12], we tend to prefer the pole fit, in both cases the new [6d] octet-decuplet mass formula (14) is satisfied practically with the same remarkable precision (16) stated in the text.

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Fig.1

CAPTION TO FIG.1

Schematic diagram representing the "external" lines 1,2 and the "internal" box in the general parametrization of a meson property; ϕ is the auxiliary state. The box contains all sorts of gluon lines and quark closed loops.