Gravitationally–Induced Three–Flavor Neutrino Oscillations as a Possible Solution to the Solar Neutrino Problem

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Abstract

Neutrinos can undergo flavor–oscillations if they possess flavor–dependent couplings to the surrounding gravitational field (the VEP mechanism). The neutrino fields can be massless, in accord with the Minimal Standard Model, but at the expense of the Einstein Equivalence Principle. We show that it is possible to explain the observed Solar Neutrino data from the various experiments using the VEP solution in a realistic three–generation framework, and further note how the three–flavor model can offer larger allowed regions of parameter space over the two–flavor models.

1 Introduction

Various explanations of the Solar Neutrino Problem (SNP) are based on the assumption that neutrinos possess two non–degenerate eigenbases in which they can be described. One of these is the flavor eigenbasis \(|\nu\rangle_W\), relevant to electroweak phenomena, while the other is diagonal in the quantum mechanical equations of motion. The first of such models was proposed by Mikheyev, Smirnov, and Wolfenstein \([1, 2]\), and is therefore dubbed the MSW mechanism. In this model, the second eigenstate must be a massive one, which implies that the neutrinos must have non–trivial masses. Such a solution saves the Standard Solar Model \([3]\), but requires an extension to the Minimal Standard Model of Particle Physics, \(i.e.\) massive neutrinos.

This compromise of the Standard Model can be saved, as was pointed out in 1988 \([4]\). Instead of having mass, if each neutrino couples differently to the (solar) gravitational field \(\phi(r)\), then the same oscillation mechanism can be obtained (up to the form of the energy dependence, the primary distinction between the massive and gravitational oscillation models). That

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is, each eigenstate $\nu_i$ has a different coupling $G_i = (1 + f_i)G$, with $G$ Newton’s constant, and each $f_i \ll 1$ a dimensionless “violation parameter”. For first generation neutrinos, we take $f_i = 0$, i.e. $G_1 = G$.

The two eigenbases are related by a matrix $V_{N_g} \in SU(N_g)$. For three flavors, $N_g = 3$; discounting CP violations in the neutrino sector, $V_3$ becomes an orthogonal three–parameter ($\theta_{12}, \theta_{13}, \theta_{23}$) matrix. With $|\nu\rangle_W = V_3|\nu\rangle_{M,G}$, the neutrino states evolve according to the equations of motion

$$i \frac{d}{d\tau} |\nu\rangle_{M,G} = H_{M,G} |\nu\rangle_{M,G}$$

$$\Rightarrow \quad i \frac{d}{d\tau} |\nu\rangle_W = H'_{M,G} |\nu\rangle_W$$

Here, $H_{M,G}$ are the diagonal Hamiltonians for MSW and VEP respectively in the mass/gravitational eigenbasis,

$$H_M = \frac{1}{2E} \text{diag} \{m_1^2, m_2^2, m_3^2\}$$

$$H_G = 2E|\phi(r)| \text{diag} \{f_1, f_2, f_3\}$$

while $H' = V_3^{-1}H V_3 + A$ is the corrected version for the electroweak interactions ($A = \text{diag} \{\sqrt{2}G_F N_e, 0, 0\}$). It is the off–diagonal nature of $H'$ which induces flavor oscillations, and the presence of $A$ creates parameter–dependent resonances. We can subtract a total factor of unity $\frac{1}{|V_{11}|^2}$ from $H$, since this yields only an unobservable phase, and hence deal only with eigenvalue differences $2E|\phi(r)| \Delta f_{21,31} (\Delta m_{21,31}^2/2E)$. 

Re–diagonalization of $H'$ by a matter–enhanced matrix $V_{3m}$ creates a new eigenbasis $|\nu\rangle_{MAT}$ in which we can describe the evolution, with $|\nu\rangle_W = V_{3m}^* |\nu\rangle_{MAT}$. For an electron neutrino $\nu_e$, created in the solar core, the averaged probability that it reaches the Earth as a $\nu_e$ is found to be [5]

$$\langle P(\nu_e \to \nu_e) \rangle = \sum_{i,j=1}^3 |(V_3)_{i1}|^2 |(P_{L2Z})_{ij}|^2 |(V_{3m})_{ij}|^2$$

$$= c_{m12}^2 c_{m13}^2 \left\{ (1 - P_1) c_{12}^2 c_{13}^2 + P_1 s_{12}^2 c_{13}^2 \right\}$$

$$+ s_{m12}^2 c_{m13}^2 \left\{ P_1 (1 - P_2) c_{12}^2 c_{13}^2 + (1 - P_1)(1 - P_2) s_{12}^2 c_{13}^2 + P_2 s_{13}^2 \right\}$$

$$+ s_{m13}^2 \left\{ P_1 P_2 c_{12}^2 c_{13}^2 + P_2 (1 - P_1) s_{12}^2 s_{13}^2 + (1 - P_2) s_{13}^2 \right\}.$$  

(4)
Here, $s_{ij}, c_{ij} \equiv \sin \theta_{ij}, \cos \theta_{ij}$ and $s_{mij}, c_{mij} \equiv \sin \theta_{mij}, \cos \theta_{mij}$ are the parameters of $V_3$ and $V_3^m$, respectively. The matrix $P_{LZ}$ has elements $P_{1,2}$ which describe the probability of non–adiabatic level crossing between $\nu_e \rightarrow \nu_\mu, \nu_e \rightarrow \nu_\tau$ (hereafter 12– and 13–transitions).

2 Behavior of $\langle P(\nu_e \rightarrow \nu_e) \rangle$ for a Double Resonance

If the $\nu_e$s are created at electron densities higher than the corresponding resonance density for either 12– or 13–transitions, then the matter–enhanced mixing angles approach the value $\theta_{12}^m, \theta_{13}^m \rightarrow \frac{\pi}{2}$. Hence, the survival probability reduces to the simpler form [6]

$$\langle P(\nu_e \rightarrow \nu_e) \rangle = P_1 P_2 c_{12}^2 c_{13}^2 + P_2 (1 - P_1) s_{12}^2 s_{13}^2 + (1 - P_2) s_{13}^2 . \quad (5)$$

To see how this is affected by the addition of the third flavor $\nu_\tau$, we can examine its limiting form for small and large $\theta_{13}$. In the former case, we have

$$\langle P(\nu_e \rightarrow \nu_e) \rangle = c_{12}^2 P_1 P_2 , \quad (6)$$

which shows energy dependence through both 12– and 13–transitions in the $P_{1,2}$ terms. Solutions to the solar neutrino problem in this case are similar to the small–angle solution in the two–flavor limit, and are of questionable statistical validity [6]. However, we note that for large $\theta_{13}$, the term $P_2 \rightarrow 0$ (adiabatic approximation for 13–transition), and so the above expression further reduces to

$$\langle P(\nu_e \rightarrow \nu_e) \rangle = s_{13}^2 . \quad (7)$$

This limiting form has interesting implications, as it suggests that the $\nu_e$ suppression is not only energy–independent (as is usually the case with large–angle oscillation solutions), but that it is also independent of the 12–transition. Figure 1 shows the allowed parameter–space overlap for the most recent solar neutrino experiment data in the large $\theta_{13}$ case. Clearly, the addition of a third flavor greatly broadens the regions from the much smaller two–flavor results (see [7] for these).
3 Comparison of $^8B$ Neutrino Fluxes

As previously mentioned, the difference between the two oscillation mechanisms resides in their energy dependence. A study of the spectrum of $^8B$ neutrinos incident on terrestrial detectors can help shed light on which suppression mechanism, if any, is at work. In the previous section, the large $\theta_{13}$ form was shown to be energy independent, while the small $\theta_{13}$ case is a function of both 12– and 13–transitions. So, we should expect to see some type of spectral distortion in the $^8B$ neutrino flux in this small angle case [6, 8]. Figure 2 shows that the attenuation of $\nu_e$s is indeed affected quite differently by each model: MSW suppresses low–energy neutrinos, while VEP suppresses higher–energy ones. Since detectors such as Kamiokande II, or SNO (when it comes online) can detect subtle variations in the high energy portion of the flux spectrum, we would expect such behavior as that in figure 2 to be a major clue as to the solution of the SNP.

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References

Figure 1: $3\sigma$ overlap for data from [3], with $\Delta f_{31} = 10^{-13}$, $s_{13}^2 = 0.4$.

Figure 2: MSW and VEP reduced fluxes yielding counting rate $R = 2.00$ SNU, with both 12– and 13–resonances allowed.