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Abstract

In this letter we re-analyze the role of chaos in the mean field simulations of spinodal decomposition. We demonstrate that, conversely to recent publications, the RPA unstable modes are only weakly non-linear and weakly coupled. The Lyapunov exponents are shown to be nothing but the largest imaginary RPA frequencies and the early mean field evolution, even of a randomly initialized trajectory, is shown to be mostly understandable within the framework of simple linear instabilities. Finally we recall how the time scales are related to the use of a reasonable range for the nuclear force.

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In recent years one-body approaches have provided a well founded framework to study the mean properties of heavy ion collisions at low energy [1]. At higher energies, the abundant production of complex fragments, which has been observed experimentally, has opened a large and challenging debate about the possible origin of such multifragmentation of the nuclear system. A proposed explanation concerns the possibility that the composite nuclear system can reach low density values, inside the spinodal instability region of the nuclear matter phase diagram. Therefore fragments would be formed through the development of the encountered dynamical instabilities.

In stable situations, nearby the nuclear matter normal density, important regularities in the mean-field dynamics, such as the presence of robust collective modes (the giant resonances) have been recognized, in particular within the Random Phase Approximation (RPA), even at high temperature, and confirmed experimentally [2]. A similar analysis, in terms of linear response, can be realized also inside the spinodal region and predicts the existence of unstable collective modes [3, 4]. However, since in unstable situations fluctuations are amplified, the survival of such regular modes in the complete mean field dynamics becomes questionable since the system may rapidly get out of the small amplitude regime. Recently the possibility of observing, after a very short time ($t \leq 20$ fm/c), a fully chaotic behaviour of the mean-field dynamics in the spinodal region has been proposed [5, 6]. Direct consequences of the dominance of chaos could be the possibility to directly populate the whole phase space in a statistical manner during the fragmentation of the system. However, this conclusion is orthogonal to the observations reported in ref. [7, 8] of a strong memory of the initial instability in the partition of the system.

In this letter we would like to solve this puzzle by re-examining the results presented in ref. [5]. By considering the same numerical simulations of the

Vlasov dynamics on a two-dimensional lattice, developed in ref.[9] and used in ref.s[5, 10] we will show that chaos does not develop over the time scale considered in [5] ($20 \text{ fm}/c < t < 70 \text{ fm}/c$). The discrepancy between our results and the interpretation of ref.[5] is shown to be due to a numerical artefact in the initialization procedure used in ref.[5], which introduces artificially small wave length components strongly dependent on the initial average density. We will demonstrate that the gross features of the early unstable mean field dynamics can be interpreted using linear response theory concepts. In particular the Lyapunov exponent of ref. [5] are shown to be nothing but the largest imaginary RPA frequencies.

Let us consider a piece of infinite unstable nuclear matter in 2D, as in ref.[5]. The mean-field potential is averaged over the y direction, so that clusterization will occur only along the x axis. It is important to notice that the mean field in the x direction is folded with the function presented in Fig.1 which corresponds to a very short range for the nuclear mean-field potential. In Fig.1 we show the imaginary RPA frequencies (analytically derived as in ref.[4]) associated with sinusoidal perturbations as a function of the wave number k , in the case $T = 0$, $\rho/\rho_0 = 0.5$, where $\rho_0 = 0.55 \text{ fm}^{-2}$ is the saturation density in two dimensions. This dispersion relation presents a maximum around $k = 1.7 \text{ fm}^{-1}$ because of the particular short range of the mean-field. The corresponding characteristic growing time, which governs the exponential amplification of such RPA mode, is approximately equal to $14 \text{ fm}/c$. The dispersion relation obtained directly from the Vlasov simulation (see ref. [11]) is also shown on Fig. 1.

As asserted in ref.[5], the main indication in the search for a chaotic behavior of a dynamical system is given by the extreme sensitivity to the initial conditions. Therefore in ref.[5] the authors studied the evolution of two very close trajectories. They initialized the system considering, at $t = 0$, a density wave, with an

amplitude $\Delta\rho_1 = 10^{-2} \rho_0$, and with a node number $n_x = 5$ ($k = 0.6 \text{ fm}^{-1}$). In the first trajectory the average density ρ was equal to $\rho = 0.5 \rho_0$, while the second trajectory had an average density equal to $0.51 \rho_0$. Then they studied the response of the mean-field dynamics to this small sinusoidal perturbation, for the two trajectories and they observed an extreme sensitivity of this response to the initial conditions (Fig.2a).

A quantitative analysis of this sensitivity is performed considering the Lyapunov exponent, defined in ref.[5] as follows:

$$\lambda(t) = \frac{\log(d(t)/d(0))}{t}, \quad (1)$$

where $d(t)$, the distance between the two trajectories $\rho^{(1)}$ and $\rho^{(2)}$, is defined as:

$$d(t) = \sum_i |\rho_i^{(1)}(t) - \rho_i^{(2)}(t)| / N_c \quad (2)$$

where the sum runs over the N_c cells in the x direction. In the considered situation the initial distance $d(0)$ results equal to 10^{-4} fm^{-2} . As shown in ref.[5], after a short time (20 fm/c) the Lyapunov exponent converges to a finite value $\lambda \approx 0.06 \text{ c/fm}$, which remains constant until around 70 fm/c. The same analysis is repeated for different density values inside the spinodal region and the obtained Lyapunov exponent is represented in Fig. 1, as a function of the initial average density of the two trajectories. Therefore the authors of ref.[5] conclude about the early appearance of chaos (after 20 fm/c) and discuss the behaviour of the system until $t = 70 \text{ fm/c}$.

Here we would like to stress that these results are not necessarily related to the presence of chaos in the dynamical evolution. In fact, even if the two considered trajectories appear very near in r -space at $t = 0$, they are actually quite different, as one can realize looking to the Fourier spectrum (see Fig.2). Indeed in the initial conditions, due to a numerical inaccuracy (see the following

text), wave numbers different than $n_x = 5$ ($k = 0.6 \text{ fm}^{-1}$) are excited with very different amplitudes and phases, in the two trajectories. In such a case it should be noticed that a positive finite Lyapunov exponent is obtained also in a linear regime. Indeed, even in the linear response approach, because of the exponential amplification of the fluctuations associated with the unstable modes, two near trajectories, initialized with some noise on various modes, will diverge and the obtained final value of λ will be approximately equal to the largest imaginary frequency associated with the most unstable modes.¹ In fact the RPA frequencies of the most important modes presented in the Fig. 1 are very close to the Lyapunov exponents, indicating the presence of simple linear instabilities and not of the dominance of chaos or bifurcations.

Let us now study in more details the two trajectories considered in ref.[5]. The spacial densities are represented in Fig. 2a) at different times. Soon the two density profiles start to diverge significantly, indicating that fragmentation occurs in a quite different way, even if the densities look very similar at the initial time. In ref.[5] this behaviour was identified as typical of a chaotic regime, signature of a strongly non-linear growth of coupled unstable modes. However we would like to stress here that, as anticipated before, because of some numerical inaccuracies in the initialization procedure used in ref.[5] in order to implement sinusoidal perturbations of the density, the two trajectories have a quite different Fourier spectrum already at $t = 0$, because the densities were artificially forced to take discrete values (see Fig. 2a). The Fourier spectra are represented in Fig. 2b). It is possible to observe that different modes are excited already in the initial conditions, in particular some very unstable modes around $k = 1.7 \text{ fm}^{-1}$. This is the explanation of the differences observed in the fragmentation pattern which has

¹It should be noticed that the same arguments hold in the case of a purely random initialization.

nothing to do with the occurrence of a prompt chaos. Indeed, we have repeated the same analysis, but avoiding the numerical problems in the initialization. The density profiles and the Fourier spectra are represented in Fig.2c) and 2d), respectively, at different times. Now it is possible to see that the Fourier spectrum at $t = 0$ exhibits only one component ($k = 0.6 \text{ fm}^{-1}$) in the two cases. Then the density profiles are very similar up to the final considered time ($t = 150 \text{ fm}/c$), whatever the densities were different. In fact the evolution is dominated by the growth of the mode initially excited, $n_x = 5$, and 5 clusters are simply formed. Performing now the analysis in terms of the Lyapunov exponent defined by eq.(1) on the corrected trajectories, we obtain λ nearly equal to zero. This shows that the considered dynamics is not chaotic until that time. More information can be gained looking at the Fourier spectra. The excited component ($k = 0.6 \text{ fm}^{-1}$) appears amplified with a characteristic growing time $\tau = 40 \text{ fm}/c$, very close to the one predicted by the linear response theory. Moreover, $k = 0.6 \text{ fm}^{-1}$ is nearly the only component present in the Fourier spectrum, up to times of the order of $100 \text{ fm}/c$, and still dominates until the end of the simulation ($t = 150 \text{ fm}/c$). Frequency doubling, which can be seen as a first step towards chaos, appears around $70 \text{ fm}/c$ but remains small. The Fourier spectra present some small differences only at the latest time. This demonstrates that the dynamics is mostly linear up to long times and then presents some sizable non linearities, but not yet chaos.

A stringent test on the linear behaviour of the system can be obtained by putting a random noise (with $\delta\rho = 0.01 \rho_0$) in the initial conditions, by propagating all the modes according to the dispersion relation presented in Fig.1 and by eventually reconstructing the density profile through the inverse Fourier transform. Therefore the comparison between this schematic purely linear evolution of decoupled unstable oscillators and the complete mean-field calculations can

provide important information on the deviation of the fragmentation dynamics from the linear regime and hence on the onset of chaos. The evolution of the density profile of one event is shown in Fig.3. It is possible to observe that the exact trajectories and the linear approximation are very similar, indicating that the early evolution can be understood on the basis of the growth of the fluctuations associated with decoupled unstable oscillators. We show just one event, but it should be noticed that our results on the validity of the linear approximation do not depend on the considered initialization.

In conclusion in this letter we have re-considered the results presented in ref.[5] and we have demonstrated that, over the time scales discussed in ref.[5], the mean field dynamics inside the spinodal region can be considered as a simple case of linear instability. We have shown that the strong sensitivity of the mean-field dynamics of a single RPA mode observed in ref.[5] was due to a numerical artefact in the initialization procedure. The Lyapunov exponents reported in ref.[5] are nothing but the largest imaginary frequency predicted by the linear response theory and do not signal the presence of chaos. This analysis explains the discrepancy between the results obtained in [5] and the observation of the dominance of linear instabilities in the early cluster formation reported in ref.[8]. It justifies the use of linear response concepts in the discussion of the early evolution of spinodal decomposition.

Finally, we would like to stress also that a realistic range for the mean-field potential is much larger than the value used in the schematic calculations of ref.s[5, 6, 10]; the dispersion relation computed for a reasonable range, as discussed in ref.s[4, 12], (see Fig. 1) presents a few important unstable modes, associated with rather long growing times and wave lengths about 3 times longer than the scales discussed in ref. [5]. Therefore, as discussed in ref.[11], in a calculation using a realistic range for the nuclear potential the validity of the

mean field approach and of the linear response will be strongly enhanced when compared with the presented results.

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Figure 1:

Upper part: The dispersion relation, for $\rho = \rho_0/2$ and $T = 0$, corresponding to the short range interaction of ref.[5], obtained analytically, in the linear response framework (dashed line), and numerically, by initializing the system with a sinusoidal perturbation and calculating its early amplification rate (squares). The short range interaction is represented in the insert on the right. The analytical dispersion relation associated with a realistic longer range interaction is also reported (full line) (see ref.s[4, 12]).

Lower part: Squares: The Lyapunov exponent, extracted from ref.[5]. The error bars are an estimation of the uncertainty on the extraction of the exponent presented in [5]. Full line: The largest imaginary frequency of the unstable collective mode, calculated solving analytically the dispersion relation, as a function of density values inside the spinodal instability region.

Figure 2:

Density profiles of two close trajectories (a), as calculated in ref.[5], presenting a discontinuous variation of the density which corresponds to high frequency components in the Fourier spectra represented in (b). The density profiles of the same two trajectories, but initialized avoiding the spurious high frequency components, are displayed in (c). The associated Fourier spectra are plotted in (d).

Figure 3:

The schematic purely linear evolution (right part), obtained considering an initial white noise, propagated according to the dispersion relation presented in Fig.1 and eventually reconstructing the density profile through the inverse Fourier transform is compared with the complete mean-field calculations (left part) for the same initial condition.

Fig 1

Fig. 2

Fig. 2

Fig. 3