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RELATIVISTIC HEAVY IONS**

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On the resonant coherent excitation of relativistic heavy ions

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Abstract

New accelerator facilities open up an interesting new field of experiments on basic channeling as well as on atomic and nuclear resonant coherent excitation (RCE) of heavy ions penetrating through aligned crystals at relativistic energies. Results of computer simulations are presented to characterize the resonant coherent excitation of atomic levels of relativistic hydrogen-like heavy ions. Nuclear resonant coherent excitation reveals interesting different characteristics compared to the corresponding atomic excitation inside crystals. An important result of our model calculations is that poorly-channeled ions have a higher nuclear excitation probability than well-channeled ions.

1 Introduction

When a beam of heavy ions penetrates a crystal along an axis or plane the motion is governed by correlated collisions which allow selected ion-solid interactions which are quite different from those in amorphous matter. Small-impact-parameter events are excluded for ions with an incident angle smaller than the corresponding critical angle for channeling [1]. This channeling phenomenon has been studied and successfully applied for heavy ions at non-relativistic velocities in many laboratories [2]. Besides the steering force for channeled particles which is well described by a mean potential formed by many atoms in the spatially periodic planes or rows, a periodic perturbation is superimposed by binary interaction with the potentials from single crystal atoms which can coherently excite atomic or nuclear levels of the projectile traveling through the crystal. Channeling is still possible for relativistic heavy projectiles because multiple scattering and dechanneling decrease much faster with increasing projectile energy than the corresponding critical channeling angles. A new generation of channeling studies becomes possible with cooled heavy-ion beams [3] taking advantage of the high phase-space density.

A review of the general features of atomic resonant coherent excitation (RCE) under channeling conditions at non-relativistic energies is given in ref. [2]. Here, we present results obtained from our calculations in order to get an insight into the basic physical processes contributing at moderate relativistic ion energies and to prepare future experiments on atomic RCE of relativistic hydrogen-like heavy ions in aligned single crystals. Also RCE of nuclear levels which has not yet been discovered experimentally will be discussed. Using the new facilities it will be possible for the first time to excite atomic and nuclear levels even of the heaviest ions at low harmonics and to achieve RCE resulting in the emission of x-rays.

2 Experimental facilities for relativistic heavy ions

The development of a new generation of complex accelerator facilities for heavy ions, which open up the relativistic velocity regime and which exploit a clever combination of several tools for beam preparation and manipulation, like e. g. efficient in-flight separators, storage rings, and cooling devices, allows for a new generation of channeling experiments. Illustrative examples are the RIKEN accelerator facilities (Japan), the GANIL complex (France), and the Heavy Ion Cyclotron Facility of JINR in Dubna (Russia), which can accelerate light-to-medium-heavy ions ($A < 130$) to energies of about 50 MeV/u. Although these energies are only poorly relativistic, the mentioned facilities are equipped with fragment separators thus allowing to prepare exotic and isotopically pure nuclear beams with interesting candidates for experiments, i. e., nuclei having very low lying excitation levels. The K4-K10 storage-ring complex of the Fleurov Laboratory for Nuclear Reactions (FLNR) of JINR in Dubna allows for the accumulation and storage of ions, and additional electron cooling allows to prepare beams with high brightness, i. e., to confine a large number of particles in a very small phase-space volume. Table 1 presents the maximum achievable relativistic γ factors and the highest possible excitation energy of the first and sixth harmonics for an ion travelling through a Si crystal parallel to the $\langle 110 \rangle$ axis.

	RIKEN, GANIL FLNR JINR	GSI	LHE JINR: Nuclotron (Supernuclotron)	BNL: AGS (RHIC)	CERN
max. γ	$\simeq 1.1$	$\simeq 3$	$\simeq 7$ (50)	$\simeq 12$ (100)	$\simeq 180$
$(\hbar\omega)_1$	1.5 keV	9.1 keV	22.4 (162) keV	38.7 (324) keV	583 keV
$(\hbar\omega)_6$	8.9 keV	55 keV	135 (970) keV	232 (1942) keV	3496 keV

Table 1: The first line gives the maximum reachable relativistic factor γ at the different laboratories, whereas the second and third line contain the maximum excitation energy in $\langle 110 \rangle$ Si crystals for the first and sixth harmonics, respectively (see eq. 1). The numbers given in brackets denote values which will be reached with planned machines, the Supernuclotron in Dubna and the RHIC collider in Brookhaven.

Still more versatile experimental conditions can be reached with the SIS-FRS-ESR complex at GSI (Germany) as outlined in ref. [3]: all ions up to and including uranium as well as any exotic fragment even far from stability can be provided with energies up to 1...2 GeV/u (depending on the mass-to-charge ratio of the ion under consideration). Especially H-like and fully-stripped projectiles are available, and, due to stochastic and electron-cooling facilities, high-quality beams with an angular divergence much smaller than the critical angle for channeling in this energy range are available. Typically transversal emittances smaller than about 0.1π mm mrad and a relative momentum spread $\delta p/p < 10^{-4}$ can be achieved. Recently experiments with decelerated and cooled bare and H-like Au and U beams, respectively, were

performed at energies as low as about 60 MeV/u. In addition, electron cooling allows for easy and especially very precise energy variation in very small steps, what is an important prerequisite for the measurement of narrow resonances. The magnetic spectrometer FRS allows for selective and precise atomic collision studies due to its high momentum resolution, the opportunity to investigate selected charge-changing events, and the useage of the ion-optical energy-loss mode for the sensitive measurement of energy-loss distributions. An very interesting feature with respect to RCE measurements is the opportunity to detect photons emitted from the projectile in coincidence with charge-changing and non-charge-changing collisions in a target as described e. g. in ref. [4]. The described versatility, selectivity and yet achievable precision makes the GSI facilities uniquely well suited for channeling experiments.

Strong relativistic and ultrarelativistic energies can be achieved at the Nuclotron and at the planned Supernuclotron at the Laboratory of High Energies (LHE) of JINR, which can provide ions from all elements, and at the AGS in Brookhaven (USA) and at CERN (Switzerland), which can provide Au and Pb ions, respectively. The latter mentioned synchrotrons can in principle provide all sorts of ions if the respective ion sources and adequate preaccelerators would be used.

The presently available facilities provide relativistic heavy ions for exciting channeling experiments. From table 1 it can be seen that at the GSI facilities in a Si (110) crystal using the first harmonics an excitation energy of about 9 keV can be transferred to the penetrating ion. This will allow to excite coherently all H-like ions up to Ni, or, e. g., to excite coherently that level of the $^{169}_{68}\text{Tm}$ nucleus which is characterized by an excitation energy $\Delta E = 8.4$ keV. In the following we will illustrate our model calculations with examples refering to these two candidates.

3 The basic conditions for RCE

Coherent excitation via the Coulomb field of crystal atoms can be used to excite atomic or nuclear levels of ions penetrating through the crystal. The ions channeling through ordered rows of crystal atoms with a velocity $v = \beta \cdot c$ experience a coherent excitation of frequency

$$\omega_n = \frac{2\pi\gamma\beta c}{d}n, \quad n = 1, 2, \dots, \quad (1)$$

where $\gamma = (1 - \beta^2)^{-1/2}$, n characterizes the harmonic number and d is the distance between atoms in a string (axis). If this frequency matches the energy difference between atomic or nuclear states

$$\hbar\omega_n = \Delta E_{ik}, \quad (2)$$

where ΔE_{ik} is the energy difference between the atomic or nuclear states i and k , the condition for RCE is fulfilled [5]. By tuning the velocity of the projectiles or changing the atomic spacing via tilting the crystal it is in principle always possible that one of

the harmonics coincides with a certain excitation level in an atom or nucleus. In a more general form, the condition for RCE can be written

$$\gamma \cdot (\vec{v} \cdot \vec{k}) = \Delta E_{i,k}/\hbar, \quad |S(\vec{k})|^2 \neq 0 \quad (3)$$

where $\vec{k}$ is the reciprocal lattice vector and $S(\vec{k})$ is the structure factor of the crystal. It follows that if the mean velocity of an ion or nucleus is parallel to the system of crystallographic planes the perturbation frequency $\omega_{nl} = \omega_n + \omega_l$, where $\omega_n = (2\pi\gamma v \cos \Psi/a) \cdot n$, $n = \pm 1, \pm 2, \dots$ is proportional to the transit frequency between two atoms in atomic strings along the OZ axis, and where $\omega_l = (2\pi\gamma v \sin \Psi/a) \cdot l$, $l = \pm 1, \pm 2, \dots$ is proportional to the transit frequency between neighbouring strings, and the angle Ψ is the angle between velocity $\vec{v}$ and the OZ axis. Thus, the planar case leads to an additional opportunity to tune smoothly the perturbation frequency by tilting the crystal with respect to the velocity of the ions $\vec{v}$.

4 Atomic RCE of relativistic Hydrogen-like heavy ions

4.1 Trajectory calculation and excitation dynamics along trajectory

The theory of atomic RCE is very complex and, therefore, still in progress. Important contributions to this subject have been made in refs. [5]-[11]. The main idea to describe atomic RCE is to use the two-level scheme in the resonant approximation and taking into account the mechanisms of broadening of the resonance, like splitting and mixing of electronic states of an ion due to the interaction with the electromagnetic field and due to the wake potential induced by the polarization of the target. Calculations carried out before use the straight-line-trajectories approximation and thus can describe only qualitatively energy and shape of the resonances observed in early experiments on atomic RCE (see [2] and references therein). Since already calculations of ion trajectories in a realistic crystal potential turn out to be a complex problem, it is obvious that the RCE process for an ensemble of ions cannot be treated analytically. A solution to this problem can be achieved by computer calculations using the approach developed in [12]. It is based on computer simulation of classical (but not straight) trajectories in aligned crystals taking into account the quantum dynamics for the ions in the complex electric and magnetic fields. The basic features of the model calculations are summarized:

- The ion trajectory $\vec{r}(t)$ in a crystal is calculated by means of the multistring model taking into account electronic energy loss and multiple scattering. The channeling potential is the sum of averaged axes potentials, see example shown in fig. 1a for $\langle 110 \rangle$ alignment in a Si crystal. The ion trajectory depends on the entrance coordinates and angles. The inelastic processes like electron loss and radiative decay of excited levels are taken into account by adding the corresponding decay constants.

- For a given trajectory one has to solve the set of coupled differential equations, which describes the quantum states of an ion moving in the electric field of the crystal. The set of differential equations follows from the time-dependent Schrödinger equation for an ion and takes into account various transitions between electronic levels which are mixed and splitted due to Stark and Zeeman effect, and wake potential depending on the ion's position in the crystal.
- The charge states of an ion penetrating through a crystal are calculated by adding the corresponding decay lengths in the set of equations which describe the quantum dynamics.
- Using calculated probabilities for the population of different excited electronic states of an ion after penetration through a crystal, it is possible to calculate the spectral-angular distribution and polarization for photons emitted by excited ions. This calculation includes the use of the spectral-angular distribution of radiation from various excited states and takes into account the Doppler effect. More complicated calculations are necessary if radiative decay of excited states occurs with a high probability inside the crystal.
- The results obtained for individual ions must be averaged over all possible trajectories depending on the incident phase-space.

The described computer code [12] was applied to calculate atomic RCE of fast Ne^{9+} ions channeled in a $\langle 111 \rangle$ Au crystal [13], and good agreement between the experimental data and our calculations was found. It is applied here to calculate some features of atomic RCE of $^{58}\text{Ni}^{27+}$ in Si $\langle 110 \rangle$ for the first harmonic number. The excitation energies for the $1S \rightarrow 2S, 2P$ transitions are $E \simeq 8.1$ keV. The required kinetic energy of the projectiles for excitation in the first harmonic number is $E_1 = 1557.6$ MeV/u. Numerical results for this case (see fig. 1b, c) which include only Stark effect and energy loss, clearly show the existence of a narrow resonance for individual trajectories which becomes broader after averaging the different ion trajectories and incident angles (see fig. 2). The various probabilities to be in an excited state have been calculated using an expansion of the distorted wave function of a projectile electron in a crystal over initial nondistorted $1S, 2S, 2P$ states. Strictly speaking, the curves shown in fig. 1b, c are valid for $L < 1 \mu\text{m}$, since for larger thicknesses it is necessary to take into account the radiative decays of excited levels.

The width of the calculated resonance as shown in fig. 2 is small and the spread of the resonance position is about 3 MeV/u. This is due to the neglected fine structure of electronic levels in the calculations. The fine structure splitting between the $2S_{1/2}, 2P_{1/2}$ and $2P_{3/2}$ levels for hydrogen-like Ni^{27+} amounts to 28.9 eV [14] and from eq. 1 one can easily derive that this leads to a shift of resonance energy of about 8 MeV/u. Thus, a doublet structure is expected in the RCE spectra of very heavy ions. The contribution of hyperfine structure effects, Zeeman effect, and wake potential can be neglected in this specific case.

What we would like to emphasize is the fact that the magnitude of the atomic RCE for hydrogen-like heavy ions in a thin crystal is quite large. A reduction can be caused

Figure 1: a) Calculated potential map of $\langle 110 \rangle$ Si; the isomeric contour lines represent cuts at 4, 7, 11, 15, 19, 25, 32, 48, 68, and 90 V. The arrowline indicates the projection of a calculated trajectory of a $^{58}\text{Ni}^{27+}$ ion on the transverse plane; the crystal thickness is $L = 1\mu\text{m}$, and the angle of incidence $\Theta_0 = 0.1 \Psi_L$, where Ψ_L is the Lindhard critical angle; the ion energy is $E_1 = 1557.6 \text{ MeV/u}$.

b) Probability to remain in the ground state $1S$ and in excited states $2S$, $2P_0$, $2P_{\pm 1}$ for a $^{58}\text{Ni}^{27+}$ ion travelling along the trajectory shown in fig. 1a in dependence on penetration length z for the case of exact resonance energy matching $E = E_1 = 1557.6 \text{ MeV/u}$.

c) Probability to remain in the ground state $1S$ in dependence on penetration depth z for a $^{58}\text{Ni}^{27+}$ ion having the same initial coordinates as in fig. 1a but for three different incident energies: $E = 0.999 E_1$ (solid line), $E = 1.000 E_1$ (dashed line), and $E = 1.001 E_1$ (dotted line). The trajectories are almost the same as shown in fig. 1a for all three energies. The dotted curve clearly shows the effect of energy loss when the resonant condition becomes fulfilled inside the crystal due to the slowing down.

Figure 2: Probability for a Ni^{27+} ion to be in any excited state $2S, 2P$ averaged over 100 different and randomly selected trajectories in $1\mu\text{m}$ $\langle 110 \rangle$ Si as a function of incident energy. The initial angular divergence is $\Delta\Theta_0 = 0.1\Psi_L$. Stark effect and energy loss are taken in to account, whereas the fine structure splitting is neglected.

by electron stripping during the penetration through the crystal, but the probability for electron loss of well-channeled hydrogen-like Ni ions in $\langle 110 \rangle$ Si at an specific energy $E = 1557.6$ MeV/u is less than 10^{-3} in a $L = 1\mu\text{m}$ thick crystal.

The radiative decay length $l_r = \gamma\tau v$ (where $\tau = 8.6 \times 10^{-16}$ s is the radiative lifetime of excited $^{58}\text{Ni}^{27+}$ in the $2P$ state) is of the order of $l_r = 5.6 \times 10^{-5}$ cm for an ion with specific energy $E = 1557.6$ MeV/u. Therefore, RCE can be observed by detecting the emitted photons with an energy of $E = \Delta E_{01}/\gamma(1 - \beta \cos \theta)$ just after the Si crystal, where θ is the angle between the ion velocity $\vec{v}$ and the direction of photon emission. Using crystals with different thicknesses ($L > l_r$ and $L < l_r$) one can study the radiation of the deexcitation both under disturbed (inside a crystal) and undisturbed (behind a crystal) conditions. The radiation characteristics of excited hydrogen-like heavy ions under RCE are considered in more detail in [15] for relativistic projectile energies; one of the very interesting results is the fact that at moderate relativistic energies a reasonable fraction of an ion beam can be excited to the $2S_0$ state whose radiative lifetime is macroscopically large ($\tau > 0.1$ s) since the dipole transition $2S_0 \rightarrow 1S_0$ is forbidden. Therefore, the new experimental possibility arises to detect the decay of resonantly excited heavy ions quite far away from the crystal and thus without any background, or after stimulation of the $2S_0 \rightarrow 2P_{\pm 1}$ transition by means of a weak external (for example magnetic) field to observe the very quick $2P_{\pm 1} \rightarrow 1S$ radiative decays.

4.2 Why is it interesting to study RCE with relativistic heavy ions?

A basic feature of the mentioned GSI, RIKEN, JINR, BNL, and CERN facilities is that they provide heavy ions with relativistic energies and therefore allow for the possibility to study atomic RCE of the heaviest ions using low number harmonics. Here, we analyze the main reasons which make relativistic RCE very attractive for experiments and compare RCE using different harmonic number.

The first reason is the well-known increase of electromagnetic excitation cross sections with increasing energy (see e. g. ref. [16] or [17]).

The second reason is related to the influence of thermal vibrations of the crystal atoms. The transition probabilities between different electronic states of an ion are proportional (like many coherent processes in crystals, see [18]) to the Debye-Waller factor $W_D(n)$

$$W_D(n) = \exp \left[- \left(\frac{2\pi n}{a} \right)^2 \cdot \overline{u^2} \right] , \quad (4)$$

which influences the strength of the n -th harmonic. Here, $\overline{u^2}$ is the mean-square amplitude of thermal vibrations. As it follows from eq. 4, the Debye-Waller factor reaches it's maximum for $n = 1$.

The third reason is the variation of trajectories with the variation of energy: with increasing energy the trajectories become more like straight lines and therefore more atoms of a crystal can take part in coherent excitation. The variation of relativistic γ -factor leads to the variation of penetration time if the crystal thickness is fixed and therefore to the variation of probability to be excited at the exit of the crystal if the resonant condition is fulfilled, due to oscillations between ground state and excited levels.

With increasing γ the radiation decay length increases, too. It is very important, because this fact allows for the possibility to investigate RCE at large distances from a crystal.

Finally, the electron-impact-ionization cross section is expected to decrease with increasing projectile kinetic energy thus allowing to investigate RCE using thicker crystals.

Using a realistic channeling potential as shown in fig. 1a and assuming that the trajectory is characterized by a certain position in the transverse plane in a $\langle 110 \rangle$ Si channel we exactly calculated position dependent matrix elements as described in section 4.1 taking into account only Stark effect (see also [12]), and then calculated excitation probabilities. These are averaged with respect to all impact parameters with crystal axes except regions with radius $\overline{u^2}$ near crystal axes, and further the ratio of excitation probability using harmonic number $n > 1$ to excitation probability using $n = 1$ was calculated. Fig. 3 shows this ratio for RCE of $^{58}\text{Ni}^{27+}$ for the transitions $1S \rightarrow 2S_0$, $1S \rightarrow 2P_0$, and $1S \rightarrow 2P_{\pm 1}$. As it follows from fig. 3, the RCE probability is larger for low harmonic numbers, whereas it decreases moderately with increasing harmonic number but thus allowing to excite resonantly a much larger number of hydrogen-like ions with excitation energies up to 50 keV. This fact makes

Figure 3: Ratio of the probabilities P_n and P_1 to excite an ion by means of the n -th and first harmonic, respectively, averaged over many trajectories. The initial state is in each case the ground state of a $^{58}\text{Ni}^{27+}$ ion penetrating through a $\langle 110 \rangle$ Si crystal, and the ratios for the various transitions are indicated by open squares ($1S \rightarrow 2S$), open circles ($1S \rightarrow 2P_0$), open triangles ($1S \rightarrow 2P_{+1}$), and full diamonds ($1S \rightarrow 2P_{-1}$).

very attractive the opportunity to use the GSI facilities for investigations of RCE with very heavy ions at relativistic energies for the first time.

5 Nuclear RCE

As already mentioned, at those specific energies which can be reached with the GSI accelerator facilities excitation energies of about 9 keV can be reached with the first harmonic in a Si $\langle 110 \rangle$ crystal. With higher harmonic numbers this is sufficient to excite resonantly nuclear levels with $E < 50$ keV, whereas thermal vibrations of the lattice atoms lead to the restriction $n \leq 6$. Table 2 presents a few possible candidates for nuclear RCE under the mentioned conditions.

First some remarks concerning nuclear RCE are given (see also [19]):

- The probability of nuclear RCE is rather small in comparison with atomic RCE due to smaller excitation cross sections, see e. g. fig. 4. Even in a 1 cm thick crystal one can expect that the probability is $P \sim 10^{-5}$ if the resonance condition is fulfilled, and much less at nonresonant condition. On the other hand, continuous velocity tuning to achieve the resonance condition is possible taking advantage of the electronic energy loss of the projectile inside a crystal [5].
- A special feature of nuclear RCE in comparison with atomic RCE is the very strong dependence of the excitation probability P on the multipolarity of the transition. This effect is related with the different dependence of the excitation probability on the impact parameter b for different multiplicities [20], and with

Table 2

Nucleus	Transition energy E (keV) and multipolarity $\Pi\lambda$	Radiative lifetime τ (ns)
^{57}Fe	14.4 (M1)	98
^{73}Ge	13.5	2.98×10^3
^{83}Kr	9.4	1.44×10^2
^{169}Tm	8.4 (M1)	4.0
^{181}Ta	6.25 (E1)	6.8×10^3
^{187}Os	9.8 (M1)	2.38

the non-occurrence of small b values for well-channeled nuclei. Lower multipolarities like E1 and M1 are more preferable for the study of nuclear RCE in aligned crystals. As predicted [20], even under nonresonant condition this fact leads to the possibility to observe the effect of the change of relative excitation probabilities of different multipolarities for channeled nuclei compared with nuclei penetrating through an amorphous target.

Figure 4: (a) Coulomb excitation cross-sections of the lowest nuclear levels (M1 transitions) of some selected nuclei in collisions with bare Ge target nuclei as a function of Lorentz-factor γ ; The calculations are based on the relativistic Coulomb excitation theory as developed by Winther and Alder [17].

(b) Coulomb excitation cross-sections of relativistic ^{169}Tm nuclei in collisions with Ge nuclei, calculated without [17] and with [21] taking into account the atomic screening of the Ge target nucleus. The results refer to M1 transitions.

A main difficulty in the theory of nuclear RCE is that one has to take into account that the magnitude of the periodic perturbation changes along the nuclear trajectory in a thick crystal. A perspective theoretical approach to describe the complex nuclear

RCE processes is performed by computer simulations as published in refs. [19, 22] showing broadening and damping of the nuclear RCE probability due to the bending of nuclear trajectories inside a thick crystal.

Here, we use a new computer simulation code for the calculations of nuclear RCE which is based on the multistring model for the calculation of nuclear trajectories in a crystal [22]. The excitation amplitude is defined as the sum of calculated excitation amplitudes resulting from collisions with separate atoms of the crystal along the whole nuclear trajectory. For M1 transitions (for E1 transitions compare refs. [19, 23]) the excitation probability $P(\text{M1})$ is calculated from

$$P(\text{M1}) = \frac{4\pi^2}{9\hbar} B(\text{M1}) \sum_{\alpha=1}^2 |S_{\text{M1}}^{\alpha}|^2, \quad (5)$$

where

$$S_{\text{M1}}^{\alpha} = \sum_{j=1}^N \exp\left(\frac{i\omega z_j}{\gamma v_z}\right) \cdot x_j^{\alpha} \cdot \int_{-a/2}^{a/2} \frac{dz}{v_z} \exp\left(\frac{i\omega z}{\gamma v_z}\right) \frac{1}{r} \frac{\partial}{\partial r} \varphi(r = \sqrt{\rho_j^2 + z^2}), \quad (6)$$

and $B(\text{M1})$ is the standard reduced transition probability, ω is the transition frequency, and a is the distance between atoms in the axis; $x_j^1 = x_j$, $x_j^2 = y_j$ are the coordinates of the collision point with the j -th atom having longitudinal coordinate z_j , $r = \sqrt{x_j^2 + y_j^2 + z^2}$, v_z is the velocity component parallel to the considered crystallographical direction, $\varphi(r)$ is the potential of the crystal atom, and N is the number of collisions in the crystal.

Further, for representative calculation, the one-string approximation (OSA) was used, i. e., we summed coherently the amplitudes during the collision with a separate axis and incoherently the amplitudes from different axes. This approach can overestimate the RCE yield in some cases, but we would like to mention that a similar approach works quite well in calculations for channeling radiation of relativistic particles in crystals.

The OSA calculation proceeds according to the following steps:

1. calculation of first part of nuclear trajectory in a complex average channeling potential for a chosen direction in a crystal using a fourth-order Runge-Kutta integration procedure for the equations of motion;
2. calculation of orbital integrals from eq. 6 for the calculated part of trajectory;
3. correction of nuclear trajectory at the end point of trajectory by adding a random value of the multiple scattering angle and then using the new value of energy decreased due to energy loss during penetration through this part of trajectory;
4. start of calculation for the next part of trajectory etc.;
5. when a nucleus transits from one axis to another one, the $P(\text{M1})$ value is calculated and stored;

6. all $P(M1)$ values resulting in collisions with separate axes are added and the excitation probability for the considered nuclear trajectory is defined;
7. the new nuclear trajectory starts (new point of incidence, new angle of incidence) and the steps 1.-6. are repeated.
8. when the trajectories of all particles have been considered, the mean value $\langle P(M1) \rangle$ is determined.

An important part of this calculation is the correct calculation of the phase shift between collisions $\exp(i\omega z_j/\gamma v_z)$ in eq. 6. This calculation includes the displacements of crystal atoms due to thermal vibrations and the change of $\gamma\beta_z$ due to energy loss, the influence of multiple scattering, and changes of β_z due to dynamics of nucleus motion in the averaged channeling potential. The new code allows to take into account all these aspects properly.

Representative results calculated with the OSA code are presented in figs. 5 and 6, where the enhancement $R(\psi)$ of excitation probability is shown for the example of ^{169}Tm in $\langle 100 \rangle$ Ge. The quantity $R(\psi) = P_{\text{coh}}(\psi)/P_{\text{am}}$ is defined as the ratio of excitation probability after penetration depth z in a crystal and the excitation probability in an amorphous target of the same thickness. In all cases the results of 1000 particles travelling on different trajectories have been averaged.

Fig. 5 shows the enhancement $R(\psi)$ for nuclear RCE of ^{169}Tm in $\langle 100 \rangle$ Ge at two different initial energies with corresponding Lorentz factors $\gamma = \gamma_{\text{res}}$ and $\gamma = 1.001\gamma_{\text{res}}$, respectively, in dependence of crystal orientation and thickness.

Figure 5: (a) Enhancement R as a function of misalignment angle ψ with respect to the $\langle 100 \rangle$ axis of Ge crystals with different thicknesses z . For the calculations the second harmonics was chosen ($n = 2$, $\gamma_{\text{res}} = \gamma_2 = 2.16$), and the computer experiment was performed with 1000 particles moving along different trajectories.

(b) Like (a) but for $\gamma = 1.001 \cdot \gamma_{\text{res}}$

The data in fig. 5a show, that the probability for RCE in a crystal exceeds the incoherent excitation probability in an amorphous target when the angle between the

nuclear beam and $\langle 100 \rangle$ axis changes from zero up to the large value about $3\psi_L$. At the same time, the orientation dependence differs drastically for $z < 1\text{ }\mu\text{m}$ and $z > 1\text{ }\mu\text{m}$. In the last case the interplay between flux redistribution in a channel due to channeling effect (depending on ψ) and coherence (the number of atoms in the axis which give rise to the amplitude depends on ψ , too) leads to the characteristic shape of the orientation dependence having a minimum near $\psi = 0$ and a maximum near $\psi = \psi_L$. Similar reasons explain fig. 5b, but in this case, since γ does not satisfy the resonance condition, the disconnection of the phases leads to the general decrease of the RCE probability. It is interesting to observe that nuclear RCE is more favourable for poorly channeled ions.

Fig. 6a shows the orientation dependence of R in a Ge crystal of greater thickness, $z = 10\text{ }\mu\text{m}$, and for different γ values. The significant decrease of RCE probability for $\gamma \neq \gamma_{\text{res}}$ for an angle of incidence $\psi \simeq \psi_L$ shows that the main reason for suppression of coherence effects is the break of the resonance condition eq. 1. A variation of alignment results more in a variation of impact parameters than in a variation of the phase multipliers $\exp(i\omega z_j/\gamma v_z)$ in the excitation amplitude.

Finally, fig. 6b demonstrates the narrow and impressive resonance for RCE of bare ^{169}Tm nuclei in dependence on γ for different initial misalignment angles of the beam with respect to the Ge $\langle 100 \rangle$ axis.

Figure 6: (a) Enhancement $R(\psi)$ as a function of misalignment angle ψ (normalized to the critical angle ψ_L for this specific case) for different γ values. The crystal thickness is $z = 10\text{ }\mu\text{m}$ in each case.

(b) Enhancement R as a function of Lorentz-factor γ in the surrounding of the resonance energy for three different misalignment angles with respect to the $\langle 100 \rangle$ Ge axis and for a crystal thickness $z = 10\text{ }\mu\text{m}$.

Comparing figs. 5a and 6a, one can see that R is larger at resonance energy at penetration depths $z = 1 \dots 10\text{ }\mu\text{m}$ and that R is larger for $\psi_0 = 1.0\psi_L$. This means that an initial angular spread of the beam of order of ψ_L still leads to strong resonance effect for nuclear RCE.

Thus, a main conclusion from our model calculations is that a slight misalignment of the crystal results in the highest probabilities for resonant excitation of the nucleus. The experimental setup could be quite similar to that for atomic RCE studies, i. e., using the detection of photons emitted from excited nuclei just behind the crystal if the lifetime of the excited level is small, or secondly, to implant the excited nuclei in matter when τ is large. Since substantial background will be a severe problem in the latter case, experiments with small radiative decaytime are more preferable. The radiative decay for the first excited nuclear level of ^{169}Tm is $\tau \simeq 4 \cdot 10^{-9}$ s, which results in the radiative decay distance from the crystal in the laboratory frame $l_r = \gamma\tau v \simeq 2.3$ m for RCE using $n = 2$ harmonic of the crystal potential. The photon energy is $E_\gamma = 8.4 \text{ keV}/\gamma(1 - \beta \cos \theta)$ depending of the emission angle θ . Combining figs. 4, 5, and 6 one can easily estimate the absolute cross sections for nuclear RCE.

6 Conclusions and outlook

Basic features of atomic and nuclear resonant coherent excitation of heavy ions and nuclei, respectively, have been pointed out when they are moving at moderate relativistic velocities through aligned crystalline targets. Model calculations have been performed in order to analyze RCE quantitatively. It turned out that the excitation at low harmonic numbers is advantageous.

The experimental study of atomic RCE in dependence on ion velocity v can contribute to the understanding of the electronic level structure in very strong electric and magnetic fields of order of $\gamma \cdot (10^9 \dots 10^{10})$ V/cm, since even a small distortion of electronic levels of an ion transforms at relativistic energies to remarkable change of shape and position of the resonance in the laboratory frame. Further, if the resonance condition is fulfilled it becomes possible via atomic RCE to obtain a high fraction of the beam after the crystal with inverse population. This is a first step towards the realization of an x-ray laser. The fascinating application of atomic or nuclear RCE as a source of monochromatic photons becomes possible.

Nuclear RCE can be considered as a new tool for nuclear spectroscopy, which allows to excite a single and distinct nuclear level due to the strong suppression of close nucleus-nucleus collisions under channeling conditions in aligned crystals. It is remarkable, that incident angles which are slightly larger than the required critical channeling angle are more efficient for nuclear RCE as shown for the M1 excitation of ^{169}Tm . Nuclear RCE has not been observed experimentally so far and therefore still exists as an exciting challenge for experimentalists.

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References

- [1] J. Lindhard, Dan. Mat. Fys. Medd. 34. No. 4 (1965), and
D.S. Gemmel, Rev. Mod. Phys. 46, 129, (1974).
- [2] S. Datz and C. D. Moak, in *Treatise on Heavy-Ion Science*, vol. 6, ed. by D. A. Bromley
(Plenum, New York, 1985).
- [3] H. Geissel, Nucl. Instr. Meth. **B67**, 120 (1992).
- [4] Th. Stöhlker et al., Rad. Eff. Def. **126**, 319 (1993).
- [5] V. V. Okorokov, Sov. Nucl. Phys. 2, 719 (1966) and
V. V. Okorokov, JETP Lett. 2, 111 (1965) and
I. L. Il'ina, V. V. Okorokov et al., Yad. Fiz. 25, 802 (1977).
- [6] O. H. Crawford, R. H. Ritchie, Phys. Rev. **A20**, 1848 (1979).
- [7] S. Shindo, Y. H. Ohtsuki, Phys. Rev. **B14**, 3929 (1976) and
Y. Yamashita, Y. H. Ohtsuki, Phys. Rev. **B22**, 1183 (1980).
- [8] V. A. Bazylev, N. K. Zhevago, Zh. Exp. Teor. Fiz. v.77, 312 (1979).
- [9] C. D. Moak, S. Datz, O. H. Crawford et.al., Phys. Rev. **A19**, 977 (1979).
- [10] D. D. Miller, H. F. Krause, J. A. Biggerstaff et.al., Nucl. Instr. Meth. **B13**, 56 (1986).
- [11] J. L. Minchile, J. M. Ugalde, P. M. Echenique, Nucl. Instr. Meth. **B48**, 21 (1990).
- [12] O. E. Krivosheev, Yu. L. Pivovarov, JETP Lett. **56** No. 5, 240 (1992).
- [13] F. Fujimoto, K. Komaki, A. Ootuka et al., Nucl. Instr. Meth **B33**, 354 (1989).
- [14] W. R. Johnson and G. Soff, At. Data Nucl. Data Tables **33**, 405 (1985).
- [15] O. E. Krivosheev, Yu. L. Pivovarov, in *Proceedings of Intern. Symposium on Radiation
of Relativistic Electrons in Periodical Structures* (6-10 Sept.1994), P.281
- [16] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975).
- [17] A. Winther, K. Alder, Nuclear Physics **A319**, 518 (1979).
- [18] M. L. Ter-Mikaelian, *High-Energy Processes in Condensed Media* (Wiley, New York,
1973).
- [19] Yu. L. Pivovarov, A. A. Shirokov, S. A. Vorobiev, Nucl. Phys. **A509**, 800 (1990).
- [20] Yu. L. Pivovarov and A. A. Shirokov, Pis'ma Zh. Exper. Teor. Fiz. v.53, 287 (1991).
- [21] Yu. L. Pivovarov, Preprint of The Niels Bohr Institute, NBI-88-46, Kopenhagen (1988).
- [22] Yu. L. Pivovarov, Yu. M. Filimonov, Izv. Akad. Nauk, Ser. Fiz., v.58, 98 (1994).
- [23] A. A. Shirokov, Yu. L. Pivovarov and S. A. Vorobiev, Yad. Fiz. v.49, 20 (1989).