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EPR EXPERIMENTS WITHOUT “COLLAPSE OF THE WAVEFUNCTION”

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ABSTRACT

We discuss how to describe “EPR experiments”, in particular those presently in preparation for $\phi \rightarrow K^0 \bar{K}^0$ or $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$, without making use of the awkward language of “the collapse of the wavefunction”. We propose that instead of the wavefunction, the *amplitude for a process* – detection included –, be given the central role. The amplitude method simplifies the considerations substantially; particularly, the lorentz covariance of the description becomes manifest. Furthermore, our covariant approach permits a unified treatment of the phase factors for all particle mixing problems. We obtain the relativistic corrections to the phase factors. Relative to a non-relativistic treatment, with “collapses” presumed to take place in the overall center of mass, we find that for $\phi \rightarrow K^0 \bar{K}^0$ these are on the order of five percent and perhaps observable. Implications for the fundamentals of quantum mechanics are briefly discussed.

In recent years, the decades-long discussion of the EPR “paradox”¹ took a new turn with the start of construction of projects² based on the suggestion³ of using EPR-type effects to obtain information about particle properties: the ϵ' parameter of K^0 decays and CP violation in the B meson system . Thus the study of EPR correlations has evolved from an illustration of some of the surprising features of quantum mechanics to a practical tool for determining physical parameters. However, the conceptual framework of “the collapse of the wavefunction” description of EPR correlations, inherited from the origin of the EPR effect as a “paradox”, leaves something to be desired. In the following, after presenting a description of the proposed experiments in the “collapse” framework and some of the difficulties of the latter, we would like to present what we feel is a much clearer approach, one which can handle all such problems in a simple way.

I. “COLLAPSE” DESCRIPTION

In the proposed experiments, one deals with the two-body decay of a resonance. The idea is to induce “the collapse of the wavefunction” for the two-body system by a measurement on one side or “arm” of the decay. It is argued that the resulting “collapse”, perhaps together with some information on the symmetry of the state, can be used to fix the state in the other arm. The newly-fixed single-particle state then undergoes a further non-trivial time evolution due to mixing effects and finally decays. The resulting pattern of decays, in their dependance on the time of “collapse” and the time of decay then provides detailed information on the parameters of the system . This further time evolution after the “collapse” is one of the new aspects relative to many discussions of the EPR “paradox”.

For our prime example we chose the system $\phi \rightarrow K^0 \bar{K}^0$; the system $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$ involves almost identical physics. The $J=1$ ϕ meson decays to two spin-zero neutral K mesons. For the non-specialist, we recall that the K^0 meson system is represented by a two dimensional vector space analogous to that of a spin one-half particle or the two polarization states for light. The particle and anti-particle $K^0, \bar{K}^0$ of opposite strangeness (flavor) may be taken as an orthogonal basis. Various linear combinations of these states, such as the states of definite CP, called K^1, K^2 , or the states of definite mass and lifetime K^S, K^L may also be introduced.

Because of the odd angular momentum ($l = 1$) in the ϕ decay the situation lends itself to interesting tests of the EPR type. The spatial state of the two K^0 mesons is antisymmetric under the interchange of the two particles, and so according to the usual rules of “statistics”, the two bosons cannot be identical. In the ingenious proposal by Lipkin³, one now forces “the collapse of the wavefunction” by observing a certain decay in one arm. This apparently forbids the same decay in the other arm, for this would lead to two identical bosonic systems in an antisymmetric state. This then seems to fix the state in the other arm, which then is taken to evolve freely from this new initial condition. For example, observing $\pi^0\pi^0$ in the first arm forbids, it is assumed, the same decay in the other arm *at the same time* and thus fixes the state in the second arm to be exactly that linear combination of K^0 ’s which *cannot* decay to $\pi^0\pi^0$ at the time of “collapse”. However, and this is essential, the subsequent time evolution of the state in the second arm *does* lead to $\pi^0\pi^0$ decays at later times.

There are some difficult points here, arising from this language of “the collapse of the wavefunction”. First of all, there has always been considerable discomfort associated with the notion of an instantaneous “collapse” or “reduction” over all space. Is this consistent with relativity?⁴ The present problem puts this question in evidence in a particularly clear way. The difficulty arises from the specification “at the same time”. We must certainly specify when the “collapse” or “reduction” takes place; and the only plausible possibility is to say “when the measurement was made”. However, this must be said for a specified lorentz frame. If the process $\phi \rightarrow K K \rightarrow (\pi^0\pi^0)(\pi^0\pi^0)$ (where each π pair has the mass of a K^0 meson) is forbidden at the same time, say in the ϕ rest frame, but allowed when the K ’s decay at different times, then by the relativity of simultaneity there will be other lorentz frames (taking the detections at space-like separation) where the decay $(\pi^0\pi^0)(\pi^0\pi^0)$ *will* occur at the same time. Must there be a preferred frame in which to perform “the collapse of the wavefunction”? Or does perhaps the wavefunction adjust itself in the new frame to give equivalent results when it is “collapsed”?

We shall see that “collapsing” in the rest frame of the ϕ (or in the analogous B problem in the rest frame of the Υ) does correspond to the correct answer, approximately. This might have been guessed by most practioners of quantum mechanics, but a principled justification seems to be lacking. Later, with the hindsight of our simpler approach we will provide such a justification. One may also conceive of situations, such as a hypothetical resonance decaying into three K^0 all with different

directions and velocities, where there is no obvious choice of frame or prescription to carry out the “collapse” at all.

A similar difficulty is that lorentz transformations can interchange time ordering for spatially separated events. The “reduction” as seen in one frame may occur *after* the other decay as seen in another frame. Can a measurement in the future “reduce” the past?

We would like to stress that these difficulties have nothing to do with particularities of the K^0 system such as CP violation or the non-zero width of the K^0 . We may convince ourselves of this by considering a hypothetical situation where CP is conserved and the K^0 does not decay. Let ϕ 's be produced at rest, say in a hydrogen bubble chamber, and decay to K^0 pairs. We arrange for a detector of strangeness by observing $\bar{K}^0 P \rightarrow \Lambda\pi$ on protons, a reaction not possible for K^0 . Then $(\Lambda\pi)(\Lambda\pi)$ events play the role of the $(\pi^0\pi^0)(\pi^0\pi^0)$ above. While symmetrically placed $(\Lambda\pi)(\Lambda\pi)$ events will not occur, asymmetric ones will. In “collapse” language we would say that the occurrence of one $(\Lambda\pi)$ leads to “the collapse of the wavefunction” yielding a K^0 on the other side which can later evolve into a K^0 . These events will look very much like the occurrence of the “impossible” $\phi \rightarrow \bar{K}^0 K^0$.

Of course, if there is a lorentz frame in which the two detections are simultaneous it is always possible to work in that frame and to simply ask for the probability of the appropriate configuration, without “reductions”. But this doesn't work for time-like separations or for more than two bodies. There ought to be a description for such problems where we need not resort to the somewhat mysterious “collapse”, one which is independent of the particular lorentz frame.

II. AMPLITUDES

These questions make it evident that there are situations where “the collapse of the wavefunction” leads to an awkward description and unclear results. We would therefore like to turn to another language for describing such problems⁵.

The great difficulty in the usual way of thinking is connected with the central role of the wavefunction or state vector. We thus propose, as advocated especially by Feynman⁶, to use the idea of the *amplitude* for a *process*, and not the wavefunction, as the central concept. The question then simply becomes that of the amplitude for a set

of correlated occurrences: no wavefunction “collapses” are necessary where no physical agency intervenes. If, as in the K^0 system, there are “interfering alternatives”⁶, they must be added to get the total amplitude.

A particular advantage of the amplitude approach is the ease of incorporating detection in the description. That is, we can quite naturally ask for the probability amplitude to start from some condition and to go to another where certain “detectors” are in certain states. This is important in the present problem for the following reason. In all experiments or observations we mentally make a “cut ” or “draw a line” between the object of experimentation or observation on the one hand and the detector or observer on the other, even though both are physical systems and subject to quantum mechanics . Usually this division is obvious and performed unconsciously. For example, in a reaction we are satisfied to say “a photon is emitted with a certain amplitude” and to square that amplitude. We then proceed to find the probability for the photon to be detected by a photomultiplier, the signal to be amplified and recorded, and so on, by classical or macroscopic physics . In the study of reactions we thus usually “draw the line” with the particles leaving the reaction center, implicitly assuming that no more quantum effects are to be expected⁷. With no further interference effects, the classical and quantum mechanics parts of the calculation simply factorize. This lack of further interference effects can serve to define what may be properly considered a “detector”.

Nevertheless, there is an element of judgement involved. Part of the subtlety in the present problem is that, due to the small mass differences for the K^0 's, interference effects can extend over macroscopic distances and we must “draw the line” somewhat further out than is usual in scattering and decay problems. Whenever there is danger of ambiguity we can always follow the process to the production of a π pair or a $\Lambda\pi$ system , etc. our “detectors” -, in order to avoid any inadmissible questions as to what particles or states we “really had”. The amplitude language is well suited to this.

Another advantage of the amplitude description is that it restores the obvious symmetry between the various parts of the process, one which is disturbed in the “collapse” language. In a two body decay the two sides are on an equal footing, and we need not be concerned with which detection was first or second and with who is “reducing” whom⁵.

III. LOCALIZED DETECTION

For the present problems we shall assume that classical space- time concepts, such as the statement that a particle was detected at a certain time and place, are applicable. This means that the particles are to be thought of as being described by a relatively well localized wavepacket; the amplitude for configurations with particles far off their classical paths will be taken to be negligible, so that with free particles we always have $x \approx vt$ according to classical kinematics⁸.

We thus propose to discuss the amplitude for a certain final particle or reaction to be detected at (x_1, t_1) , while another is detected at (x_2, t_2) and so forth.

The opposite of localized detection would be detection with very high momentum resolution, so that the detection point is not well localized. In such a case momentum space methods, as with the familiar feynman graphs, would apply. However, this is not our interest here.

The situation we have to deal with may then be characterized as follows. At the origin some wave packets are produced, each one moving towards one of the detectors. Let us fix our attention on the wavepacket with central momentum $\mathbf{p}$, which reaches detector 1 at (x_1, t_1) . The wavepacket, however, is not in a mass eigenstate, but in a linear combination of them. Upon decomposing it into mass eigenstates, we will have two (or possibly more) component packets with the same central momenta but, due to the different masses, different velocities.

If the component packets separate greatly, that is if they move apart so that they no longer overlap upon reaching the detector⁹, the situation is straightforward and there are no interesting interference effects; therefore we confine ourselves to the case where there is significant overlap. It is, nevertheless, important to recognize for the following that because of the different velocities the centers of the wavepackets may be shifted relative to one another, even though they are still largely overlapping.

Note that we have neither assumed that the energy nor the momentum has a sharp value. For localized detection we need wavepackets, which are uncertain in both.

IV. THE PHASE FACTOR

Our basic quantity of interest is the phase acquired by the mass eigenstate in traveling: due to the different phases accumulated on the way, the states recombined at the detector can be different than those originally produced.

We base our method on the principle that given a classical path or trajectory for a free particle between two space-time points, there is a phase factor $e^{i(\mathbf{p}\mathbf{x}-Et)} = e^{-im\int ds}$ to be associated with it¹⁰. Here m is the particle mass and ds the proper time interval, whose integral we call τ , the total proper time for the particle to travel between the two points. The resulting phase may be regarded as a consequence of elementary quantum mechanics plus relativity: $m\tau = m\sqrt{t^2 - x^2}$ is simply the classical action.

Accordingly, when we are concerned with several states of different mass there is for every mass eigenstate a phase factor

$$e^{-i m^n \tau^n} \tag{1}$$

in the amplitude. The index n refers to the eigenstate in question, so that m^n is the mass of eigenstate n .

The index on τ , however, involves a more subtle question. Its understanding is essential for a correct general treatment of particle mixing phenomena. With it we can also understand completely relativistic problems, such as neutrino oscillations. The subtlety arises from the fact that the detection point (x_1, t_1) cannot in general lie on the idealized classical path for *all* the mass eigenstates: the idealized classical path is followed by the center of the wave packet, and as remarked above, even though the wavepackets for the various mass eigenstates are taken to overlap at the detector, their centers are in general displaced with respect to each other. However, since we assume a local detection mechanism we must refer all phase factors to a common point (x_1, t_1) . If we take this point to coincide with the center of one of the wavepackets, say that for mass eigenstate r (for “reference”), the phase factor for the n -th eigenstate must be evaluated, in general, at a point which is *off* the classical path for particle n . This phase factor for n is $e^{i(\mathbf{p}\mathbf{x}_1 - E^n(p)t_1)}$, where $E^n(p) = \sqrt{(m^n)^2 + \mathbf{p}^2}$, where we recall that $\mathbf{p}$ is the central momentum of the packet. To evaluate $(\mathbf{p}\mathbf{x}_1 - E^n(p)t_1)$ we may, since it is an invariant, transform to the rest frame of n . There $E^n = m^n$,

so that we have $-m^n t$ for the phase, with t the time of detection as seen in the rest frame of n . This invariant time is what we call τ^n .

We may also find τ^n by saying that since we take (x_1, t_1) on the classical path for r , we can re-express (x_1, t_1) in terms of $(\mathbf{p}^r, E^r)$, so that for another mass eigenstate n we have,

$$-m^n \tau^n = (\mathbf{p}\mathbf{x}_1 - E^n(p)t_1) = \frac{p^n \cdot p^r}{m^r} \tau^r, \quad (2)$$

where τ^r refers to the (on path) proper time for r .

With this understanding for τ^n , then, Eq (1) gives a general covariant expression for the phase factor relevant in all mixing problems. This also includes particle mixing in single- armed configurations, although this is not our main concern here. When relativistic effects are unimportant, τ is of course the ordinary time and the index n plays no role. In addition, for the decays $\phi \rightarrow K^0 K^0$ or $\Upsilon(4S) \rightarrow B^0 B^0$ the smallness of the mass differences relative to the masses among the K 's or B 's means that the particle label on τ may be ignored. On the other hand for relativistic problems like neutrino oscillations the label is important. In a frame where two mass eigenstates n and n' are relativistic, we find from Eq (2), $m^n \tau^n - m^{n'} \tau^{n'} \approx \frac{(m^n)^2 - (m^{n'})^2}{2E} t$. This is the usual expression for neutrino oscillations. Note that this applies even if one neutrino is relativistic in the rest frame of the other, quite a different situation than for $\phi \rightarrow K K^{11}$.

Finally, there is the question of the instability of the K^0 or other particles, which we handle in the usual way by allowing the masses to be complex. The lifetime of a particle is then governed by the proper time, as is to be expected.

V. GENERAL AMPLITUDE AND EXAMPLES

To treat any problem involving classical propagation and local detection we thus take the particles to be produced at a given space-time point, decompose them into states of definite mass, propagate them with the phase factor to the points of detection, and employ the amplitudes for the detection reactions. We then add the alternative amplitudes, if any, so calculated. Thus the amplitude $A(D_1, D_2, \dots, D_n)$ for a process with an n -body final state detected at points $1, 2, \dots, n$ in the channels $D_1, D_2, \dots, D_n$ will be a sum of terms with different intermediate particles $A, B, C, \dots, L$

$$A(D_1, D_2, \dots, D_n) \sim \sum A_{ABC\dots L}^P \epsilon^{-im^A \tau_1^A} \epsilon^{-im^B \tau_2^B} \dots \epsilon^{-im^L \tau_n^L} A_{AD_1}^1 A_{BD_2}^2 \dots A_{LD_n}^n, \quad (3)$$

where A^P is the production amplitude for the configuration where the first named particle A goes to point 1, B to point 2 and so forth and $A_{AD_1}^1$ is the amplitude for detection at point 1 in the channel D_1 . Appropriately normalized, this expression can be written as an equality. If the constituent A 's are invariants the whole expression is an invariant.

As a first example we take p-wave decay to two spinless particles ; that is, the $\phi \rightarrow KK$ or $\Upsilon(4S) \rightarrow BB$ cases. For the production of two particles A and B we have the amplitude A_{AB}^P , and for their detection at spacetime point 1 in the channel D_1 we have the amplitudes $A_{AD_1}^1$ or $A_{BD_1}^1$; and similarly for point 2. The total amplitude for production, propagation and detection via D_1 at 1 and D_2 at 2 is then proportional to

$$A_{AB}^P (\epsilon^{-im^A \tau_1^A} \epsilon^{-im^B \tau_2^B} A_{AD_1}^1 A_{BD_2}^2 - \epsilon^{-im^B \tau_1^B} \epsilon^{-im^A \tau_2^A} A_{BD_1}^1 A_{AD_2}^2). \quad (4)$$

The minus sign corresponds to the antisymmetry of the spatial state: that is we have used $A_{AB}^P = -A_{BA}^P$. The meaning of the indices on τ was discussed above.

This formula deals with the question "in which lorentz frame should we perform the collapse?" by providing a covariant expression referring to no overall frame of reference. For the ϕ case, A and B are to be identified with K^L and K^S , the states of definite mass and lifetime. The fact that these are not orthogonal in the two-state K^0 subspace need not concern us; this will manifest itself in the relationships of various decay and production channels, in the structure of the A 's.

We now specialize further to ϕ decay with identical systems on opposite sides, such as decay to double π^0 pairs. We then have $D_1 = D_2 = \pi^0 \pi^0$, and since the decay amplitude is independent of position, $A^1 = A^2$. There is no significant separation of wavepackets, so we must consider interfering alternatives according to whether we have K^S, K^L or K^L, K^S in the two arms. Eq (4) becomes

$$A(\phi \rightarrow K^S K^L) (\epsilon^{-im^S \tau_1^S} \epsilon^{-im^L \tau_2^L} - \epsilon^{-im^L \tau_1^L} \epsilon^{-im^S \tau_2^S}) A(K^S \rightarrow \pi^0 \pi^0) A(K^L \rightarrow \pi^0 \pi^0) \quad (5)$$

This corresponds to Lipkin's expression³ for identical systems with, however, the difference that the relativistic $m\tau$ appears in the phase factors. The expression gives vanishing of the amplitude at equal proper times, where $\tau_1^L = \tau_2^L, \tau_1^S = \tau_2^S$. This is the invariant meaning of the statement that both K^0 's cannot decay to the same state "at the same time": it turns out to mean at equal proper times. Thus it is indeed in the ϕ rest frame (but not in other frames in general), where both K^0 's have the same time dilatation factor, that we cannot have the same decays for both K^0 's simultaneously.

It is worth noting that the phase $im\tau$ differs from the non-relativistic iEt suggested by the "collapse" approach by a factor $(m/E)^2$, which in ϕ decay differs from one by $4.8 \cdot 10^{-2}$, or about five percent. This is small, but in sufficiently accurate experiments it will be important to use the full relativistic phase factor, which follows from the basic assumptions of our approach¹¹. In $\bar{P}P$ annihilation, where the K^0 are fast, the relativistic effect is important: $(m/E)^2$ becomes $\approx 1/4$.

If we now consider the non-identical decays $D_1 = (\pi^0\pi^0)$ and $D_2 = (\pi^+\pi^-)$ we find, dropping the superscripts in view of the very small K^0 mass difference, at equal τ 's the amplitude

$$A(\phi \rightarrow K^S K^L) e^{-i(m^S + m^L)\tau} \\ [A(K^S \rightarrow \pi^0\pi^0)A(K^L \rightarrow \pi^+\pi^-) - A(K^L \rightarrow \pi^0\pi^0)A(K^S \rightarrow \pi^+\pi^-)] \quad (6)$$

Since this expression is non-vanishing only to the extent that the K^L and K^S decay amplitudes are not exactly proportional to each other, this process may be used to find the so-called ϵ' parameter, which characterizes this lack of proportionality³.

All that has been said applies equally well to $\Upsilon(4S) \rightarrow BB \rightarrow (l^\pm X^\mp)(f^{CP})$, a process which will be the main focus of upcoming B factory experiments on CP violation. The B 's are neutral, $l^\pm$ is a lepton while $X^\mp$ is a non-leptonic system of opposite charge; f^{CP} is a final hadronic (approximate) CP eigenstate like $J/\psi K^S$.

In "collapse" language, the first B to decay forces the remaining B to be, "at the same time", that linear combination of B^0 mass eigenstates (called B^{heavy} and B^{light}) which cannot decay in the same way, just as in the argument described previously for ϕ . The subsequent time evolution of the remaining B then proceeds from this initial condition by the usual mixing formalism. The calculation must be performed

separately according to whether the semi-leptonic or the hadronic decay came first.

In our amplitude approach, without “collapses,” we simply invoke the amplitude Eq (4) for the entire sequence of events. Particle A in Eq (4) is then B^{heavy} , and particle B is B^{light} . Since the B^{heavy} , B^{light} mass difference Δm is tiny compared to the masses themselves, we may safely drop the superscripts on the proper times, just as for the K^0 mesons. Assuming as usual that a single Feynman diagram dominates the decay of a definite flavor B^0 into f^{CP} , we find¹² that the probability for the entire sequence of events is proportional to

$$e^{-\Gamma(\tau_2+\tau_1)}\{1 \pm \eta \sin\phi \sin[\Delta m(\tau_2 - \tau_1)]\} \quad (7)$$

Here, τ_1 is the proper time of the B decay to $(l X)$, and τ_2 that of the B decay to f^{CP} , while Γ is the common decay width of B^{light} and B^{heavy} , η is the CP parity of f^{CP} , and ϕ is the phase of a product of quark mixing matrix elements whose identity depends on the choice of f^{CP} . The plus and minus signs correspond to the cases of positive and negative leptons respectively. A CP-violating asymmetry results from the difference of the $\pm$ cases, which can yield clean information on the phase ϕ , thereby testing the Standard Model hypothesis that CP violation is caused by the phases of the quark mixing matrix elements.

Eq (7) verifies the expression¹³ based on the “collapse” approach, with the difference that the time variables are now proper times, and not (presumably) times in the $\Upsilon(4S)$ rest frame. To be sure, the B mesons are sufficiently nonrelativistic that this is a negligible distinction. Thus the amplitude approach shows that the expression obtained by collapsing the wave function remains numerically accurate, even when the possible effects of relativity are not neglected. More importantly, this approach obtains that expression without invoking any “collapse”, with its attendant puzzles¹⁴.

In addition to the case of more than two bodies, as covered by Eq (3), one can imagine generalization to other amusing applications. For example, we can have a “cascade” where a decay into one mixing system is followed by another, such as $\Upsilon \rightarrow B^0 \rightarrow K^0$, where the B ’s and the K ’s manifest themselves by decays or interactions at definite space-time points. There is then a phase factor like Eq (1) between each detection.

VI. WAVEFUNCTIONS VS AMPLITUDES

Particularly in connection with relativistic invariance and simultaneity, “the collapse of the wavefunction” threatens to be not only awkward but actually inconsistent; that is, to give inequivalent answers in different lorentz frames. However, this threat is more apparent than real. As mentioned above, the wavefunction can transform so that after a lorentz transformation equivalent results appear. For example, we learned from the amplitude approach that it is only in the ϕ rest frame that the two K^0 ’s are forbidden to decay in the same way at the same time. Thus there appears to be a preferred frame in which to do “the collapse of the wavefunction” : here, the ϕ frame. However, analysis shows that there is a reason for this preference in the wavefunction approach: it is only in this frame that the wavefunction is anti-symmetric¹⁵. Consequently, simultaneous identical decays also become possible in the wavefunction approach, once we leave the ϕ rest frame.

Similarly, in the $\Upsilon \rightarrow BB$ case, for example, one can refine the wavefunction approach to take the motion of the B mesons into account and find the relativistic corrections; however it would have been difficult to make the adaptation without already knowing the answer from the amplitude approach. More generally, we know that relativistic quantum field theory can be developed in terms of the second quantized wavefunction or state vector and in that theory we may speak of particles produced and detected at various space-time points in a covariant manner.

Nevertheless, the superiority of the amplitude approach seems clear. This is not only because of the greater simplicity of calculation, deriving from the fact that the amplitude is an invariant while the wavefunction is not. By choosing to focus on the probability amplitude, something attached to a *process*⁶, we avoid the conceptual difficulties associated with a reified, “existing” wavefunction . The latter is now relegated to the role of a secondary quantity, one describing changes in the amplitude. Since there is no longer any wavefunction to “collapse” in the first place, the psychological and philosophical discomforts associated with said “collapse” disappear. Also, on a pedagogical level there is less danger of the frequent confusion of probability amplitude waves with the waves of some physical entity like the electric field. Concerns with bizarre constructs like enigmatic parallel universes, worlds haunted by the restless prowlings of possibly dead cats, disappear, cats and all. The best answer, finally, to the “question of the collapse of the wavefunction” is that there is no wavefunction.

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3. H. J. Lipkin. Phys. Rev. **176** 1715 (1968). I. Dunietz, J. Hauser and J. L. Rosner. Phys. Rev. **D35** 2166 (1987).
4. However, the possibility that the authors of ref. 1 did not discuss difficulties connected with relativity because at the time of writing they were unaware of the latter would seem to be excluded.
5. The question of how to handle a sequence of observations in quantum mechanics has arisen in various contexts. Our problem is in some ways similar to that of the production of a particle track. In considering two successive ionizations one can say the first collision "reduces the wavepacket" and proceed to calculate the probability of the second, starting from this new initial condition. Or one can consider the probability for two ionizations, viewed as a single process. See W. Heisenberg, *The Physical Principles of the Quantum Theory*, Dover, NY, Chapt. V.1., also L.I. Schiff, *Quantum Mechanics*, McGraw Hill, NY 1949, sec. 30. Another case in point is quantum optics, when we wish to know the probability of counting a photon given that one has already been counted at some earlier time. R.J. Glauber handled this question by the artifice of a set of detectors which are turned on and off, in *Quantum Optics and Electronics, Les Houches 1964*, Eds. C. DeWitt, A. Blandin and C. Cohen Tannoudji, Gordon and Breach, N.Y., Lecture V.

6. The first sections of Vol. III of *The Feynman Lectures on Physics*, by R. P. Feynman, R. B. Leighton and M. Sands, Addison-Wesley, Reading MA, (1965), contain an exposition of quantum mechanics in terms of amplitudes; as well as R. P. Feynman and A.R. Hibbs, *Quantum Mechanics and Path Integrals*. See also *The Principles of Quantum Mechanics* by P.A.M. Dirac, Oxford (1935), second edition, section 33.
7. J. von Neuman in *Mathematische Grundlagen der Quantenmechanik*, Dover, N.Y. 1943, chapt.VI, sect. 2, discusses "moving the line". See also Heisenberg's comments in the above mentioned discussion of producing a track.
8. "Local detection " of course also includes decays in vacuum, where the point of decay is determined by tracing classical trajectories back from devices which may be located elsewhere. In this case, "l" means the inferred coordinates (x_1, t_1) where the decay products appeared.
9. Such a separation of mixing particles according to mass is a real possibility for neutrinos from a supernova burst. See P. Reinartz and L. Stodolsky, *Z. f. Physik* **C27** 507 (1985).
10. L. Stodolsky . *Genl. Relativity and Gravitation* **11** 391 (1979). In principle, gravity influences the phase factor, as discussed in this reference. When there is a gravitational field present our phase $m \int ds$ receives an additional contribution so that $m \int ds \approx (\mathbf{p}\mathbf{x} - Et) + \frac{1}{2} \int h_{\mu\nu} p_\mu dx_\nu$, where $h_{\mu\nu}$ is the deviation of the metric tensor from its flat (Minkowski) value. Thus, there is an effect according to whether the arms of the $\phi \rightarrow K\bar{K}$ experiment are parallel or perpendicular to the earth's field. The effects are of course extremely small because of the small K^0 mass difference.
11. Other authors have also written the phase factors with the proper time, but not with the particle label on τ . Although this was entirely appropriate and not important for the cases under consideration (in the center of mass), the particle label is in general necessary, as we have explained; T. Kamae, T. Kifume and K. Tsumemoto, *Prog. Theo. Phys.* **41** 1267 (1969); I. Bigi et al. in *CP Violation*, C. Jarlskog, ed., World Scientific, Singapore (1989); R. Blankenbecler (unpublished).
12. B. Kayser, talk presented at the Moriond Workshop on Electroweak Interactions and Unified Theories, March 1995; to be published in the Proceedings.

13. A. Carter and A. Sanda, Phys. Rev. **D23** 1567 (1981); I. Bigi and A. Sanda, Nucl. Phys. **B193** 85 (1981).
14. We have also applied the amplitude approach to $\Upsilon(4S) \rightarrow BB \rightarrow (l^\pm X^\mp)(f)$, where f is not a CP eigenstate, and to processes where the B meson pair is in a symmetric state, rather than the antisymmetric one of $\Upsilon(4S)$ decay, confirming previous results without invoking any “collapses”. Details of these analyses will be reported elsewhere.
15. In checking this statement one must be careful to perform lorentz transformations on mass eigenstates only.