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The Sun's hydrostatic structure from LOWL data

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ABSTRACT

Recent observations with the LOWL instrument have for the first time provided a uniform set of frequencies of sufficient extent to allow detailed inversion for the structure of much of the Sun's deep interior. In addition, improvements in the description of the required physics motivates the calculation of new solar models. Here we present results of inversions of the LOWL data against several reference models using up-to-date physics. In models including the gravitational settling of helium, the sound speed and density agree with the Sun to within substantially better than 1%. An even better agreement between the model and the Sun results from a moderate increase in the rate of helium diffusion. We also find that the seismic data support theoretical calculations which indicate that settling of heavier elements has reduced the present surface value of Z by about 8% relative to its mean interior value.

Subject headings: Sun: interior — Sun: oscillations — Sun: abundances — Diffusion

1. Introduction

As is well known, the observed solar p-mode frequencies provide an extremely useful diagnostic of the internal structure of the Sun, and permit us to test in considerable detail the physics used in the theory of stellar structure. In particular, one can use the frequencies to infer the variation of sound speed with depth (Christensen-Dalsgaard *et al.* 1985). Over the past ten years, the data and the inversion techniques have improved considerably, but the structure of the energy-generating core remains rather uncertain. Few of the observed modes are sensitive to this region, namely only the most deeply penetrating low-degree p modes. Furthermore, the frequencies of these modes and of the higher-degree modes used in the inversions have been determined from different observations: the frequencies at low degree coming from whole-disk measurements while the majority of the data come from resolved measurements. Any systematic differences between the different datasets may be interpreted in the inversions as being due to structural features in the Sun. Such systematic differences might arise from different instrumental characteristics, differences in analysis techniques, or from temporal variations in the solar p modes if the different observations are not exactly contemporaneous. The difficulties of determining the core structure from such inhomogeneous datasets are well illustrated by Gough & Kosovichev (1993) and Gough, Kosovichev & Toutain (1995) (see also Basu *et al.* 1995a).

The recently-constructed LOWL instrument has been designed specifically to make possible the accurate measurement of low- and intermediate-degree modes with the same instrument (Tomczyk *et al.* 1995b). Thus we can perform inversions for solar structure extending into the core using a homogeneous set of frequency data. Observations with the instrument started in February 1994, and we now have a homogeneous set of solar p-mode frequencies with degrees $l = 0 - 80$ for a six-month observing run.

The input physics used in modelling the solar interior has also been refined over the past decade. The opacities and equation of state have been revised, leading to better agreement with the observed frequencies (e.g. Christensen-Dalsgaard, Däppen & Lebreton 1988; Dziembowski, Pamyatnykh & Sienkiewicz 1992; Berthomieu *et al.* 1993; Gabriel 1994). The inclusion of the gravitational settling of helium and heavier elements in the models also appears to bring

the model closer to the Sun (e.g. Cox, Guzik & Kidman 1989; Christensen-Dalsgaard, Proffitt & Thompson 1993; Basu & Antia 1994), although Guenther & Pinsonneault (1993) found no obvious improvement in straight frequency comparisons between the Sun and models including helium settling.

We have computed models using recent physics and carried out inversions of the LOWL data using these models as reference. The results confirm the clear improvement in the models resulting from the inclusion of helium settling and demonstrate the sensitivity of the comparison with the observations to even quite modest changes in the physics or input parameters. In particular, a modest increase in the rate of helium diffusion leads to a model that appears to agree with the Sun to within 0.15% in sound speed and 0.5% in density, throughout most of the inner 80% of the solar radius.

2. Observations and inversion technique

Our data were derived from observations made between February 26 and August 26 1994 using the LOWL instrument situated on the Mauna Loa, Hawaii. For details of the instrument see Tomczyk *et al.* (1995b); details of the basic data analysis were given by Schou (1992) and Tomczyk, Schou & Thompson (1995a). Our dataset comprises 702 modes with degrees $l = 0 - 80$ and frequencies in the range 1500 μHz to 3500 μHz .

The inversion technique is based upon linearizing differences between the Sun and a reference solar model. The differences in frequencies between the Sun and model can then be related to structural differences between the two. Specifically, the frequency differences can be expressed as weighted integrals of differences in some pair of structural variables. We then perform an inversion to estimate the structural differences. Because the problem has been linearized, a number of different linear inversion techniques can be used: we have used the Subtractive Optimally Localized Averages method of Pijpers & Thompson (1993). The precise implementation of the method is as described by Basu *et al.* (1995a): for further details on the structure of the problem, see Basu *et al.* (1995b). We have performed inversions for sound speed, formulating the problem in terms of sound-speed and density differences. If we assume that the equation of state of the Sun is the same as that in the reference model, and that the abundances of elements

TABLE 1
CHARACTERISTICS OF SOLAR MODELS

Name	X_0	X_s	X_c	T_c 10^6 K	ρ_c g cm^{-3}	$d_b/R_\odot$	Φ^{Cl}	Φ^{Ga}	^8B
STD ^a	0.7059	0.7339	0.3319	15.689	155.59	0.293	8.67	133.5	6.27
STD-ND ^b	0.7038	0.7038	0.3442	15.611	151.65	0.279	7.98	130.1	5.72
NS-OPAC ^c	0.7050	0.7332	0.3315	15.757	153.86	0.293	9.28	136.0	6.79
NS-AGE ^d	0.7049	0.7327	0.3371	15.666	153.94	0.292	8.47	132.5	6.11
NS-DIFF ^e	0.7057	0.7325	0.3462	15.677	151.94	0.293	8.58	133.0	6.20
NS-Z ^f	0.7174	0.7452	0.3452	15.547	154.27	0.289	7.46	127.4	5.32

NOTE.—(a) Standard model with helium settling. (b) No helium settling. (c) Increased core opacity. (d) Assumed age of present Sun 4.5 Gyr. (e) Increased helium diffusion coefficient. (f) Heavy-element abundance $Z = 0.018$. X_0 , X_s and X_c are, respectively, the initial, present surface and present central hydrogen abundances, T_c and ρ_c are central temperature and density, $d_b/R_\odot$ is the depth of the convection zone in units of the solar radius $R_\odot$. The columns marked Φ^{Cl} and Φ^{Ga} give the neutrino capture rates in the chlorine and gallium radio-chemical neutrino detectors in units of SNU, while the last column gives the flux of ^8B neutrinos, in units of $10^6 \text{ cm}^{-2} \text{ s}^{-1}$, solely to which the Kamiokande electron-scattering experiment is sensitive.

heavier than helium are known precisely, then one can also formulate the problem in terms of differences in density and helium abundance, for example: with this formulation we have performed inversions for density.

3. The solar models

The basic calculation of the solar models was essentially as presented by Christensen-Dalsgaard (1982), but including in most cases also helium diffusion and settling as described by Christensen-Dalsgaard, Proffitt & Thompson (1993). Thermodynamic quantities were obtained from interpolation in tables based on the MHD equation of state (Hummer & Mihalas 1988; Mihalas, Däppen & Hummer 1988; Mihalas *et al.* 1990), using the MHD(F) case (for details, see Christensen-Dalsgaard & Däppen 1992). We used OPAL opacities, including spin-orbit interaction and calculated with the meteoritic iron abundance, at temperatures higher than 10^4 K (e.g. Iglesias, Rogers & Wilson 1992) and Kurucz tables at lower temperature (Kurucz 1991); interpolation in density and temperature was carried out by means of a sophisticated procedure incorporating a nine-parameter interpolating function (Houdek & Rogl 1993). Convection was treated according to the Böhm-Vitense

(1958) mixing-length formulation. Nuclear-reaction parameters were largely as given by Bahcall & Pinsonneault (1995); the initial abundances of the CNO elements were taken from Grevesse & Noels (1993). Microscopic diffusion and settling coefficients for helium were calculated using the expressions given by Christensen-Dalsgaard *et al.* (1993); we did not take into account diffusion of heavier elements. Except where otherwise noted, the heavy-element abundance was $Z = 0.02$ and the age of the present Sun was taken to be 4.6 Gyr.

Evolution was started from a chemically homogeneous zero-age main-sequence model. The initial hydrogen abundance X_0 and the mixing-length parameter α_c were adjusted to obtain a model of solar luminosity and radius, taken to be $3.846 \cdot 10^{33} \text{ erg s}^{-1}$ and $6.9599 \cdot 10^{10} \text{ cm}$, respectively, for the model of the present Sun. Tests based on models with differing numbers of mesh points or time steps show that the numerical relative errors in sound speed and density are unlikely to exceed 10^{-4} (see also Christensen-Dalsgaard 1991).

Compared to the models considered by Christensen-Dalsgaard *et al.* (1993) the major changes result from improvements in the equation of state and a reduction

Fig. 1.— (a) Inferred relative differences in sound speed squared between the Sun and reference model, in the sense Sun minus model, for the diffusive model STD (triangles) and for the nondiffusive model STD-ND (squares). Vertical error bars denote the $1\text{-}\sigma$ standard errors; horizontal bars denote the radial resolution. The errors and resolution are similar for each of the two reference models. (b) As panel (a), but showing the inferred relative density differences.

in the assumed abundance of the CNO elements. The latter change tends to reduce the energy generation rate, leading to an increase in the hydrogen abundance required to calibrate the model, with consequent changes also to the structure of the convection zone. Some characteristics of the computed models are listed in Table 1.

Adiabatic oscillation frequencies were calculated following the procedure described by Christensen-Dalsgaard & Berthomieu (1991); in particular, the frequencies were obtained from Richardson extrapolation. The relative numerical error in the computed frequencies was below 10^{-5} . The combined numerical errors in the model and frequency calculations contribute by less than 5×10^{-5} to the values of $\delta c^2/c^2$ inferred from the inversion.

4. Results

We first invert for the differences between the Sun and two “standard” models, one without helium diffusion STD-ND and the other including helium diffusion STD. Figure 1 shows inferred sound-speed differences (panel a) and density differences (panel b) using either model STD (triangles) or model STD-ND (squares). We are only able to form localized kernels for radii $0.05 \lesssim r/R_\odot \lesssim 0.75$; the upper limit of the range could be moved closer to the surface if

we had higher-degree data. For both sound speed and density, it is striking that the inferred differences are much smaller when the reference model incorporates gravitational settling of helium. This strengthens the conclusion reached by Christensen-Dalsgaard, Proffitt & Thompson (1993), based on only an asymptotic inversion for sound speed. The improvement results partly from the increased convection-zone depth, evident from a comparison of Models STD-ND and STD in Table 1; this in turn comes about due to the increased hydrogen abundance, and hence increased opacity, near the base of the convection zone. The increase in the hydrogen abundance also directly contributes to increasing the sound speed below the convection zone and, through effects in the hydrogen and helium ionization zones, to the increase in the density in the convection zone. Although the differences with the nondiffusive model are small by most astronomical standards, we are able effectively to rule out this model because of the high precision of the frequencies.

Note, however, that even for the diffusive model the inferred differences are significant, and there are systematic deviations from zero. In particular, both the sound-speed and density differences indicate that the Sun’s core structure is imperfectly modelled. The negative density differences in $r \lesssim 0.2R_\odot$ indicate that the Sun is less centrally condensed than the

Fig. 2.— (a) Inferred relative differences in sound speed squared between the Sun and the following reference models: STD (thick continuous line), NS-DIFF (dotted line) and NS-Z (long dashed line). Standard $1\text{-}\sigma$ errors are shown on the inversion relative to model STD: the errors on the other inversions are similar. For clarity the inversion results are represented with continuous lines rather than discrete points. (b) As panel (a), but showing the inferred relative density differences.

model: in fact, given the relatively poor radial resolution in that region, the difference between Sun and model might locally be considerably larger than the extremal values obtained from the inversion. It is of interest therefore to investigate the effect of further modifications of the input physics. One can reduce the model's core density by a modest increase in the core opacity or by reducing the assumed age of the Sun. These effects are illustrated by models NS-OPAC and NS-AGE of Table 1: the core opacity in the former has been increased by 4.7% while in the latter the model has been evolved to match the observed solar radius and luminosity at an age of 4.5 Gyr rather than at our standard assumed age of 4.6 Gyr. In both cases, this is sufficient to create a modest decrease in the central density relative to our standard diffusive model STD and to reduce the absolute magnitude of the inferred density differences $\delta\rho/\rho$ in $r < 0.2R_\odot$ to less than 0.2%. In both cases, however, the agreement between Sun and model in the convection zone is made much worse, the inferred $\delta\rho/\rho$ being increased to 0.6% just below the convection zone, and to even larger values in the convection zone itself. Likewise the agreement in sound speed is actually made worse between $0.6R_\odot$ and $0.7R_\odot$.

A rather better agreement with observation can be obtained if we increase diffusion, by multiplying

the hydrogen-abundance gradient term in Eq. (1) of Christensen-Dalsgaard, Proffitt & Thompson (1993) by a factor three, while leaving the settling velocity unchanged. The improved agreement in terms both of density and of sound speed can be judged from Fig. 2. Admittedly this is an *ad hoc* modification, for which we do not offer any physical justification; but we note that it is similar to the modification proposed by Schatzman *et al.* (1981) in an attempt to solve the solar neutrino problem and account for observed surface abundances, and intended as a representation of rotationally induced turbulent diffusion. Schatzman *et al.* considered different values of the factor Re^* multiplying the diffusion coefficient, in the range 50 – 200, to achieve a substantial reduction in the ^{37}Cl neutrino flux. In the present case it can be seen by comparing models STD and NS-DIFF in Table 1 that the effects on the neutrino fluxes are not significant. Differences in other quantities between these two models are illustrated in Fig. 3. It may be noted from the figure that the sound-speed and density model differences in the core are greater than 1%. No hint of these substantial differences can be seen in the results of the inversions in Fig. 2, however. The rather poor radial resolution of the inversion in the core smooths out any sharp features: thus we cannot rule out the possibility that similar large but localized

differences exist between the Sun and our models in the central regions.

The standard heavy-element abundance Z that we have adopted for all the models discussed so far has been 0.02. However, a recently published observational determination of $Z/X = 0.0245$ (Grevesse & Noels 1993) would indicate that one should adopt instead a somewhat lower value of Z . The principal effect of a modest reduction in the heavy-element abundance on the solar structure will come about through the associated reduction in opacity: to match the observed solar luminosity one then needs to increase the initial hydrogen abundance. To investigate this effect, we have constructed a model with opacities appropriate to a value $Z = 0.018$ (NS-Z), the characteristics of which are listed in Table 1. Such a model has a lower central temperature and lower neutrino fluxes than does the standard model STD, albeit not low enough to provide a solution to the neutrino problem. However, this model is in much *worse* agreement with the Sun than either STD or NS-DIFF. A likely resolution of this puzzle comes from the gravitational settling of elements heavier than helium, which has not been included in any of our models. This would lead to the heavy-element abundance in the interior being somewhat higher than the observationally de-

termined surface value. The magnitude of the heavy-element diffusion in a solar model has been estimated by Proffitt (1994). His results indicate that the surface Z value is lower than the mean interior value by about 8%. Roughly similar results were obtained by Bahcall & Pinsonneault (1995). Thus a surface value of $Z = 0.018$ would correspond roughly to an internal value of $Z = 0.02$. This then is consistent with our finding that a model with $Z = 0.02$ everywhere gives better agreement with the seismic measurements than a model with a global value of $Z = 0.018$. Taken with the latest surface Z/X determination, this therefore appears to be confirmation of the magnitude of heavy-element diffusion predicted by Proffitt (1994).

5. Conclusion

With the recent improvements in the observations of solar oscillation frequencies and the computation of solar models there is strikingly good agreement between the “standard” solar models including helium settling and solar structure as inferred from helioseismic inversion. Nonetheless, there remain discrepancies which substantially exceed the observational errors. Amongst the modifications tested here we found that an increase in the helium diffusion coefficient was most successful in reducing the differences in sound speed and density between the model and the Sun. This may be an indication of additional weak mixing of the solar interior. However, we cannot exclude that the remaining discrepancies might be eliminated instead through, for example, a suitable change in opacity.

We attribute much of the apparent success of this investigation to the use of a homogeneous set of frequencies, thus avoiding possible systematic errors resulting from combining data obtained with different instruments and at different epochs. On the other hand, data from whole-disk instruments (eg. Elsworth et al. 1994, Fossat 1995) with their intrinsic very high stability, remain an important source of frequencies of low-degree modes. Thus it is of crucial importance to find ways of combining data from such instruments with results from spatially-resolved observations which minimize systematic shifts caused by differences in observing periods or in the characteristics of the instruments and data analysis. Such data combination will also be indispensable for the full utilization of the ground-based helioseismic networks or the instruments on the SOHO satellite.

Fig. 3.— Differences NS-DIFF minus STD, at fixed distance r from the center. The solid and short-dashed lines show relative differences in squared sound speed and density, respectively, whereas the long-dashed line shows the difference in hydrogen abundance X .

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