Seismic Detection of Nuclearites

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Abstract

We make use of Monte Carlo methods to determine the mass range over which "nuclearites," nuggets of stable strange quark matter, should they in fact exist and have densities in the 10^{14} g/cm^3 range as expected, could be detected seismically. We assume an isotropic, maxwellian galactic distribution and take into account the sun's velocity with respect to the galactic center of mass. We consider a model with 287 actual seismic stations, 48 of which have sufficient sensitivity to detect 1 kT of TNT with 1% coupling at 5000 km. We assume a single average global sound propagation speed of 10 km/s. We assume 5% coupling to seismic waves for nuclearites.

A nuclearite event should have a distinctive signal because of the large ratio (4 to 80) of nuclearite speed to the speed of sound in the Earth. Detection of a nuclearite would require at least six station sites to fix its impact time and location and its (vector) velocity. We require seismic detection of signals by at least seven stations in order to separate nuclearite events from random spurious coincidences. The result is that about a twelfth of nuclearites with mass below one metric ton and about a third of those below ten metric tons could be detected.

I. Introduction

Witten in 1984 [1] pointed out that strange quark matter is more likely to be stable than non-strange quark matter since conversion to strange quarks (so long as $m_S < m_N / 3$) lowers the Fermi energy. This fact was known to others [2]. Witten, however, went on to suggest that nuggets of strange quark matter could be produced in phase transitions in the early universe or in supernova explosions and gave a possible scenario for the former, modified versions of which are still under debate [3]. He raised the possibility that such nuggets could solve the cosmological dark matter problem [4] by evading the bound on the cosmological baryon density from the abundance of primordial deuterium [5]. Fahri and Jaffe [6] considered in some detail the properties of such nuggets as a function of nucleon number (A). de Rujula and Glashow [7] considered terrestrial effects, on land, in the sea, and in the air, from incident strange quark nuggets (nuclearites) with masses from nuclear to about 10^9 g where the maximum possible flux compatible with the likely local halo dark matter density drops below one per year. One of the effects they considered was that of the seismic signals from nuclearite passage through the Earth. The purpose of this paper is to examine in more detail the possibility of detecting those signals. Although the results are given for the case of the flux distribution expected from the galactic dark matter halo, they can be used to estimate sensitivity to other possible sources, for example the case of a relatively nearby collision of two strange stars.

II. The Model

A. Nuclearite Distribution

We assume that quark nuggets have a standard maxwellian distribution in the galactic rest system given by:

$$f_g \sim v^2 dv e^{-3/2(v/v_0)^2} \theta(v_{\max} - v) \quad (1)$$

with $v_0 = 270$ km/s and $v_{\max} = 500$ km/s. We then follow procedures developed over the past decade by a number of authors [8] in connection with the dark matter problem, to compute the distribution with

respect to velocity magnitude and direction, geographic location, and time for nuclearites incident on the Earth. We take the velocity of the local standard of rest (LSR) as 250 km/s with respect to the galactic center of mass. We take the nuclearite density to be $\rho_N = 10^{14} \text{ g cm}^{-3} \rho_{N14}$. The numerical calculations take $\rho_{N14} = 3.6$. They take $(\pi/2, 0)$ as the LSR velocity direction in galactic latitude and longitude and $(\alpha_s, \delta_s) = (5.54, 0.840)$ in right ascension and declination in equatorial coordinates. We add a further 14 km/s peculiar motion of the sun with respect to the LSR with $(\alpha_p, \delta_p) = (0.925, 0.436)$ in galactic coordinates. We transform the nuclearite velocity to equatorial coordinates using a routine (see appendix) based on the transformation formulas given in Lang [9] (checked against Collar [10]). The final velocity of the nuclearites with respect to the Earth is then :

$$\begin{aligned} v_x &= v_N \cos \delta_N \cos \alpha_N - v_s \cos \delta_s \cos \alpha_s - v_E \sin \omega_2 t \\ v_y &= v_N \cos \delta_N \sin \alpha_N - v_s \cos \delta_s \sin \alpha_s + v_E \cos \omega_2 t \cos \epsilon \\ v_z &= v_N \sin \delta_N - v_s \sin \delta_s - v_E \cos \omega_2 t \sin \epsilon \end{aligned} \quad (2)$$

In Eq(2), ω_2 is the reciprocal of the number of seconds in a year. ϵ is the obliquity of the ecliptic (0.409 radians) the angle between the plane of the Earth's orbit and that perpendicular to the axis of rotation.

Following the work of Griest [11] finding the effect to be negligible, we do not attempt to take into account the distortion of the velocity distribution by the gravitational fields of the sun and Earth in any detail. We do, however, discard in the calculation all Monte Carlo events with $|v|$ (with respect to the Earth) less than 43 km/s. We note that a signal detected in a seismic analysis or in a dark matter particle search that could be seen to be generated by a particle with lesser velocity would perforce need to be of solar system origin.

B. The Simplified Earth

We consider a spherical Earth of radius 6378 kilometers, with uniform density ρ (in grams). In a more ambitious calculation layered densities as used by, for example Collar in ref 12, would be possible. The numerical calculations take $\rho = 5.5 \text{ g/cm}^3$. We take the speed of sound in the Earth to be

a uniform 10 km/s. In a much more ambitious calculation, dispersion, reflections, and refraction might be modeled. We do not include any signal attenuation (other than geometric spreading) because our calculations are done for responses in the 1 Hz range where the Q -type absorption in Earth's interior is low for distances involved.

We considered a set of 278 seismic stations drawn from a list of those in operation in the mid-1980's. Many more stations operated during this time period; we selected stations based on sensitivity and geographic distribution. Improved stations are currently in operation so sensitivity to nuclearites will increase somewhat but retrospective analysis will be limited by past seismic detection capabilities. We assume the best selected stations (Class 1) to be sensitive to seismic signals carrying (isotropically) the energy from an explosion of 1 kiloton of TNT, in which 1% of the energy release is into 1 Hz seismic waves, at a distance of 5000 km. This corresponds to capability to detect ground motion of 0.5 - 1.0 nanometers. We define two further classes of less sensitive stations, with a minimum detectable signal strength for stations of the three classes as follows:

$$S_1 = \frac{4.2 \times 10^{17} \text{ ergs}}{4\pi (5 \times 10^8 \text{ cm})^2} = 0.133 \text{ erg cm}^{-2} \text{ s}^{-1} \quad (3)$$

$$S_2 = 100S_1$$

$$S_3 = 100S_2$$

C. Nuclearite Signal

We follow slavishly the model of Reference 7 with regard to nuclearite interaction with the Earth. We take nuclearite energy loss to be given by

$$\frac{dE_N}{dx} = -A\rho v^2 \quad (4)$$

$$v(x) \cong v(0) e^{-A\rho x/M} \quad (5)$$

so that

$$\frac{dE_N}{dt} = -A\rho(v(0))^3 e^{-3A\rho x/M} \quad (6)$$

We will find in the calculations of section III that the values of M that give detectable signals are for the most part sufficiently large that the stopping length ($M/(3A\rho)$) is comfortably over the distance traveled through the Earth (essentially always by more than a factor of ten).

We compute the signal received at a seismic station as a result of the energy loss of Eq(6) in the same fashion as Cerenkov radiation is computed by Jackson [13]. The rate at which energy reaches the seismic station is given by

$$\frac{\partial^3 E_s(r,t)}{\partial^2 A \partial t} = \frac{1}{4\pi} \int_0^{t_1} dt' \frac{dE_N}{dt'} \int d^3 r' \frac{\delta\left[\frac{|\vec{r} - \vec{r}'|}{c} - (t - t')\right] \delta(\vec{r}' - \vec{v}_N t')}{|\vec{r} - \vec{r}'|^2} \quad (7)$$

Where $(\vec{r}, t)$ are the position and time at the receiver, $(\vec{r}', t')$ those at the nuclearite, $\vec{v}_N$ its velocity, and 0 and t_1 , its Earth entry and exit times. (Here we have taken the origin at the nuclearite entry point .) Following Jackson [13], we let $\vec{X} = \vec{r} - \vec{v} t$, and solve, given t , for the time t' when the δ -function of Eq(7) shows that the signal was emitted:

$$(t - t')_{\pm} = \frac{1}{v^2 - c^2} \{ \vec{v} \cdot \vec{X} \pm [(\vec{v} \cdot \vec{X})^2 - (v^2 - c^2) X^2]^{1/2} \} \quad (8)$$

Note that, because of the labeling of the two solutions in Eq(8), we have $t_+ < t_-$ and that $\vec{v} \cdot \vec{X}$ is always negative. Eq(7) gives the usual Cerenkov radiation cone ($\cos \alpha = (1 - \frac{c^2}{v^2})^{1/2}$). At time $t' = t_{\pm}$, the quantity $|\vec{r} - \vec{v} t'_{\pm}|^2$ is given by

$$|\vec{r} - \vec{v} t'_{\pm}|^2 = \frac{c^2}{(v^2 - c^2)^2} \{ \vec{v} \cdot \vec{X} \pm [(\vec{v} \cdot \vec{X})^2 - (v^2 - c^2) X^2]^{1/2} \}^2 \quad (9)$$

and the derivative of the argument of the first delta function in Eq (7) is :

$$\begin{aligned} \kappa &= 1 + \frac{d}{dt} |\vec{r} - \vec{v} t| = \left\{ 1 + \frac{1}{c} \frac{d}{dt} |\vec{X} + \vec{v} (t - t')| \right\} t' = t_{\pm} \\ &= 1 - \frac{1}{c} \frac{\vec{v} \cdot \vec{X} + v^2 (t - t')_{\pm}}{|\vec{r} - \vec{v} t'_{\pm}|} \end{aligned} \quad (10)$$

We need the arrival time at the station of the first signal. The argument of the square bracket in Eq(8) first becomes positive at t_F found by solving the equation $(\vec{v} \cdot \vec{X})^2 - (v^2 - c^2)X^2 = 0$

$$t_F = \frac{\vec{v} \cdot \vec{r}}{v^2} \frac{1}{vc} \left(1 - \frac{c^2}{v^2}\right)^{1/2} [r^2 v^2 - (\vec{v} \cdot \vec{r})^2]^{1/2} \quad (11)$$

Inserting in Eq(8) gives the corresponding time when the signal was emitted.

$$t'_F = t_F + \frac{\vec{v} \cdot \vec{X}}{(v^2 - c^2)} \quad (12)$$

t'_F is greater than zero if $\hat{r} \cdot \hat{v} > \frac{c}{v}$

For $t'_F < 0$, no signal could have been emitted at t'_F ; the first signal must have been emitted at $t' = 0$ and is received at $t = \frac{r}{c}$. There are thus three cases for t_F , the time of arrival of the first signal at the station at $\vec{r}$:

- (i) $t'_F < 0$; $t_F = \frac{r}{c}$
 - (ii) $0 < t'_F < t_1$; t_F given by Eq (11)
 - (iii) $t'_F > t_1$; $t_F = \frac{|\vec{r} - \vec{v}t_1|}{c}$
- (13)

Putting these results together gives the signal strength at the station:

$$\begin{aligned} \frac{\partial^3 E}{\partial A \partial t} &= \theta[(\vec{v} \cdot \vec{X})^2 - \frac{(v^2 - c^2)}{X^2}] \frac{1}{4\pi} \\ &\left\{ \frac{1}{\kappa_+} \frac{\theta(t_+) \theta(t_1 - t_+)}{|\vec{r} - \vec{v}t_+|^2} + \frac{1}{\kappa_-} \frac{\theta(t'_-) \theta(t_1 - t'_-)}{|\vec{r} - \vec{v}t'_-|^2} \right\} \frac{dE_n}{dt'} \\ &\equiv \tilde{S}(t) \frac{dE_n}{dt'} \end{aligned} \quad (14)$$

D. Nuclearite Detection

Nuclearite passage through Earth gives a much different set of signals than an Earthquake or a nuclear explosion, as emphasized by De Rujula and Glashow [7]. The latter are point events with time of signal receipt at station proportional to station distance from the site of the event. Nuclearite passage is a line event with time of signal receipt (roughly) proportional to distance from the line. Determining the location of a point event requires four variables, which can be taken as latitude, longitude, depth, and time of explosion. Detecting a nuclearite requires determining six variables which can be taken as entry point time, latitude and longitude, plus velocity magnitude and direction. Given signals from six stations it should be possible to solve for a "best fit" nuclearite line. However there would, in any search, be a significant background from "random" station reports. Seismic stations report a number of signals that are never "associated" into Earthquakes less than, but of the order of magnitude of, the number so associated. Thus a minimum requirement for detection of a nuclearite would appear to be detection of its signal by 7 stations. If approximately the same straight line was determined by the seven, six-station, best-fit straight lines, the event would appear to be a convincing nuclearite candidate. We therefore take signal detection by seven stations as our minimum requirement for detectability.

III. Calculations

We generated Monte Carlo nuclearites as described in Section (II, A); we computed their signals as described in Section (II, C). We approximated the one Hertz signal as follows: We generated a hundred random nuclearites. For each, we integrated the signal at each of the 278 stations for one second beginning at t_F , the time the signal first reaches the station. We then computed the ratio

$$R_s = \frac{S(t = t_F + 0.25s)}{\int_{t_F}^{t_F + 1} S(t)dt} \quad (15)$$

in each case. We found R_s averaged over the 100 nuclearites and 278 stations to be about 1.0 (with a maximum of 1.03, a minimum of 0.99 and a standard deviation of about 0.01). The integration was

done by 50 point Simpson's rule, with the 25 point approximations deviating by less than 1% from the 50 point results. The signal has a square root divergence as $t \rightarrow t_F$; integral results from the transformations $t - t_F = y^2$ and $t - t_F = y^3$ agree to within three standard deviations and have been combined.

To find the minimum detectable mass for a given nuclearite we use Eq(14)

$$\frac{\partial^3 E}{\partial A \partial t} = \tilde{S}(t) \left| \frac{dE_n}{dt} \right| \quad (16)$$

with $\left| \frac{dE_n}{dt} \right| = A \rho_E (v(0))^3 f_c$ where f_c is the assumed coupling to seismic waves.

$$A = \pi \left(\frac{3M}{4\pi \rho_N} \right)^{2/3} = 5.6 \times 10^{-6} \left(\frac{M_t}{\rho_{N14}} \right)^{2/3} \text{ cm}^2$$

$$\rho_E = \rho_g [1 \text{ g cm}^{-3}], \quad v(0) = 200 \frac{\text{km}}{\text{s}} v_2$$

This gives

$$\frac{dE_v}{dt} = 4.5 \times 10^{16} \frac{\text{erg}}{\text{s}} \left(\frac{M_t}{\rho_{N14}} \right)^{2/3} \rho_g v_2^3 f_c \quad (17)$$

where v_2 is in units of 200 km/s, M_t is in units of metric tons, ρ_{N14} is in units of $10^{14} \text{ g cm}^{-3}$, and ρ_g is in units of g cm^{-3} . Let $\tilde{S}_k = 4.5 \times 10^{16} \tilde{S}(t_2)$. $\tilde{S}_k$ is the signal at the station k for the given geometry, a one ton nuclearite, and unit value for the other factors in Eq(17); it is in units of $\text{erg cm}^{-2} \text{s}^{-1}$, and is of order 1 for a distance to the line of closest approach of order 1000km. With a sensitivity, for each class i station, of $S_i \text{ erg cm}^{-2} \text{s}$ (See Eq(3)), we have for the minimum nuclearite mass detectable by station k of class $i(k)$.

$$S_{i(k)} = \left(\frac{M_t}{\rho_{N14}} \right)^{2/3} f_c \rho_g v_2^3 \tilde{S}_k \quad (18)$$

α

$$M_t = \left[\frac{S_i}{f_c \rho_g v_2^3 \tilde{S}_k} \right]^{3/2} \rho_{N14} \quad (19)$$

With the numerical assumptions above, the minimum mass nuclearite with a given geometry, detectable at a given station of class 1, would be

$$M_t = 13.4 \left(\frac{200}{v_N} \right)^{9/2} \left(\frac{1}{S} \right)^{3/2} \left(\frac{0.01}{f_c} \right)^{3/2} \quad (20)$$

In the results of Section IV below we took $f_c = 0.05$. Nuclear explosions tend to have $f_c \sim 0.01$, while chemical explosions tend to have double that value. We expect that the lower temperature of the nuclearite passage is likely to lead to increased coupling, but we would anticipate an uncertainty of at least a factor of two about the five percent selected. We can not rule out a significantly higher coupling, but we have no basis for increasing this initial estimate.

IV. Results

A. Frequency of Detectable Nuclearites

We present here the results from generating 120,000 Monte Carlo nuclearite geometries (over the "year" 1991).

In Table 1 we give: (a) the "raw data" for the distribution of values for the minimum detectable masses (MDM), $N(M_t)$, found from Eq(20) where $\tilde{S}_k$ is the seventh strongest signal among all 278 stations for the Monte Carlo generated geometry; and (b) the cumulative fraction of events that would be detected for $M < M_t$ ($R(\log_{10} M)$). Probably the most important result of this paper is the fact that, apparently, the chances are only about one in three of detecting ten ton nuclearites [$R(\log_{10} 10 = 1) = 0.334$] and less than one in ten of detecting nuclearites under 2.5 tons ($R(\log_{10} 2.5 = 0.4)$). This sets the scale for the difficulty in detecting nuclearites seismically and the extent to which one can expect significant limits on a nuclearite mass distribution from the absence of such detection.

The final column of Table 1 shows that seismic nuclearite detection would be most sensitive to a SQM dark matter distribution of 4 ton nuggets (since, for given dark matter density ρ_M , if that density is achieved with nuggets of mass M , the number of such nuggets is proportional to M^{-1}).

The limit on the number of incident nuclearites per unit time is given by

$$\frac{dN(M)}{dt} = \frac{\pi R_E^2 \rho_{DM} \bar{v}}{M} \quad (21)$$

where it is assumed that the (halo) dark matter density of about $5 \times 10^{-25} \text{ g cm}^{-3}$ is saturated by nuggets of mass M . Our Monte Carlo gives a velocity averaged over all generated geometries of 330 km/s.

Equation (21) then gives a rate of

$$\frac{dN}{dt} = \frac{\pi (6378)^2 \times 10^{10} \times 5 \times 10^{-25} \times 3.3 \times 10^7 \times \pi \times 10^7}{4 \times 10^6} \cong 160 / \text{year} \quad (22)$$

With a probability of detection of 0.168 in Table 1, one could expect to detect as many as 25 four-ton nuclearite events per year if four ton strange quark nuggets were to saturate the halo dark matter density. If ten percent of the dark matter density were distributed in nuggets over the mass range from 0.25 to 100 tons we would expect about an event per year. Note that, by Eq. (20), these results scale as $(f_c/0.05)^{3/2}$.

B. Event Geometry

We now describe in more detail the characteristics of the third of the 120,000 Monte Carlo events for which the minimum detectable mass was under ten tons. Figure 1 gives the distribution of minimum detectable masses for the 41,804 events. In Figure 2 we show the monthly distribution of events; the peak in July of Reference[8] is apparent. Figure 3 gives the speed distribution. The speed averaged over those low mass events is 25% higher than that averaged over the entire 120,000 events since, for a given geometry, the minimum detectable mass varies as the inverse cube of the velocity.

Figure (4) gives the distribution in right ascension and declination of the velocity vectors. The peak in declination is in part due to the direction of motion of the sun with respect to the rest system of the galaxy. The variation in right ascension is not related to the motion since there is effectively an average over time which is one of the random variables. Rather, that variation is due to the distribution of detection stations. Seismic stations are located on the 40% of the surface of the Earth that is land, and there are relatively few stations in the southern hemisphere. Additionally, nuclearite detection depends mostly on the class one stations. For the events generated, class one stations comprised 98.5% of the 1.2×10^6 positions on the 120,000 "top 10" (in signal strength to sensitivity ratio) lists. The locations of each of the 48 class one stations are given in Table 2 along with number of appearances on

top 10 station lists for event geometries with minimum detectable masses (MDM) under ten tons. Note that Southern hemisphere stations have less appearances than Northern; if the restriction of minimum detectable mass to under ten tons is removed Southern stations appear more frequently by over a factor of two since there are fewer to share detection of nuclearites passing through the south. The peaks in Figure 4a at zero and π apparently represent nuclearites entering or exiting the Earth with velocity vector heading toward or away from the relatively dense concentration of class one stations in Europe.

The geographic distribution of entry points is given in Table 3 for geometries for which the minimum detectable mass is under ten tons. As noted above, entry points in the Southern hemisphere are suppressed because (a) the sun is moving north and (b) there are fewer class one stations in the Southern hemisphere. Note in Table 3 the high number of entry points per square degree (5) for Europe. Figures 5 - 7 give the counterparts to Figures 1, 3, 4 for the 3000 nuclearite geometries for which the minimum detectable mass is under ten tons with entry points in Europe as a specific geographic example. Note the lower particle speeds in Figure 6 in comparison to those of Figure 3 as a result of the above average detection capability. Similarly, note the more rapid fall with increasing mass for the same reason. Note that, in Figure 7, velocity right ascension can be zero (or 2π); such cases correspond to nuclearites heading south. In Figure 7b, however, there is a sharp cut off in velocity vector declination since straight lines can't both penetrate the Earth in Europe and head north.

C. Point Sources

We note briefly that there is a possibility that supernova explosions generate quantities of strange quark nuggets. A supernova at 1000 light years 10,000 years ago would be such a source if it produced nuggets with velocities on the order of $0.1c$. Such nuggets could be detected down to the milligram level in view of the $v^{9/2}$ scaling of Eq (20). The Earth would be expected to intercept about one in 10^{24} of such nuggets. Depending on the mass and velocity distribution of nuggets produced, detecting such a supernova signal is possible. Similar remarks apply to collisions of strange quark stars.

D. Identifiability

The question of whether a nuclearite passing through the Earth could be correctly identified as such is not a simple one and detailed discussion of it is beyond the scope of the paper; however a few preliminary remarks can be made. Summary records of significant seismic detection are sent electronically to The U. S. Geological Society (USGS) from most of the world's seismic stations including all the class one stations cited above. The USGS fits, wherever possible, sets of reports to seismic waves originating in point events (Earthquakes or nuclear explosions). It retains the record of

the "unassociated events" that cannot be associated with other events to fit the pattern expected from a point event. There is, however, an important exception: stations in the U. S. are linked electronically and only report seismic events that can be associated with point sources. As pointed out by de Rujula and Glashow [7] the key to nuclearite detection is the difference between the pattern of arrival times expected from a nuclearite and that expected from a point event. That difference is illustrated in Figure 8. It is thus expected that nuclearite seismic signals should be found among unassociated events.

For normal incidence of the nuclearite the difference between the point and line events becomes dependent on (a) detections significantly far enough from the entry point for Earth-surface curvature effects to become important in determining travel times and (b) sufficiently reliable models for seismic wave propagation to exploit such difference. For non-normal incidence, there is a question as to whether initial signal arrival times might frequently belong to Cases (i) and (iii) of Eq (13). Since these correspond to the initial signals originating at the entry and exit points respectively, to the extent that, for a particular geometry, several stations signals are in classes (i) and (iii), they run risk of being associated into (false) earthquakes. A priori, the frequency of cases (i) and (iii) are one sixth each as can be seen as follows: for a nuclearite with velocity parallel to the positive z-axis entering the Earth at $z = -(R^2 - r^2)^{1/2}$, stations with $\cos \theta < -z/R$ are in class (i); those with $\cos \theta > +z/R$ are in class (iii). Averaging over impact parameter $\vec{r}$ gives 1/6 each for the relative frequency of class (i) and (iii). This is found by the Monte Carlo program when averaged over all stations for all geometries. However if one restricts to the top seven stations and to the case of MDM under ten tons the combined frequency for cases (i) and (iii) falls to below ten percent.

While we have not yet begun a full search in it for nuclearite events, the USGS has permitted us to copy their record of unassociated events for the period 1981-1993 [14]. There are about three million events in this record with an event defined as a report from a single station. For the same period there are about six million events associated into Earthquakes. In examining the unassociated events for possible nuclearite candidacy, a first test is time bunching. The average time between 3×10^6 events spread over 4×10^8 seconds is about two minutes so that random bunches of seven or more within ten minutes should be quite rare. We have, however, found that some unassociated events are, in fact, bunched into short time periods [15]. Many of these reported signals should have been associated with known Earthquakes, but for some reason remained in the unidentified category. Geological significance of remaining groups of unidentified record is not clear; there are no current predictions of such events of which we are aware. They could imply a significant, as yet not understood, background

to the nuclearite search. This uncertainty, the absence of unassociated events from U. S. class one stations and the need for development of appropriate algorithms for testing the nuclearite hypothesis on sets of events all make it difficult to predict the final detection sensitivity attainable from extant data. Recognition of at most a few hundred reports, which might be attributed to nuclearite passages, among the three million unassociated reports is a demanding task.

V. Conclusions

We conclude that seismic detection of strange quark nuggets with masses in the range between a few tenths of a ton and a few tens of tons is feasible with the current network of seismic stations and the cooperation of USGS. The chances for such detection would be improved by availability of unassociated events from all U. S. and other class one stations. There may be significant backgrounds to identifying the nuclearite signal so that successful detection (or bounding) will likely require generating relatively sophisticated computer programs.

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Appendix

For completeness we record here a convenient variation on the formulas of Lang[9] for the transformations used here. If λ is the transformation from S to S', specified by Euler angles φ, θ, ψ , it is of the form

$$\lambda = \lambda(\varphi) \lambda(\theta) \lambda(\psi) \quad (\text{A1})$$

following the form and conventions of Goldstein[16]. λ^{-1} can be written as

$$\lambda^{-1} = \lambda(-\varphi) \lambda(-\theta) \lambda(-\psi) = \lambda(\pi - \varphi) \lambda(\theta) \lambda(\pi - \psi) \quad (\text{A2})$$

The declination $\delta_{\tilde{z}}$ and right ascension $\alpha_{\tilde{z}}$ of the $\tilde{S}$ z-axis, $\tilde{z}$, in the System S are given by

$$\delta_{\tilde{z}} = \pi/2 - \theta, \quad \alpha_{\tilde{z}} = \varphi + \frac{3\pi}{2} \equiv \varphi - \frac{\pi}{2} \pmod{2\pi} \quad (\text{A3})$$

The declination δ_z and right ascension $\tilde{\alpha}_z$ of the S z-axis, z, in the system $\tilde{S}$ are given by

$$\delta_z = \theta, \quad \tilde{\alpha}_z = \frac{\pi}{2} - \psi \quad (\text{A4})$$

From (A3) and (A4) we see that if we write

$$\mu(\tilde{\alpha}_z, \delta_z, \alpha_{\tilde{z}}) = \lambda(\psi, \theta, \varphi) \quad (\text{A5})$$

Then we have

$$\mu(\alpha_{\tilde{z}}, \delta_{\tilde{z}}, \tilde{\alpha}_z) = [\lambda(\psi, \theta, \varphi)]^{-1} \quad (\text{A6})$$

Using $(\tilde{\alpha}_z, \delta_z, \alpha_{\tilde{z}})$ we can relate (δ, α) in $\tilde{S}$ to (δ, α) in S by equating projections along: (a) the line of nodes; (b) $\hat{\tilde{z}}$; and (c) $\hat{z}$, obtaining

$$\cos \tilde{\delta} \sin (\tilde{\alpha} - \tilde{\alpha}_z) = - \cos \delta \sin (\alpha - \alpha_z) \quad (\text{A7.a})$$

$$\sin \tilde{\delta} = \sin \delta_z \sin \delta + \cos \delta_z \cos \delta \cos (\alpha - \alpha_z) \quad (\text{A7.b})$$

$$\sin \delta = \sin \delta_z \sin \tilde{\delta} + \cos \delta_z \cos \tilde{\delta} \cos (\tilde{\alpha} - \tilde{\alpha}_z) \quad (\text{A7.c})$$

Table Captions

1. Distribution of 120, 000 Monte Carlo nuclearite geometries according to the common log of minimum detectable mass (MDM), M_t , in metric tons, necessary for detection by station with seventh strongest signal (weighted by class capability); Cumulative fraction $R(\log M_t)$ of events detected for $M < M_t$. (The values of M_t correspond to the upper end of each interval); and relative detection probability, $R(\log M_t)/M_t$.
2. Class one stations by conventional symbol, latitude and longitude, ordered according to number of appearances among the "top 10" detecting stations list of each Monte Carlo nuclearite.
3. Geographic distribution of entry points.

Table 1

Monte Carlo Results

Log ₁₀ M ₁	Number of events	Cumulative fraction of total events	Relative Detection Probability
-1.2	9		0.003
-1.0	26		0.008
-0.8	69	0.001	0.014
-0.6	200	0.003	0.026
-0.4	513	0.007	0.044
-0.2	1043	0.016	0.063
0	2031	0.032	0.084
0.2	3449	0.061	0.100
0.4	5345	0.106	0.109
0.6	7462	0.168	0.109
0.8	9281	0.245	0.100
1.0	10,678	0.334	0.086
1.2	11,612	0.431	0.070
1.4	11,939	0.530	0.055
1.6	11,613	0.627	0.041
1.8	10,180	0.712	0.029
2.0	8,747	0.789	0.020
2.2	7,027	0.844	0.014
2.4	5,459	0.889	0.009
2.6	3,988	0.922	0.006
2.8	2,879	0.946	0.004
3.0	2,101	0.964	0.002
3.2	1,486	0.976	0.002
3.4	1,006	0.985	0.001
3.6	659	0.990	0.001
3.8	475	0.994	
4.0	267	0.996	
over 4.0	446		

Table 2
Class One Stations

Station Symbol	Latitude	Longitude	Number of Appearances on the 40,000 "top-10" Lists
bae	-0.2763E+00	-0.8346E+00	6109.0
bwa	-0.6007E+00	0.2590E+01	6243.0
lpb	-0.2883E+00	-0.1188E+01	6362.0
lz-bv	-0.2836E+00	-0.1195E+01	6438.0
nwao	-0.5746E+00	0.2046E+01	6754.0
wb0	-0.3451E+00	0.2346E+01	6797.0
wr0	-0.3482E+00	0.2347E+01	6821.0
asar	-0.4128E+00	0.2337E+01	6859.0
bul	-0.3513E+00	0.4992E+00	6965.0
boco	0.7999E-01	-0.1292E+01	7289.0
bng	0.7737E-01	0.3234E+00	7681.0
ksrs	0.6632E+00	0.2234E+01	7753.0
bcao	0.7622E-01	0.3241E+00	7788.0
fbc	0.1112E+01	-0.1195E+01	7929.0
res	0.1303E+01	-0.1656E+01	8355.0
tlo	0.6955E+00	-0.6981E-01	8373.0
ara0	0.1214E+01	0.4451E+00	8385.0
ali	0.6693E+00	-0.8435E-02	8416.0
gma	0.1142E+01	-0.2813E+01	8583.0
chto	0.3278E+00	0.1727E+01	8604.0
npo	0.1130E+01	-0.2564E+01	8691.0
cmo	0.1132E+01	-0.2580E+01	8728.0
cmc	0.1184E+01	-0.2009E+01	8789.0
pmr	0.1075E+01	-0.2602E+01	8815.0
eka	0.9655E+00	-0.5498E-01	8871.0
gba	0.2374E+00	0.1351E+01	8940.0
eil	0.5157E+00	0.6100E+00	9055.0
n01a0	0.1062E+01	0.1888E+00	9063.0
nra0	0.1060E+01	0.2012E+00	9190.0
fia0	0.1072E+01	0.4550E+00	9279.0
ash	0.6624E+00	0.1018E+01	9295.0
dsh	0.6728E+00	0.1200E+01	9301.0
tlg	0.7552E+00	0.1351E+01	9343.0
ltx	0.5119E+00	-0.1810E+01	9473.0
gar	0.6807E+00	0.1227E+01	9538.0
hel	0.1050E+01	0.4355E+00	9580.0
sftx	0.5222E+00	-0.1822E+01	9587.0
rsnt	0.1090E+01	-0.2000E+01	9587.0
anto	0.6964E+00	0.5721E+00	9700.0
tuc	0.5637E+00	-0.1934E+01	9726.0
gg-	0.8671E+00	0.1955E+00	9742.0
gea0	0.8522E+00	0.2391E+00	9913.0
bea	0.6437E+00	-0.2039E+01	10020.0
abq	0.6096E+00	-0.1857E+01	10090.0
ffc	0.9550E+00	-0.1780E+01	10129.0
lar	0.7208E+00	-0.1843E+01	10233.0
dug	0.7013E+00	-0.1969E+01	10705.0
ban	0.8931E+00	-0.2018E+01	10742.0

Table 3

SUMMARY OF MONTE CARLO BY REGIONS

<u>Region</u>	<u>Definition</u>	<u>Number of Events *</u>
World		41904
Northern Hemisphere		40385
Southern Hemisphere		1519
Eastern Hemisphere		20313
Western Hemisphere		21519
Europe	35° N to 75° N 10° W to 25° E	2923
Asia	5° N to 90° N 60° E to 150° E	8925
North America	15° N to 90° N 45° W to 165° W	13,802
Pacific Basin	50° N to 80° S 130° W to 180° W	2579
Australia - New Zealand	10° S to 70° S 100° E to 180 E	210
North Polar Region	70° N to 90° N	7
South Polar Region	70° S to 90° S	2

*Threshold mass 10 metric tons or less

Figure Captions

- Figure 1. Distribution in minimum detectable mass (MDM) of the 41, 904 event geometries for which that mass was less than 10 tons (low MDM events).
- Figure 2. Monthly distribution of the 41, 904 event geometries for low MDM events.
- Figure 3. Nugget speed distribution for the event geometries for low MDM events.
- Figure 4. (a) Right ascension of velocity vector and (b) Declination of velocity vector for the low MDM events.
- Figure 5. Distribution in MDM of the 2923 low MDM events with entry points in Europe.
- Figure 6. Speed distribution for low MDM events with entry points in Europe.
- Figure 7. (a) Right ascension and (b) declination of velocity vector for low MDM events with European entry points.
- Figure 8. Difference between point and line events. An Earthquake at time zero at the entry point would give arrival times $t_1 < t_2 < t_3$ and no signal from 4 and 5. A nuclearite would give $t_4 < t_2 < t_1 < t_3 < t_5$.

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Fig 1

Fig 2

Fig 3

Fig 4a

Fig 5

Fig 4b

Fig 5

Fig 6

Fig 7a

Fig 7b

DIFFERENCE BETWEEN POINT AND LINE EVENTS

Fig 8