Strangeness in the Scalar Form Factor of the Nucleon

Hyun-Chul Kim *, Andree Blotz †, C. Schneider ‡, and Klaus Goeke §

Institute for Theoretical Physics II,

P.O. Box 102148, Ruhr-University Bochum,

D-44780 Bochum, Germany

(July, 1995)

Abstract

The scalar form factor of the nucleon and related physical quantities are investigated in the framework of the semibosonized SU(3) Nambu-Jona-Lasinio soliton model. We take into account the rotational $1/N_c$ corrections and linear $m_s$ corrections. The strangeness content of the nucleon in the scalar form factor is discussed in detail. In particular, it is found that the $m_s$ corrections play an essential role of reducing the $\langle N|\bar{s}s|N\rangle$ arising from the leading order and rotational $1/N_c$ contributions. We obtain the $\sigma_{\pi N}(0) = 40.80$ MeV, $\Delta \sigma = \sigma_{\pi N}(2m^2_\pi) - \sigma_{\pi N}(0) = 18.18$ MeV and $\langle r^2\rangle_N^{S} = 1.50$ fm$^2$. The results are in a remarkable agreement with empirical data analyzed by Gasser, Leutwyler, and Sainio [3].

*E-mail address:kim@hadron.tp2.ruhr-uni-bochum.de

†Present address: Department of Physics, State University of New York, Stony Brook, 11794, U.S.A.

‡E-mail address:carstens@elektron.tp2.ruhr-uni-bochum.de

§E-mail address:goeke@hadron.tp2.ruhr-uni-bochum.de
I. INTRODUCTION

Since Cheng [1] showed that there is a factor-of-two discrepancy between the empirical data for the pion-nucleon sigma term ($\Sigma_{\pi N}$) and the naive estimates of the $\sigma$-term from the mass spectrum, there have been a great deal of discussions and disputes about the $\Sigma_{\pi N}$ and $\sigma$ term (see Ref. [2,3] and references therein). Donoghue and Nappi [4] suggested that the discrepancy is due to the presence of strange quarks in the nucleon, i.e. $\langle N|\bar{s}s|N\rangle \neq 0$ and showed that $\langle N|\bar{s}s|N\rangle$ contributes almost 30% to the quark condensate in the nucleon, making use of the Skyrme model and bag model. At the first thought, it seems to be reasonable, since Cheng used the Zweig rule, i.e. neglected $\langle N|\bar{s}s|N\rangle$. However, one serious question arises: a large fraction of the nucleon mass then stems from strange quarks if one follows Ref. [4], which contradicts the quark model. Another assumption was that the ratio $m_s/\bar{m}$ is off by a factor of two, which means that the first order perturbation theory collapses. However, this kind of suggestion would lead to a breakdown of the Gell-Mann-Okubo mass formula which predict the masses of hadrons in a few percent.

Motivated by these contradictions, Gasser, Leutwyler and Sainio [3] recently reanalysed the $\sigma$ term prudently, taking advantage of newly accumulated and better $\pi N$ scattering data and considering the strong $t$-dependence of the scalar form factor $\sigma(t)$ ($\sigma(2m^2) - \sigma(0) \simeq 15$ MeV). The results of Ref. [3] were $\sigma = 45 \pm 8$ MeV and $\Sigma \simeq 60$ MeV. The $y = 2\langle N|\bar{s}s|N\rangle/\langle N|\bar{u}u + \bar{d}d|N\rangle$, a share of $\langle N|\bar{s}s|N\rangle$ in the $\sigma$ term, was about 0.2, so that the corresponding contribution of the term $\langle N|\bar{s}s|N\rangle$ to the nucleon mass was about 130 MeV.

In the meanwhile, the efforts to understand the $\sigma$ term puzzle theoretically have continued [5–7]. However, the bone of contention still lies in the role of strange quarks, more specifically the contribution of the $\langle N|\bar{s}s|N\rangle$ to the $\sigma$ term. Recently, several works insist that there is no need to introduce a portion of strange quarks to explain the $\sigma$ term discrepancy. Bass [6] proposed that based on the Gribov confinement the value of the $\sigma$ term can be explained without need to invoke large strangeness content of the nucleon. Ball, Forte and
Tigg [5] also suggested that with the correct understanding of the baryon matrix element the \( \sigma \) term (identified with \( \sigma_8 = \bar{m} \langle N | \bar{u}u + \bar{d}d - 2\bar{s}s | N \rangle \) ) can be reproduced without violating the Zweig rule. Hence, following these arguments, strange quarks do not contribute to the nucleon mass. Though it should be small, it is still important to consider the contribution of strange quarks to the \( \sigma \) term, in line with recent experiments indicating the fact that strange quarks might play an important role of explaining the properties of the nucleon [8,9].

It is the object of the present work to study the strangeness contribution to the \( \sigma \) term in the framework of the semi-bosonized SU(3) Nambu-Jona-Lasinio soliton model (often called as the chiral quark soliton model). In our model, the nucleon is understood explicitly as \( N_c \) valence quarks coupled to the polarized Dirac sea bound by a non-trivial chiral mean field configuration. The proper quantum numbers of the nucleon can be acquired by the semiclassical quantization [10,11] performed via integrating over the zero-mode fluctuations of the pion field around the saddle point. It allows the nucleon to carry proper quantum numbers such as spins and isospins. The SU(3) NJL soliton model has a merit in that it interpolates between the nonrelativistic naive quark model and the Skyrme model. It enables us to study the interplay between these two different models [12]. The model is quite successful in describing the static properties of the baryons and their form factors [13–15].

The outline of the paper is as follows: In the next section, we sketch the basic formalism for the scalar form factor in SU(3) NJL soliton the model. In section 3, we present the numerical results and discuss about them. In section 4, we summarize the present work and remark the conclusion.

II. FORMALISM

The scalar form factor \( \sigma(t) \) is defined as a condensate of \( u \) and \( d \) quarks in the nucleon:

\[
\sigma(t) = \bar{m} \langle N(p') | \bar{u}u + \bar{d}d | N(p) \rangle
\]

(1)
with $\bar{m} = (m_u + m_d)/2 \simeq 6$ MeV. The $t$ denotes the square of the momentum transfer. Our model is characterized by a low-momenta QCD partition function in Euclidean space given by the functional integral over pseudoscalar meson and quark fields:

$$Z = \int \mathcal{D}\Psi \mathcal{D}\Psi^\dagger \mathcal{D}\pi^A \exp \left( - \int d^4x \Psi^\dagger iD\Psi \right)$$  \hspace{1cm} (2)

where

$$iD = \beta(-i\partial + MU^\gamma + \hat{m}), \quad U^\gamma = e^{i\pi^A \lambda^A}. \hspace{1cm} (3)$$

$\lambda^a$ are SU(3) Gell-Mann matrices normalized as $\text{Tr}\lambda^a \lambda^b = 2\delta^{ab}$. The $\hat{m}$ denotes the current quark mass matrix for which we take the form $\text{diag}(m_u, m_d, m_s)$, where $m_u, m_d$ and $m_s$ are the corresponding current quark masses of the up, down and strange quark, respectively. Here, we assume that isospin symmetry is not broken, i.e. $m_u = m_d = \hat{m}$. The $M$ stands for the momentum-dependent dynamical mass arising from the spontaneous chiral symmetry breaking. The momentum-dependence of the $M$ introduces the ultra-violet cut-off. However, we shall regard it as a constant for simplicity. Instead, we employ a simple proper-time regularization. The differential operator $iD$ is expressed in Euclidean space in terms of the Euclidean time derivative $\partial_\tau$, the Dirac one-particle Hamiltonian $H(U)$ and symmetry breaking part [17]:

$$iD = \partial_\tau + H(U) + h_{sb}$$  \hspace{1cm} (4)

with

$$H(U) = \frac{\bar{\alpha} \cdot \nabla}{i} + \beta M_\alpha U + \beta \bar{m} 1, \quad h_{sb} = \beta \mu_0 1 + \beta \mu_8 \lambda_8. \hspace{1cm} (5)$$

Here, we have made the famous embedding Ansatz for the pseudoscalar fields $U^{\gamma^b}$ and $U$ is expressed by

$$U = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix}. \hspace{1cm} (6)$$
The $U_0$ expresses the SU(2) chiral background field $U_0 = \exp i[\bar{\sigma} \cdot \tau P(r)]$ with the hedgehog Ansatz. $P(r)$ denotes the profile function with proper boundary conditions. $\mu_0$ and $\mu_8$ are defined by $\mu_0 = (M_s - M_u)/3$ and $\mu_8 = -(M_s - M_u)/\sqrt{3}$. $M_s$ and $M_u$ are constituent quark masses of the $s$ and $u$ quarks respectively. The $M_u$ is used as an input parameter, while the $M_s$ is determined by the gap equation [17]. The current strange quark mass $m_s$ is also settled in the same way. We treat the explicit symmetry breaking term $h_{sb}$ perturbatively.

The hadronic matrix elements of the $\pi N$ $\sigma$-term is related to the correlation function

$$\sigma(t) \sim \langle 0 | J_N(\vec{x}, -\frac{T}{2}) \hat{\sigma} J_N^\dagger(\vec{y}, -\frac{T}{2}) | 0 \rangle$$

at large Euclidean time $T$. $\hat{\sigma}$ is the quark operator for the $\sigma$ term, defined by $\hat{\sigma} = \bar{\sigma}(\vec{u} + \vec{d})$. $J_N$ is the nucleon current constructed from $N_c$ quark fields [10]

$$J_N(x) = \frac{1}{N_c!} \epsilon_{i_1 \cdots i_{N_c}} \Gamma_{J_3 TT^3 Y}^{\sigma, \sigma_{N_c}} \psi_{\alpha_1 i_1}(x) \cdots \psi_{\alpha_{N_c} i_{N_c}}(x).$$

$\alpha_1 \cdots \alpha_{N_c}$ denote spin–flavor indices, while $i_1 \cdots i_{N_c}$ designate color indices. The matrices $\Gamma_{J_3 TT^3 Y}$ are taken to endow the corresponding current with the quantum numbers $J J_3 TT^3 Y$. The $J^3_B$ plays the role of creating the baryon state.

The integral over the quark fields are trivial. The integral over the pseudo-Goldstone boson fields can be performed by the saddle point method in the large $N_c$ limit. In order to find the quantum $1/N_c$ corrections, it is important to take into account the small oscillations of the pseudo-Goldstone bosons around the saddle point and the zero modes. The zero modes are taken into account by the soliton expressed by $\bar{U}(\vec{x}, x_4) = A(x_4) U(\vec{x} - \vec{Z}) A^\dagger(\vec{x}_4)$ with an SU(3) unitary matrix $A(t)$. Hence, the collective action $S_{eff}$ becomes

$$S_{eff} = -N_c S_p \log (i D)$$

$$= -N_c S_p \log \left[ \partial_\tau + H(\bar{U}) + A^\dagger(x_4) \hat{A}(x_4) - i \beta \hat{\vec{Z}} \cdot \nabla 
+ A^\dagger(x_4) h_{sb} A(x_4) - \xi(y) \beta A^\dagger(x_4) \frac{1}{\sqrt{3}}(\sqrt{2} \lambda_0 + \lambda_8) A(x_4) \right]$$

with the angular velocity $A^\dagger(x_4) \hat{A}(x_4) = i \Omega_E = i \Omega^a_E \lambda^a / 2$. $S_p$ denotes the functional trace. The $\xi$ stands for the external scalar field, with regard to which we make a functional derivative so as to obtain the sigma form factor:
\[
\sigma(t) = -N_c \frac{\delta}{\delta \xi(z)} \text{Sp} \log \left\{ \partial_x + H(\hat{U}) + A^\dagger(x_4) \hat{A}(x_4) - i\beta \hat{Z} \cdot \nabla \\
+ A^\dagger(x_4) h_{\beta\alpha} A(x_4) - \xi(y) \beta A^\dagger(x_4) \frac{1}{\sqrt{3}} (\sqrt{2} \lambda_0 + \lambda_8) A(x_4) \right\}
\]

(10)

It is known that there is the dependence of the \( \sigma \) term on the regularization scheme [18]. However, we want to stress the fact that we have employed the proper-time regularization and have evaluated possible physical observables such as mass splittings, magnetic moments, axial constants and electromagnetic form factors within the same scheme and same values of input parameters \(^1\). Hence, we stick to the proper-time regularization for the \( \sigma \) term and make use of the same input parameters without adjusting. However, we shall not be here bothered by going through all the tedious technical details arising from the regularization (see Ref. [16] for details).

Having taken into account the rotational \( 1/N_c \) corrections and linear \( m_s \) corrections, we arrive at

\[
\sigma(t) = \Sigma_{SU(2)}(t) \langle 2 + D_{88}^{(8)}(A) \rangle_N \\
+ \frac{2\tilde{m}}{\sqrt{3}I_1} K_1(t) \langle D_{88}^{(8)}(A) R_i \rangle_N + \frac{2\tilde{m}}{\sqrt{3}I_2} K_2(t) \langle D_{88}^{(8)}(A) R_i \rangle_N \\
- \frac{\tilde{m} \mu_b}{\sqrt{3}} \left[ N_1(t) - K_1(t) \frac{K_1}{I_1} \right] \langle D_{88}^{(8)}(A) D_{88}^{(8)}(A) \rangle_N \\
- \frac{\tilde{m} \mu_b}{\sqrt{3}} \left[ N_2(t) - K_2(t) \frac{K_2}{I_2} \right] \langle D_{88}^{(8)}(A) D_{88}^{(8)}(A) \rangle_N \\
- \frac{\tilde{m} \mu_b}{3\sqrt{3}} N_0(t) \langle D_{88}^{(8)}(A) (D_{88}^{(8)}(A) + 1) \rangle_N - \frac{8\tilde{m} \mu_0}{3} N_0(t),
\]

(11)

where

\[
\Sigma_{SU(2)}(t) = N_c \int d^3x \ j_0(Qr) \left[ \Psi_{val}(x) \beta \Psi_{val}(x) - \sum_n \frac{1}{2} \text{sign}(E_n) R(E_n) \Psi_n^\dagger(x) \beta \Psi_n(x) \right],
\]

\[
K_1(t) = \frac{N_c}{6} \sum_{n,m} \int d^3x j_0(Qr) \int d^3y \left[ \frac{\Psi_n^\dagger(x) \bar{\tau} \Psi_{val}(x) \cdot \Psi_m^\dagger(y) \beta \bar{\tau} \Psi_n(y)}{E_n - E_{val}} \right]
\]

\(^1\)In fact, we have only one free parameter, \( i.e. \) the constituent up-quark (down-quark) mass. However, it is more or less fixed to around 420 MeV by the mass splitting [13].
\[ \mathcal{K}_2(t) = \frac{N_c}{6} \sum_{n,m} \int d^3 x \, j_0(Qr) \int d^3 y \left[ \frac{\Psi_n^*(x) \Psi_m(x) \Psi_{m}^o(y) \beta \Psi_{n}^o(y) R_\lambda(E_n, E_m)}{E_m - E_{val}} \right. \\
+ \frac{1}{2} \Psi_n^*(x) \Psi_m(x) \beta \Psi_{m}^o(y) \beta \Psi_{n}^o(y) R_{\lambda}(E_n, E_m^0) \biggr], \]

\[ \mathcal{N}_1(t) = \frac{N_c}{6} \sum_{n,m} \int d^3 x \, j_0(Qr) \int d^3 y \left[ \frac{\Psi_n^*(x) \beta \Psi_{m}^o(x) \Psi_{m}^o(y) \beta \Psi_{n}^o(y)}{E_m - E_{val}} \right. \\
+ \frac{1}{2} \Psi_n^*(x) \beta \Psi_{m}^o(x) \Psi_{m}^o(y) \beta \Psi_{n}^o(y) R_{\beta}(E_n, E_m^0) \biggr], \]

\[ \mathcal{N}_2(t) = \frac{N_c}{6} \sum_{n,m} \int d^3 x \, j_0(Qr) \int d^3 y \left[ \frac{\Psi_n^*(x) \beta \Psi_{m}^o(x) \Psi_{m}^o(y) \beta \Psi_{n}^o(y)}{E_m - E_{val}} \right. \\
+ \frac{1}{2} \Psi_n^*(x) \beta \Psi_{m}^o(x) \Psi_{m}^o(y) \beta \Psi_{n}^o(y) R_{\beta}(E_n, E_m^0) \biggr], \]

\[ \mathcal{N}_0(t) = \frac{3N_c}{2} \sum_{n,m} \int d^3 x \, j_0(Qr) \int d^3 y \left[ \frac{\Psi_n^*(x) \beta \Psi_{m}^o(x) \Psi_{m}^o(y) \beta \Psi_{n}^o(y)}{E_m - E_{val}} \right. \\
+ \frac{1}{2} \Psi_n^*(x) \beta \Psi_{m}^o(x) \beta \Psi_{m}^o(y) R_{\beta}(E_n, E_m^0) \biggr]. \quad (12) \]

The subscripts \( i \) and \( p \) in the collective part are \( i = 1, 2, 3 \) and \( p = 4, 5, 6, 7 \), respectively. \( I_i \) and \( K_i \) are respectively the moments of inertia and anomalous moments of inertia [13]. When \( t \to 0 \), \( \mathcal{K}_i(t) \) become \( K_i \). The \( \Sigma_{SU(2)} \) corresponds to the \( \pi N \) sigma term in \( SU(2) \) [13] at \( t = 0 \), which can be obtained by the Feynman-Hellman theorem

\[ \Sigma_{SU(2)} = \tilde{m} \frac{\partial E(\tilde{m})}{\partial \tilde{m}} \bigg|_{\tilde{m} = 0}, \quad (13) \]

where \( E \) stands for the classical soliton energy. The regularization functions \( R(E_n), R_{\lambda}(E_n, E_m), R_{\beta}(E_n, E_m) \) \(^2\) are defined by

\[ R(E_n) = \int \frac{du}{\sqrt{\pi u}} \phi(u; \Lambda_n) |E_n| e^{-uE_n^2}, \]

\[ R_{\lambda}(E_n, E_m) = \frac{1}{2} \text{sign}(E_n) - \text{sign}(E_m), \]

\[ R_{\beta}(E_n, E_m) = \int_0^\infty \frac{du}{2\sqrt{\pi u}} \phi(u; \Lambda_n) \frac{E_n e^{-uE_n^2} - E_m e^{-uE_m^2}}{E_n - E_m}, \quad (14) \]

\(^2\)\( R_{\lambda}(E_n, E_m) \) is not actually a regularization function, since \( K_i \) come from the imaginary part of the action. It does not depend on the cut-off parameter.
respectively. The $\langle i \rangle_N$ stands for the expectation value of the Wigner $D$ functions in collective space spanned by $A$. The expectation values of the $D$ functions can be evaluated by SU(3) Clebsch-Gordan coefficients found in Refs. [19,20]. With SU(3) symmetry explicitly broken by $m_s$, the collective part is no longer SU(3)-symmetric. Therefore, the eigenstates of the hamiltonian are not in a pure octet or decuplet but mixed states. Since we treat the strange quark mass $m_s$ perturbatively, we can obtain the mixed SU(3) baryonic states as follows:

$$|8, N\rangle = |8, N\rangle + c_{i0}^N|I0, N\rangle + c_{27}^N|27, N\rangle$$

(15)

with

$$c_{i0}^N = \frac{\sqrt{5}}{15}(\bar{\sigma} - r_1)I_2m_s, \quad c_{27}^N = \frac{\sqrt{6}}{75}(3\bar{\sigma} + r_1 - 4r_2)I_2m_s.$$  

(16)

The constant $\bar{\sigma}$ is related to the $\Sigma_{SU(2)}$ by $\Sigma_{SU(2)} = 2/(m_u + m_d)\bar{\sigma}$. $r_i$ denotes the ratio $K_i/I_i$.

Since the Cheng-Dashen point is out of the physical region, it is necessary to extrapolate to the region $t > 0$. This can be done by the analytic continuation of the $|q|$, i.e. $|q| \rightarrow i|\bar{q}|$ so that we may have the positive $t$ up to the Cheng-Dashen point ($t = 2m_s^2$). The analytic continuation above the threshold $t = 4m_s^2$ is not valid in our model, since above this threshold, the correlation between mesonic clouds is getting important [22]. Hence, in this work, we only evaluate the scalar form factor from the Cheng-Dashen point to the physical channel (space-like region: $t < 0$).

III. NUMERICAL RESULTS AND DISCUSSION

In order to calculate the $\sigma_{8N}(t)$ numerically, we take advantage of the Kahana-Ripka discretized basis [26]. Figure 1 shows the scalar form factor as a function of the constituent quark mass $M = M_u = M_d$. The $\sigma(t)$ decreases as the $M$ increases, in particular, below $t = 0$. As a result, the difference between the $\sigma(2m_s^2)$ and $\sigma(0)$ changes drastically when we increase the $M$ from 370 MeV to 450 MeV, as shown in Table 1. We select the $M = 420$ MeV.
for the best fit as we did for other observables. The error bar presented in Fig. 1 stands for the empirical analysis due to Gasser, Leutwyler, and Sainio [3], i.e. $\sigma(0) = 45 \pm 8$ MeV. Our numerical prediction is in a remarkable agreement with Ref. [3]. It is also interesting to see how the $m_s$ corrections contribute to the scalar form factor. As shown in Fig. 2, the $m_s$ corrections are very small. At $t = 0$, the $m_s$ corrections contribute to the $\sigma$ term about 2% which is negligible. However, the $m_s$ corrections play a significant role of reducing remarkably the large strangeness contribution $\langle N | s \bar{s} | N \rangle$ arising from the leading term and rotational $1/N_c$ corrections. With the $m_s$ corrections taken into account, we obtain $y = 0.27$ in case of the $M = 420$ MeV, which agrees with the empirical value $y \simeq 0.2$ [3] within about 30%, whereas we have $y = 0.48$ without the $m_s$ corrections. It is already known that the explicit symmetry breaking term quenches the $\langle N | s \bar{s} | N \rangle$ [27–29].

The difference $\Delta \sigma = \sigma(2m_s^2) - \sigma(0)$ we have obtained is 18.18 MeV. This value is very close to what Gasser and Leutwyler extracted [21], $\Delta \sigma = 15.2 \pm 0.4$ MeV. The tangent of the scalar form factor at $t = 0$ is known to be related to the scalar square radius. It is almost two times larger than the electric one, i.e. the $\langle r^2 \rangle_S \simeq 1.6 \text{fm}^2$ while $\langle r^2 \rangle_E \simeq 0.74 \text{fm}^2$. The prediction of our model for the $\langle r^2 \rangle_S$ is 1.5 fm$^2$ which is almost the same as obtained by Gasser and Leutwyler. It implies that the tail of the scalar density is of great importance. In Fig. 3 we can find a long-stretched and strong tail in the sea contribution to the scalar density. This tail is due to the mesonic clouds arising from the Dirac sea polarization. Moreover, the sea contribution in the scalar density is large, compared with the other densities such as electromagnetic densities [23–25].

The other interesting quantities are presented in table 1. $\sigma_0$ is the condensate of the singlet scalar quark operator in the nucleon: $\sigma_0 = \bar{m} \langle N | \bar{u}u + \bar{d}d + \bar{s}s | N \rangle$ $R_s$ is defined by $R_s = \langle N | s \bar{s} | N \rangle / \langle N | \bar{u}u + \bar{d}d + \bar{s}s | N \rangle$. 

9
IV. SUMMARY AND CONCLUSION

We have discussed the scalar form factor with related quantities in the SU(3) NJL soliton model. The results we have obtained are in a good agreement with empirical data [3,21]. The reliable strangeness contents of the nucleon in the scalar channel is obtained by taking into account the $m_s$ corrections, since they suppress the excess of $\langle N|\bar{s}s|N\rangle$ due to the leading order and rotational $1/N_c$ contributions. In contrast to Refs. [5,6] suggesting no strangeness contribution, our model favors $y = 0.27$. The large value of the $\langle r^2 \rangle^S_N$ is caused by the pronounced long ranging tail which can be identified with the pion and kaon clouds.

ACKNOWLEDGEMENT

The authors would like to thank M. Polyakov for helpful discussions. This work has partly been supported by the BMFT, the DFG, the COSY-Project (Jülich) and Department of Energy grant DE-FG02-88ER40388. One of us (AB) would like to thank the Alexander von Humboldt Foundation for a Feodor Lynen grant.
TABLE I. The physical quantities related to the scalar form factor. The empirical data come from Ref.[3,17].

<table>
<thead>
<tr>
<th>$M$</th>
<th>370 MeV</th>
<th>420 MeV</th>
<th>450 MeV</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_s$ [MeV]</td>
<td>0</td>
<td>156.75</td>
<td>0</td>
<td>148.49</td>
</tr>
<tr>
<td>$\sigma_{\pi N}$ [MeV]</td>
<td>43.09</td>
<td>44.71</td>
<td>40.01</td>
<td>40.80</td>
</tr>
<tr>
<td>$\sigma_0$ [MeV]</td>
<td>53.25</td>
<td>49.25</td>
<td>49.58</td>
<td>46.24</td>
</tr>
<tr>
<td>$\sigma_S$ [MeV]</td>
<td>22.77</td>
<td>35.63</td>
<td>20.87</td>
<td>29.92</td>
</tr>
<tr>
<td>$y$</td>
<td>0.47</td>
<td>0.20</td>
<td>0.48</td>
<td>0.27</td>
</tr>
<tr>
<td>$R_s$</td>
<td>0.19</td>
<td>0.09</td>
<td>0.19</td>
<td>0.12</td>
</tr>
<tr>
<td>$\Delta\sigma$ [MeV]</td>
<td>32.29</td>
<td>33.37</td>
<td>18.36</td>
<td>18.18</td>
</tr>
<tr>
<td>$\langle r^2 \rangle^S_N$</td>
<td>1.94</td>
<td>1.87</td>
<td>1.56</td>
<td>1.50</td>
</tr>
</tbody>
</table>


[17] C. Schneider, Diplomarbeit, Ruhr-Universität Bochum, unpublished (1994);
    C. Schneider, A. Blotz, and K. Goeke, in preparation.


