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Abstract

We study the effect of the photon formation length on the interference of two bremsstrahlung photons in the simplest case of radiation emitted by only two charged particles. Most of interference patterns obtained can be interpreted in terms of the photon formation length. After having analyzed the simplest situations we discuss the interference of bremsstrahlung photons in more realistic and more complicated situations. A possibility of studying the effect in synchrotron radiation is briefly sketched.

 

1 Introduction

The Hanbury-Brown and Twiss (HBT) intensity interferometry [1] has become since the paper by Goldhaber et al. [2] a tool which is frequently being used in particle physics, for recent reviews see e.g. [3]. The concepts of the formation time and length has been introduced on the basis of studies by Landau, Pomeranchuk and Migdal [4] of emission of soft photons by fast charged particles multiply scattered in matter. Results of these calculations follow from classical or quantum electrodynamics and are due to characteristic features of bremsstrahlung emission by charged particles, in particular by the dependence of the energy and angular spectra of photons on the trajectory of the charged particle. The concept of the photon formation length is not inserted by hand into the calculation, it is however extremely useful when analyzing and interpreting the results obtained and it is most valuable in leading intuition when estimating or evaluating the expected results in similar problems.

The interferometry of bremsstrahlung photons has also some specific features connected with the characteristic angular and energy spectra of amplitudes for bremsstrahlung emission by charged particles. In the present paper we are using terms like “photon formation length in HBT interferometry” to denote simply those features of the HBT interferometry of bremsstrahlung photons which are connected with specific features of amplitudes for emission of soft photons by charged particles. In some situations the notion of the formation length permits to understand intuitively the results obtained, but there is no guarantee that a simple and straightforward application of the concept of the formation length will intuitively explain all of possible cases of HBT interference of bremsstrahlung photons.

In analyzing the HBT interference of bremsstrahlung photons we shall use the standard form of the amplitudes for emission of soft photons and assumptions about the coherence and incoherence of amplitudes and the concept of the formation length comes in only when results obtained are analyzed.

The interferometry of two photons has been studied in Refs. [5, 6, 7, 8]. The authors have not touched the question of a possible influence of photon formation length on the interference patterns. In our recent paper [9] we have shown by a numerical study that the radius of a sphere emitting from its surface charged particles appears larger when observed by the HBT interferometry of soft photons radiated by these charged particles. In a simplified language we can say, in the sense discussed above, that the photon formation length influences the effective radius of region emitting charged particles, which in turn radiate photons.

Although the results of Ref. [9] are qualitatively, and in a good approximation even quantitatively correct, the basic relations describing the HBT interference of bremsstrahlung need better motivation and clarification of the physics behind the assumptions used. Moreover the final relations used in Ref. [9] have to be corrected in the contribution of two photons radiated by the same charged particle. We shall discuss these issues in detail below. In the present paper we shall put emphasis on the way in which the photon formation length influences the interference patterns of two bremsstrahlung photons. For this purpose we shall describe in detail the simplest case of two photons radiated by two charged particles. It turns out that an important role is played by the coherence or incoherence of relevant amplitudes

and we shall pay attention to this issue. We shall first analyze a simplified model of multiparticle production - the case when there are two neutral particles in the initial state and the final state consists of two non-identical charged particles plus a variable number of neutral ones. The assumptions made are then illustrated by a simple model of two sources emitting charged particles from two specified points.

The paper is organised as follows. In the next Section we describe the simple situation of bremsstrahlung radiation by two charged particles in the simplified model of multiparticle production and in the case of two sources and discuss the coherence of amplitudes. In Sect. 3 the interference of two photons in this simple situation is analysed in detail. To study the influence of the photon formation length on interference patterns we consider in Sect. 4 a situation in which charged particles, radiating two photons, are absorbed some time after their production. The case when the two photons are radiated by more charged particles is discussed in Sect. 5. Some considerations about the possibilities of observing the HBT interference effects in synchrotron radiation are presented in Sect. 6. The last Section contains comments and conclusions.

2 Interference of bremsstrahlung radiation by two charged particles

For the sake of simplicity and definiteness, let us consider as an example a collision of two neutral particles N_1 and N_2 leading to a formation of a complex many particle system decaying into two non-identical charged particles 'a' and 'b' and a set of neutral particles $c_1, c_2, \dots, c_n$ where n varies from collision to collision:

$$N_1 + N_2 \rightarrow a + b + c_1 + c_2 + \dots + c_n \quad (1)$$

The complexity and the dimension of the system are supposed to justify the concept of particles 'a' and 'b' being produced near space-time points x_a and x_b and with momenta close to p_a and p_b [10, 11]. The symbol for the amplitude $M(a, b; c_1, c_2, \dots, c_n)$ will be understood below as including implicitly also the four-momenta p_a, p_b and positions x_a, x_b . In places where it is unavoidable we shall write the amplitude explicitly as $M(a(p_a, x_a), b(p_b, x_b), \dots)$. The amplitude for the emission of a single bremsstrahlung photon by one of the charged particles in a situation when x_a, x_b and the momenta of charged particles p_a, p_b **after** the emission of photons are specified is given as

$$\begin{aligned} T(k, \varepsilon) = & M(a(p_a + k, x_a), b(p_b, x_b), \dots) B(k, a) \\ & + M(a(p_a, x_a), b(p_b + k, x_b), \dots) B(k, b) \end{aligned} \quad (2)$$

where $B(k, a) \equiv B(k, \varepsilon, a)$ is the amplitude for the emission of a photon with momentum k and polarization ε by particle 'a'. In the first term on the r.h.s. of Eq.2 the symbol $a(p_a + k, x_a)$ indicates that the particle 'a' is produced from the complex intermediate system with the off-the-mass-shell momentum $p_a + k$ and after having radiated the photon reaches the final momentum p_a . Note that the final-state momenta of charged particles 'a' and 'b' are the same in both parts of the amplitude in Eq.2 and the two parts of the amplitude for a specified set of momenta of neutral

particles add coherently. Because of the complexity of the system the phase of the amplitude M is **assumed** to depend strongly on the momenta of particles emerging from the system - prior to radiation of the photon. That means that the difference of phases of the two amplitudes in the r.h.s. of Eq.2 will fluctuate strongly depending on the way how the rest of the energy and momentum is distributed among the neutral particles $c_1, c_2, \dots, c_n$ and the two parts of the amplitude, differing in the place, x_a or x_b , from where the four-momentum k has been taken away, become incoherent after integration over momenta of neutral particles and summation over their number.

The argument depends substantially on two issues: on the complexity of the system and on the magnitude of the momentum of the radiated photon. For simple amplitudes and for no neutral particles in the final state the two terms will remain coherent. In the same way for a given system the two parts of the amplitude will become coherent for sufficiently small momentum of the photon. A natural conjecture consists in assuming that for any complex system there exists a parameter κ such that for photon momenta $|\vec{k}| < \kappa$ the two parts of the amplitude in Eq.2 are approximately coherent and for $|\vec{k}| > \kappa$ approximately incoherent. The parameter κ is expected to decrease with increasing complexity of the system.

For $|\vec{k}| < \kappa$ the four-momentum k in the amplitudes M in Eq.2 can be neglected and we obtain the amplitude $T(k, \varepsilon)$ corresponding to the coupling of the photon to the total current, calculated without taking the photon emission into account. This is what is usually denoted as the classical approximation. The question of the coherence or incoherence of the two amplitudes in Eq.2 is not a question of the wave length of the emitted photon, it is the question of interference of two complex states differing in from where the energy has been taken away. This is the same question as by the HBT interferometry of charged identical particles and we shall assume that the values of κ in both cases are approximately the same, perhaps in region of a few tens to one hundred of MeV/c. In what follows we shall limit ourselves to situations where the photon momentum is larger than κ . We **assume** therefore that after forming $|T|^2$, integrating over the momenta of neutral particles and summing over their number n the two contributions in the r.h.s of Eq.2 become incoherent. The probability to detect a single photon with momentum k_1 (and polarization ε_1) is then given as

$$P(k_1; p_a, p_b, x_a, x_b) = \langle |M|^2 \rangle \{ |B(k_1, a)|^2 + |B(k_1, b)|^2 \} \quad (3)$$

where $\langle |M|^2 \rangle$ denotes the integration over the momenta of neutral particles and summing over their number, with p_a, p_b and x_a, x_b being held fixed. The momenta p_a and p_b can be in principle imagined as held fixed - the momenta can be measured and we can keep only the events with specified momenta of charged particles in the final state, the accuracy of the measurement being in principle much higher than the magnitude of the momentum of the photon. On the other hand the positions x_a and x_b are unknown and the two terms in Eq.3 have finally to be integrated over the density of sources corresponding to the given complex system. Bremsstrahlung of charged particles created in different pairs of points x_a, x_b is assumed to be incoherent. In this way we obtain the single photon distribution $P(k_1)$. The amplitude for the radiation of two bremsstrahlung photons with (k_1, ε_1) and (k_2, ε_2) is then

given as

$$\begin{aligned}
T(k_1, k_2) = & M(a(p_a + k_1, x_a), b(p_b + k_2, x_b), \dots) B(k_1, a) B(k_2, b) \\
& + M(a(p_a + k_2, x_a), b(p_b + k_1, x_b), \dots) B(k_2, a) B(k_1, b) \\
& + M(a(p_a + k_1 + k_2, x_a), b(p_b, x_b), \dots) B(k_1, a) B(k_2, a) \\
& + M(a(p_a, x_a), b(p_b + k_1 + k_2, x_b), \dots) B(k_1, b) B(k_2, b) \quad (4)
\end{aligned}$$

In studying the HBT interference we are mostly interested in the region of small values of $\Delta k = k_1 - k_2$. The (off-the-mass-shell) momenta of particles 'a' and 'b' in the first two amplitudes in Eq.4 differ only by Δk and we **assume** that these two amplitudes add coherently. The amplitudes in the third and fourth terms differ in momenta of charged particles before the radiation of photons by $k_1 + k_2$ between themselves and by k_1, k_2 with respect to the first two terms. All of k_1, k_2 and $k_1 + k_2$ are supposed to be larger than κ . We shall therefore **assume** that the third and fourth terms add incoherently. In this way we are rather speaking about the region of hard bremsstrahlung, not about the limit $k \rightarrow 0$. Under the same assumptions as made above for the case of a single photon the probability to register two photons is given as

$$\begin{aligned}
P(k_1, k_2; p_a, p_b, x_a, x_b) = & \langle |M|^2 \rangle \left\{ |B(k_1, a)B(k_2, b) + B(k_2, a)B(k_1, b)|^2 \right. \\
& \left. + |B(k_1, a)B(k_2, a)|^2 + |B(k_1, b)B(k_2, b)|^2 \right\} \quad (5)
\end{aligned}$$

This probability corresponds again to the situation in which both momenta and positions x_a, x_b are held fixed. The observed probability distribution is obtained by integrating over x_a and x_b . In contradistinction to the phase of M the $|M|$ is supposed to be a slowly varying function of its arguments and in order to simplify formulas we shall put $\langle |M|^2 \rangle$ equal to one both in Eq.3 and Eq.5. The correlation function corresponding to the idealized situation when both the momenta p_a, p_b and the coordinates x_a, x_b are kept fixed is then given as

$$C(k_1, k_2) = \frac{P(k_1, k_2)}{P(k_1) P(k_2)} \quad (6)$$

where

$$\begin{aligned}
P(k_1, k_2) = & |B(k_1, a)B(k_2, b) + B(k_2, a)B(k_1, b)|^2 \\
& + |B(k_1, a)B(k_2, a)|^2 + |B(k_1, b)B(k_2, b)|^2 \\
P(k_1) = & |B(k_1, a)|^2 + |B(k_1, b)|^2 \\
P(k_2) = & |B(k_2, a)|^2 + |B(k_2, b)|^2 \quad (7)
\end{aligned}$$

The Eqs.6 and 7 are modifications of well known [2] basic relations for the HBT interferometry of identical pions for the case of two sources. Differences between the two cases consist in:

- i) for identical pions the last two terms in $P(k_1, k_2)$ are missing since the emission of two pions from the same source is usually not considered,
- ii) for pions instead of $B(k_1, a)$ we have simply $\exp(-ik_1 \cdot x_a)$. For bremsstrahlung a strong correlation between k_1 and p_a is present,

- iii) in the case of bremsstrahlung one integrates not only over the positions of sources but also over the momenta of charged particles,
- iv) for photons a limit of the four-vector k going to zero is possible, whereas this is impossible for a massive particle.

For identical pions the correlation function $C(k_1, k_2)$ in Eq.6 is simply $1 + \cos[(k_1 - k_2) \cdot (x_a - x_b)]$, for the bremsstrahlung photons the expressions are a bit more complicated and will be discussed shortly.

Note that we would have obtained the same result by assuming that particles 'a' and 'b' were produced by two independent sources with random and uncorrelated phases ϕ_a and ϕ_b . In such a situation the amplitude for radiating a single photon becomes

$$M(k_1) = e^{i\phi_a} B(k_1, a) + e^{i\phi_b} B(k_1, b) \quad (8)$$

Making use of $\langle e^{i(\phi_a - \phi_b)} \rangle = \delta_{ij}$ we find immediately the expression for $P(k_1)$ given by Eq.7. The physics behind this elegant argument using random and uncorrelated phases is given by mechanisms leading to coherence and incoherence of amplitudes discussed above. The amplitude for the radiation of two photons becomes

$$\begin{aligned} T(k_1, k_2) = & e^{i(\phi_a + \phi_b)} [B(k_1, a)B(k_2, b) + B(k_2, a)B(k_1, b)] \\ & + e^{i\phi_a} B(k_1, a)B(k_2, a) + e^{i\phi_b} B(k_1, b)B(k_2, b) \end{aligned} \quad (9)$$

The absence of correlation between sources, expressed by $\langle e^{i\phi_a} \rangle = \langle e^{i\phi_b} \rangle = 0$ leads immediately to $P(k_1, k_2)$ in Eq.7. Note that the phase factors are connected with sources (charged particles) and not with photons.

For n_c charged particles in the final state the results in Eq.7 are generalized to

$$\begin{aligned} P(k_1, k_2) = & \sum_{i>j} |B(k_1, i)B(k_2, j) + B(k_2, i)B(k_1, j)|^2 \\ & + \sum_i |B(k_1, i)B(k_2, i)|^2 \end{aligned} \quad (10)$$

We shall use Eq.10 later on, now we shall study in more detail the case of two charged particles with momenta (in the final state) p_a and p_b emitted from space-time points x_a and x_b .

Radiation of two photons is described by four amplitudes shown schematically in Fig.1. In cases c) and d) the graphs represent sums of two amplitudes with interchanged photon momenta.

Suppose that particle 'a' moves along the z-axes with velocity v , and particle 'b' moves in the opposite direction with velocity of the same magnitude.

For momenta $\vec{k}_1$ and $\vec{k}_2$ of the two photons we shall use the notation

$$\vec{k}_1 = \vec{K} + \vec{q}, \quad \vec{k}_2 = \vec{K} - \vec{q} \quad (11)$$

and choose $\vec{K}$ perpendicular to the z-axes and $\vec{q}$ parallel to the z-axes. The kinematics is shown in Fig.2. The amplitude of radiation of photon with momentum $\vec{k}_1$ and polarization $\vec{\epsilon}_1$ by particle 'a' is given as

$$B(k_1, a) = Q_a e^{-i\vec{k}_1 \cdot \vec{x}_a} \frac{\vec{\epsilon}_1 \cdot \vec{v}}{\omega \left(1 - \frac{\vec{k}_1 \cdot \vec{v}}{\omega}\right)} \quad (12)$$

where Q_a is the charge of the particle 'a'. This amplitude can be obtained either from classical electrodynamics [13] or from Feynman diagrams [14]. We shall use below the amplitude in the soft photon limit. This contains the most important factor in the denominator responsible for the characteristic energy and angular dependence of bremsstrahlung radiation. The next higher order correction in the development in photon momentum gives vanishing contribution for radiation of real photons of pseudoscalar pions, see e.g. Eqs.(2.3) and (2.4) in Lichard [15], for fermions a part of the contribution of the higher order gives a vanishing contribution and a part is non-factorizable and requires a detailed knowledge of the strong amplitude, see Sect.III in Lichard [15] and Appendix A in Ref. [16]. Note that in these references one speaks about massive virtual photons and to obtain expressions for real photons one has to put $k^2 = 0$. The present state of understanding of soft photon production in collisions at high energies is reviewed by Lichard in Ref.[12]. In the standard notation used when working with Feynman diagrams the amplitude $B(k_1, a)$ is written as

$$B(k_1, a) = Q_a e^{-i\vec{k}_1 \cdot \vec{x}_a} \frac{\varepsilon_1 \cdot p}{k_1 \cdot p} \quad (13)$$

where $\varepsilon_1 = (0, \vec{\varepsilon}_1)$, $p = (E, \vec{p})$ and $k_1 = (\omega_1, \vec{k}_1)$. It is easy to show that terms in Eqs.12 and 13 are up to the sign identical. In classical electrodynamics the polarization of the field enters as $\vec{n} \times (\vec{n} \times \vec{v})$ where $\vec{n} = \vec{k}/\omega$ is the direction of the motion of photon, whereas in Feynman diagrams the polarization of the photon enters as $\vec{\varepsilon} \cdot \vec{p}$ where $\vec{p}$ is the momentum of the charged particle. The polarization of the photon with momentum $\vec{k}_1$ in Fig.2 is perpendicular to $\vec{k}_1$ and lies within the plane given by $\vec{k}_1$ and $\vec{v}$. It is easy to see that in this situation $\vec{\varepsilon} \cdot (\vec{n} \times (\vec{n} \times \vec{v}))$ and $\vec{\varepsilon} \cdot \vec{p}$ are proportional. Using the notation of Eq.11 and kinematics of Fig.2 we obtain for all the four amplitudes

$$\begin{aligned} B(k_1, a) &= Q_a e^{-iq \cdot z_a} \frac{\vec{\varepsilon}_1 \cdot \vec{v}}{\omega(1 - \frac{qv}{\omega})} \\ B(k_1, b) &= Q_b e^{-iq \cdot z_b} \frac{-\vec{\varepsilon}_1 \cdot \vec{v}}{\omega(1 + \frac{qv}{\omega})} \\ B(k_2, a) &= Q_a e^{iq \cdot z_a} \frac{\vec{\varepsilon}_2 \cdot \vec{v}}{\omega(1 + \frac{qv}{\omega})} \\ B(k_2, b) &= Q_b e^{iq \cdot z_b} \frac{-\vec{\varepsilon}_2 \cdot \vec{v}}{\omega(1 - \frac{qv}{\omega})} \end{aligned} \quad (14)$$

3 Interference patterns of two bremsstrahlung photons radiated by two charged particles

We shall start with discussing in some detail the simplest situation shown in Fig.2, using the notation of Eq.11. The amplitudes are given by Eq.14 and the correlation function as defined by Eq.6 is calculated on the basis of Eqs.7. To simplify the notation denote

$$\eta \equiv \frac{qv}{\omega} \quad (15)$$

Assuming that $|Q_a| = |Q_b|$ the charges and expressions $\vec{\varepsilon}_1 \cdot \vec{v}$, $\vec{\varepsilon}_2 \cdot \vec{v}$ cancel in the final expressions for $C(\vec{k}_1, \vec{k}_2)$ so we shall omit them for the moment. With this

simplification we have

$$P(k_1) = P(k_2) = \left(\frac{1}{1-\eta}\right)^2 + \left(\frac{1}{1+\eta}\right)^2 \quad (16)$$

$$P(k_1, k_2) = \left(\frac{1}{1-\eta^2}\right)^2 + \left(\frac{1}{1-\eta^2}\right)^2 + \left| e^{iq\Delta z} \left(\frac{1}{1+\eta}\right)^2 + e^{-iq\Delta z} \left(\frac{1}{1-\eta}\right)^2 \right|^2 \quad (17)$$

where $\Delta z = z_a - z_b$. Denoting $\Delta k = 2q = |\vec{k}_1 - \vec{k}_2|$ we finally obtain

$$C(k_1, k_2) = 1 + \frac{1}{2} \frac{(1-\eta^2)^2}{(1+\eta^2)^2} \cos(\Delta k \Delta z) \quad (18)$$

The effective dimension of the emission region is defined by the behaviour of the correlation function for small values of Δk .

As a reference we shall use the correlation function corresponding to the interference of bremsstrahlung photons by charged particles with velocities $v \ll c$. In this case we have

$$C_{\text{ref}}(k_1, k_2) = 1 + \frac{1}{2} \cos(\Delta k \Delta z_{\text{eff}}) \quad (19)$$

Requiring the same second derivative of $C(k_1, k_2)$ and $C_{\text{ref}}(k_1, k_2)$

$$\left. \frac{d^2 C(k_1, k_2)}{d(\Delta k)^2} \right|_{\Delta k=0} = \left. \frac{d^2 C_{\text{ref}}(k_1, k_2)}{d(\Delta k)^2} \right|_{\Delta k=0} \quad (20)$$

we obtain from Eqs.18 and 19

$$(\Delta z_{\text{eff}})^2 = (\Delta z)^2 + 2 \frac{v^2}{\omega^2} \quad (21)$$

This result offers an attractive interpretation [9]: $1/\omega$ is the formation time of a photon with momentum perpendicular to the velocity of radiating particle. This corresponds to vector $\vec{K}$ in the kinematics of Fig.2. The term v/ω is the distance the charged particle travels during the emission of the photon – the formation length of the photon. The Eq.21 says simply that formation lengths of photons are added to the dimension of the region emitting charged particles. Before taking this interpretation literally one has to study more complicated but still reasonably transparent situations and analyse the effect in more detail.

We shall now turn to the situation shown in Fig.3.

The calculation proceeds along the same lines as above. This leads to the formula for effective dimension

$$(\Delta z_{\text{eff}})^2 = \sin^2 \vartheta (\Delta z)^2 + 2 \left(\frac{v}{\omega} \right)^2 \frac{1}{1 - v^4 \cos^4 \vartheta} \left(v^2 \sin^2(2\vartheta) \frac{v^2 \cos^2 \vartheta}{1 - v^4 \cos^4 \vartheta} - \cos(2\vartheta) \right) \quad (22)$$

The dependence of the Δz_{eff} on the angle ϑ is presented in Fig.4. This dependence of Δz_{eff} on ϑ is not so simple to interpret. Its understanding requires analysis of further simple situations including the case of unequal velocities of charged particles in Fig.2 and an analysis of the behaviour of interference patterns with respect to Lorentz boosts along the z -direction. In discussing the simple situation in Fig.2 we have assumed that both particles were emitted at the same time. The simultaneity is not a boost-invariant concept and when studying Lorentz boosts the time dependence of particle emissions has to be built into the scheme explicitly. We shall not go into this analysis here. And, of course, there is also a possibility that this is one of cases which show limitations of the simple picture of how the photon formation length enters into the interference patterns.

The next simple situation is shown in Fig.5. The role played by polarizations of photons becomes essential and we have to discuss this issue first. For simplicity consider the two charged particles as shown in Fig.5 emitting two photons with momenta $\vec{k}_1$ and $\vec{k}_2$. According to classical electrodynamics [13] the polarization of bremsstrahlung photon with momentum $\vec{k}_1$ emitted by particle 'a' is given

$$\vec{\varepsilon}_{1a} = \frac{\vec{k}_1 \times (\vec{k}_1 \times \vec{v}_a)}{|\vec{k}_1 \times (\vec{k}_1 \times \vec{v}_a)|} \quad (23)$$

and similarly for other polarizations $\vec{\varepsilon}_{1b}$, $\vec{\varepsilon}_{2a}$, $\vec{\varepsilon}_{2b}$. Suppose now that the photon with momentum $\vec{k}_1$ is registered by the detector which accepts only the polarization $d_{1\zeta}$. The amplitude for the emission of photon '1' by particle 'a' and its detection then reads

$$B(k_1, a) = Q_a e^{-i\vec{k}_1 \cdot \vec{x}_a} \langle d_{1\zeta} | \varepsilon_{1a} \rangle \frac{\vec{\varepsilon}_{1a} \cdot \vec{v}_a}{\omega \left(1 - \frac{\vec{k}_1 \cdot \vec{v}_a}{\omega}\right)} \quad (24)$$

The detector registering the photon '2' accepts only the polarization $d_{2\zeta}$ and the corresponding amplitude is constructed along the same lines as $B(k_1, a)$ in Eq.24. Summing over polarizations accepted by detectors we obtain for $P(k_1, k_2)$ as given by Eq.7 the following expression

$$\begin{aligned} P(k_1, k_2) = & |B(k_1, a)B(k_2, a)|^2 + |B(k_1, b)B(k_2, b)|^2 \\ & + |B(k_1, a)B(k_2, b)|^2 + |B(k_1, b)B(k_2, a)|^2 \\ & + 2\Re \left\{ B(k_1, a)B(k_2, b)B^*(k_1, b)B^*(k_2, a) \langle \varepsilon_{1b} | \varepsilon_{1a} \rangle \langle \varepsilon_{2a} | \varepsilon_{2b} \rangle e^{-i\Delta\vec{k} \cdot \Delta\vec{x}} \right\} \end{aligned} \quad (25)$$

The interference term contains the scalar products of polarizations given by Eq.23, what in general diminishes the magnitude of the interference term. Expressing the same otherwise, we can say that two photons with polarizations $\vec{\varepsilon}_{1a}$ and $\vec{\varepsilon}_{1b}$ are only partially coherent, the degree of coherence being given by the scalar product $\langle \varepsilon_{1b} | \varepsilon_{1a} \rangle$.

This phenomenon, in different context, is discussed in detail in Ref. [17].

After the treatment of polarizations according to the Eq.25 the calculation of the correlation function corresponding to the situation in Fig.5 becomes straightforward. From $C(\vec{k}_1, \vec{k}_2)$ one can calculate via Eq.20 the effective dimension of the source. Results are presented in Fig.6. The vector $\vec{K}$ is now perpendicular to $\vec{v}_b$ and $\Delta\vec{k}$

is parallel to $\vec{v}_b$. The most naive expectation for the dimension of the source in the $\vec{v}_b$ direction is $\Delta z_{\text{eff}}[\varphi = 0](1/2)(1 + \cos \varphi)$. This curve is also plotted in Fig.6. Although the agreement is not perfect, the qualitative trend is reproduced. The issue also requires more detailed study.

4 HBT interference of photons radiated by charged particles with short paths of flight

We shall now briefly discuss the HBT interference of photons radiated by charged particles in the situation shown in Fig.7.

The correlation function $C(k_1, k_2)$ is calculated via Eqs.2-6 but the amplitudes $B(k_1, a), \dots, B(k_2, b)$ in Eq.14 get modified. The modifications are easily obtained from the expression for the emission of photons by a classical particle moving along the trajectory $\vec{r}(t)$ [13, 18].

$$\omega \frac{dN}{d^3k} = \frac{\alpha}{4\pi^2} \left| \int_{-\infty}^{\infty} \vec{n} \times (\vec{n} \times \vec{v}(t)) e^{i\omega[t - \vec{n} \cdot \vec{r}(t)]} dt \right|^2 \quad (26)$$

where $\alpha = e^2/\hbar c = 1/137$, $\vec{k}$, ω is the momentum and energy of the radiated photon and $\vec{n} = \vec{k}/\omega$ is the direction in which the photon moves.

For the photon radiated by a charged particle starting at $t = 0$ from $\vec{r}_a$ with velocity $\vec{v}_a$ and absorbed after the time Δt the corresponding integral in Eq.26 gives

$$\left| \int_0^{\Delta t} \vec{n} \times (\vec{n} \times \vec{v}(t)) e^{i\omega[t - \vec{n} \cdot (\vec{r}_a + \vec{v}_a t)]} dt \right|^2 = e^{-i\vec{k} \cdot \vec{r}_a} \frac{1}{i\omega(1 - \vec{n} \cdot \vec{v}_a)} \left[e^{i\omega(1 - \vec{n} \cdot \vec{v}_a)\Delta t} - 1 \right] \quad (27)$$

The amplitude $B(k_1, a)$ given by Eq.12 is then modified to

$$B'(k_1, a) = e^{-i\vec{k}_1 \cdot \vec{r}_a} \frac{\vec{\epsilon}_1 \cdot \vec{v}_a}{\omega(1 - \vec{k}_1 \cdot \vec{v}_a/\omega)} \left[1 - e^{i\omega(1 - \vec{n} \cdot \vec{v}_a)\Delta t} \right] \quad (28)$$

and the amplitudes in Eq.14 are modified along the same lines. For Δt smaller than the photon formation time $t_f = 1/[\omega(1 - \vec{k}_1 \cdot \vec{v}_a/\omega)]$ the amplitude $B'(k_1, a)$ becomes

$$B'(k_1, a) \approx -i Q_a e^{-i\vec{k}_1 \cdot \vec{r}_a} \vec{\epsilon}_1 \cdot \vec{v}_a \Delta t \quad (29)$$

which corresponds to the radiation of a dipole. The amplitude in Eq.29 has lost its angular dependence characteristic for bremsstrahlung and the correlation function as calculated via Eq.6 leads to Eq.18 without the factor $(1 - \eta^2)^2/(1 + \eta^2)^2$. The second term in the r.h.s. of Eq.21 is also lost and the effective radius Δz_{eff} is equal to Δz .

The correlation function, calculated with amplitudes in Eq.28 and the corresponding effective dimension of the source have been computed numerically and are presented in Figs.8a,b.

The calculations have been done only for $d < 10$ fm. Note, that the formation length of photon with $\omega = 10$ MeV is 20 fm.

This example demonstrates that the characteristic angular dependence of the bremsstrahlung plays an essential role in obtaining the result given in Eq.(21).

5 HBT interference of photons radiated by many charged pions

We shall now consider a situation in which N charged particles $i = 1, 2, \dots, N$ are emitted simultaneously from positions $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_N$, their charges and momenta being $Q_1, Q_2, \dots, Q_N$ and $\vec{p}_1, \vec{p}_2, \dots, \vec{p}_N$. Each source emits the charged particle with an amplitude like in Eq.12, all phases are supposed to be random and uncorrelated. For definiteness we can think of N charged particles emitted from the surface of an excited system, produced in a heavy ion collision. Using the notation of Eq.13 and denoting ϕ_j the random phase of j -th charged particle, the amplitude for single photon radiation by N charged particles becomes

$$A(k, \varepsilon) = \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{i\phi_j} e^{ik \cdot x_j} Q_j \frac{\varepsilon \cdot p_j}{k \cdot p_j} \quad (30)$$

The probability for single photon emission is given as

$$P(k, \varepsilon) = \langle |A(k, \varepsilon)|^2 \rangle \quad (31)$$

where the brackets denotes averaging over random and uncorrelated phases. Using

$$\langle e^{i(\phi_j - \phi_l)} \rangle = \delta_{jl} \quad (32)$$

we find

$$P(k, \varepsilon) = \frac{1}{N} \sum_{j=1}^N Q_j^2 \left(\frac{\varepsilon \cdot p_j}{k \cdot p_j} \right)^2 \quad (33)$$

In the next step we calculate the emission of two photons with momenta k_1, k_2 and polarizations $\varepsilon_1, \varepsilon_2$. The amplitude consists of two types of contributions, in the former the two photons are radiated by two different charged particles, in the latter both photons are radiated by the same particle. Noting that the phase factors $\exp(i\phi_j)$ are connected with sources of charged particles and not with photons we obtain

$$A(k_1, \varepsilon_1, k_2, \varepsilon_2) = \frac{1}{N} \left[\sum_{j \neq l} e^{i\phi_j} e^{i\phi_l} e^{ik_1 \cdot x_j} e^{ik_2 \cdot x_l} Q_j \frac{\varepsilon_1 \cdot p_j}{k_1 \cdot p_j} Q_l \frac{\varepsilon_2 \cdot p_l}{k_2 \cdot p_l} + \sum_j e^{i\phi_j} e^{i(k_1 + k_2) \cdot x_j} Q_j \frac{\varepsilon_1 \cdot p_j}{k_1 \cdot p_j} Q_j \frac{\varepsilon_2 \cdot p_j}{k_2 \cdot p_j} \right] \quad (34)$$

The probability $P(k_1, \varepsilon_1, k_2, \varepsilon_2)$ is calculated according to

$$P(k_1, \varepsilon_1, k_2, \varepsilon_2) = \langle |A(k_1, \varepsilon_1, k_2, \varepsilon_2)|^2 \rangle \quad (35)$$

by using the following rules for averaging over phases

$$\langle e^{i(\phi_j - \phi_{j'})} \rangle = \delta_{jj'} \quad (36)$$

$$\langle e^{i(\phi_j + \phi_l - \phi_{j'})} \rangle = 0 \quad (37)$$

$$\langle e^{i(\phi_j + \phi_l - \phi_{j'} - \phi_{l'})} \rangle = \delta_{jj'} \delta_{ll'} + \delta_{jl'} \delta_{lj'} \quad (38)$$

Note that in Eq.38 we consider only $j \neq l$ and $j' \neq l'$, the case of two photons radiated by the same charged particle is already included in the rules 36 and 37.

The probability $P(k_1, \varepsilon_1, k_2, \varepsilon_2)$ is then obtained in the following form

$$P(k_1, \varepsilon_1, k_2, \varepsilon_2) = \frac{1}{N^2} \sum_{j,l=1}^N Q_j^2 \left(\frac{\varepsilon_1 \cdot p_j}{k_1 \cdot p_j} \right)^2 Q_l^2 \left(\frac{\varepsilon_2 \cdot p_l}{k_2 \cdot p_l} \right)^2 + \frac{1}{N^2} \sum_{j \neq l} e^{i(k_1 - k_2) \cdot (x_j - x_l)} Q_j^2 Q_l^2 \frac{\varepsilon_1 \cdot p_j}{k_1 \cdot p_j} \frac{\varepsilon_2 \cdot p_j}{k_2 \cdot p_j} \frac{\varepsilon_1 \cdot p_l}{k_1 \cdot p_l} \frac{\varepsilon_2 \cdot p_l}{k_2 \cdot p_l} \quad (39)$$

which can be rewritten as

$$P(k_1, \varepsilon_1, k_2, \varepsilon_2) = P(k_1)P(k_2) + |G(k_1, \varepsilon_1, k_2, \varepsilon_2)|^2 \quad (40)$$

where

$$|G(k_1, \varepsilon_1, k_2, \varepsilon_2)|^2 = \frac{1}{N^2} \sum_{j \neq l} e^{i(k_1 - k_2) \cdot (x_j - x_l)} Q_j^2 Q_l^2 \frac{\varepsilon_1 \cdot p_j}{k_1 \cdot p_j} \frac{\varepsilon_2 \cdot p_j}{k_2 \cdot p_j} \frac{\varepsilon_1 \cdot p_l}{k_1 \cdot p_l} \frac{\varepsilon_2 \cdot p_l}{k_2 \cdot p_l} \quad (41)$$

Assuming that charges of all particles are only +e or -e, we have the same Q_j^2 for all the charged particles and the charges in numerator and denominator are cancelled. The correlation function finally becomes

$$C(k_1, \varepsilon_1, k_2, \varepsilon_2) = 1 + \frac{\sum_{j \neq l} e^{i(k_1 - k_2) \cdot (x_j - x_l)} \frac{\varepsilon_1 \cdot p_j}{k_1 \cdot p_j} \frac{\varepsilon_2 \cdot p_j}{k_2 \cdot p_j} \frac{\varepsilon_1 \cdot p_l}{k_1 \cdot p_l} \frac{\varepsilon_2 \cdot p_l}{k_2 \cdot p_l}}{\sum_{j=1}^N \left(\frac{\varepsilon_1 \cdot p_j}{k_1 \cdot p_j} \right)^2 \sum_{l=1}^N \left(\frac{\varepsilon_2 \cdot p_l}{k_2 \cdot p_l} \right)^2} \quad (42)$$

Note that the probability for radiation of two photons can be rewritten as

$$P(k_1, \varepsilon_1, k_2, \varepsilon_2) = \frac{1}{N^2} \left[\sum_{j > l} \left| e^{ik_1 \cdot x_j} e^{ik_2 \cdot x_l} Q_j \frac{\varepsilon_1 \cdot p_j}{k_1 \cdot p_j} Q_l \frac{\varepsilon_2 \cdot p_l}{k_2 \cdot p_l} + e^{ik_2 \cdot x_j} e^{ik_1 \cdot x_l} Q_j \frac{\varepsilon_2 \cdot p_j}{k_2 \cdot p_j} Q_l \frac{\varepsilon_1 \cdot p_l}{k_1 \cdot p_l} \right|^2 + \sum_j \left| Q_j \frac{\varepsilon_1 \cdot p_j}{k_1 \cdot p_j} Q_j \frac{\varepsilon_2 \cdot p_j}{k_2 \cdot p_j} \right|^2 \right] \quad (43)$$

what can be interpreted as shown in Fig.9.

The first term in Eq.43 corresponds to the sum of probabilities of two photons emitted by two different charged particles, whereas the second term is sum of the emissions of two photons by the same particle.

Note that in our preceeding paper [9] we have incorrectly assigned an independent phase not to the sources of charged particles but to radiated photons. Instead of Eq.34 we have written only the first term and the sum over j, l included also the term $j = l$. The procedure for averaging over phases has been based on the rule - Eq.9 of Ref. [9]

$$\langle \exp i(\phi_i + \phi_j - \phi_k - \phi_l) \rangle = \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \quad (44)$$

This has enhanced the contribution of two photons radiated by the same charged particle. The results for cases considered in Ref. [9] are not considerably changed

since for large N the contributions of two photons from the same charged particle is suppressed by combinatorial factor anyway. The averaging over phases as given by Eq.44 leads to correct results provided that it is modified to

$$\langle \exp i(\phi_i + \phi_j - \phi_k - \phi_l) \rangle = \delta_{ik}\delta_{jl} + (1 - \delta_{ij})\delta_{il}\delta_{jk} \quad (45)$$

In our opinion the procedure based on Eqs.34 - 43 is transparent and correct conceptually and leads to the result in Eq.43 which is intuitively easily understandable.

In the next step we have to sum over polarizations of the photons. Following the same procedure as for photons radiated by two particles we obtain the correlation function

$$C(k_1, k_2) = 1 + \frac{\sum_{j \neq l} e^{i(k_1 - k_2) \cdot (x_j - x_l)} \langle \varepsilon_{1l} | \varepsilon_{1j} \rangle \langle \varepsilon_{2j} | \varepsilon_{2l} \rangle \frac{\varepsilon_{1j} \cdot p_j}{k_1 \cdot p_j} \frac{\varepsilon_{2j} \cdot p_j}{k_2 \cdot p_j} \frac{\varepsilon_{1l} \cdot p_l}{k_1 \cdot p_l} \frac{\varepsilon_{2l} \cdot p_l}{k_2 \cdot p_l}}{\sum_{j=1}^N \left(\frac{\varepsilon_{1j} \cdot p_j}{k_1 \cdot p_j} \right)^2 \sum_{l=1}^N \left(\frac{\varepsilon_{2l} \cdot p_l}{k_2 \cdot p_l} \right)^2} \quad (46)$$

where ε_{1j} is the true polarization of photon with momentum k_1 radiated by the j -th charged particle as defined by Eq.23.

We shall now consider spherical “fireball” emitting from its surface charged pions with energy of 400 MeV in random directions within the half-space tangential to the surface of the sphere in each of points. In the simulation we have taken 100 of pions emitted from randomly chosen points at the surface of the sphere with radius R .

In Figs. 10 and 11 we present the correlation function and effective radius obtained from HBT interference of bremsstrahlung photons.

The effective radius has been determined by comparing the results obtained by using Eq.46 with the “reference” correlation function defined as

$$C_{\text{ref}}(\Delta \vec{k}) = 1 + \kappa \exp \left(-(\Delta k)^2 \cdot r_{\text{eff}}^2 \right) \quad (47)$$

Parameter κ is chosen to fit the value of correlation function given by Eq.46 for $\Delta k = 0$.

The comparison has been done for Δk close to zero and r_{eff} has been determined by requiring the same second derivative of $C(\Delta k)$ following from Eq.46 and from $C_{\text{ref}}(\Delta k)$ at $\Delta k = 0$.

In Figs. 12 and 13 we show results of the same calculation under the assumption that velocities of all pions are perpendicular to the surface of the sphere.

6 A toy model of HBT correlation of synchrotron radiation

Recent most interesting experimental work with the synchrotron radiation at LEP [19, 20] has demonstrated the increasing possibilities of this type of studies. Moreover the results obtained in Ref. [20] have in fact demonstrated in a very clean way the Landau - Pomeranchuk - Migdal effect, completing previous results obtained at SLAC [21].

The synchrotron radiation is an incoherent sum of radiation due to individual charged particles, each particle radiating independently [22]. This indicates that the two photon interferometry of synchrotron radiation is a radiation of chaotic sources and may be described within the HBT approach. As a toy model of effects which might be expected in the case when both beams in a collider radiate photons through windows in both tubes we shall consider the following situation.

N particles of the same charge are emitted from the point $\vec{r}_a(0, 0, z_a)$ and their velocities $\vec{v}_a(0, 0, v)$ are parallel with the z -axis. At the same time $t = 0$, M particles of the opposite charge are emitted from the point $\vec{r}_b(0, 0, z_b)$ with velocity $\vec{v}_b(0, 0, -v)$. The amplitude of each of particles contains a random phase factor $\exp(i\phi_i)$ uncorrelated with phase factors of other particles. The situation is the same as in Fig.2 with the only difference that we have N particles moving right and M particles moving left.

The HBT correlation function is given by the general expression Eq.46. Using the notation of Eq.14 we obtain the probability for the radiation of single photon

$$\begin{aligned} P(\vec{k}_1) &= N|B(k_1, a)|^2 + M|B(k_1, b)|^2 \\ P(\vec{k}_2) &= N|B(k_2, a)|^2 + M|B(k_2, b)|^2 \end{aligned} \quad (48)$$

The joint probability distribution becomes

$$\begin{aligned} P(\vec{k}_1, \vec{k}_2) &= \sum_{n=1}^{N+M} |B(k_1, n)B(k_2, n)|^2 \\ &+ \sum_{n=1}^{N+M} \sum_{\substack{l=1 \\ l < n}}^{N+M} |B(k_1, n)B(k_2, l) + B(k_1, l)B(k_2, n)|^2 \end{aligned} \quad (49)$$

where in the summation from $n = 1$ to $n = N + M$ in the terms for $n = 1, \dots, N$ we have $n = a$ and for $n = N + 1, \dots, N + M$ we have $n = b$. In this way we obtain

$$\begin{aligned} P(k_1, k_2) &= N|B(k_1, a)B(k_2, a)|^2 + M|B(k_1, b)B(k_2, b)|^2 \\ &+ 2N(N-1)|B(k_1, a)B(k_2, a)|^2 + 2M(M+1)|B(k_1, b)B(k_2, b)|^2 \\ &+ NM|B(k_1, a)B(k_2, b) + B(k_1, b)B(k_2, a)|^2 \end{aligned} \quad (50)$$

For $N = M = 1$ we are back at the result given in Eq.7 and in the simplest kinematics of Fig.2 we obtain the correlation function of Eq.18.

For the case $N = M \gg 1$ we find, still for the simplest kinematics of Fig.2

$$C(k_1, k_2) = 1 + \frac{1}{2} \frac{(1 - \eta^2)^2}{(1 + \eta^2)^2} [1 + \cos(\Delta k \cdot \Delta z)] \quad (51)$$

For $N, M \gg 1$ and $M/N = \gamma$ we obtain

$$C(k_1, k_2) = 1 + \frac{(1 - \eta^2)^2 [\gamma^2 + 1 + 2\gamma \cos(\Delta k \cdot \Delta z)]}{\gamma [(1 + \eta)^4 + (1 - \eta)^4] + (\gamma^2 + 1)(1 - \eta^2)^2} \quad (52)$$

In both cases the effective radius increases with the increasing photon formation length $l_f = \frac{v}{\omega}$. In the former case we have

$$(\Delta z)_{\text{eff}}^2 = (\Delta z)^2 + 4 \frac{v^2}{\omega^2} \quad (53)$$

which differs only by a factor of 2 in front of the second term in the r.h.s. in comparison with the case on $N = M = 1$ as given in Eq.21.

In a more detailed analysis one should include the time factor describing in detail the passage of particles below the windows and include the length of windows, the rear side of windows playing the role of absorbers as in the discussion in Sect. 4. With the present simplified model we wish just to point out that the HBT interferometry of synchrotron radiation of colliders is probably the simplest and the cleanest way study the effects of the photon formation length as defined by HBT interferometry.

7 Comments and conclusions

In the present paper we have studied mainly the HBT interference of bremsstrahlung photons emitted by two charged particles. The coherence and incoherence of amplitudes for radiation of soft photons has been motivated by a simple model of multiparticle production when there are only two charged particles in the final state and numerous neutral ones. The resulting interference patterns are given by the specific energy and angular dependence of bremsstrahlung emission by charged particles and by the complexity of the intermediate state which justifies the coherence properties of different contributions to the amplitudes.

The results obtained can be in some cases easily interpreted in terms of the photon formation length and time, in the case of results presented in Figs.4 and 6 the interpretation requires further work, or alternatively this case might be indicating limits of such a straightforward interpretation.

In order to justify the assumptions of coherence and incoherence made, it would be most interesting to have models with increasing complexity which would permit us to understand how the incoherence develops and what are the values of the parameter κ which specifies the border between coherent and incoherent contributions. The problem is not a new one, it is with us since the classical paper [2]. The problem is connected with the one of a rapid thermalization in hadron collisions at very high energies. In this field an important progress has been achieved recently, for literature one could consult the paper by Elze [23]. According to the discussion in Sect.2 the coherence or incoherence of amplitudes in HBT interferometry is that of coherence of complex systems formed by taking different amounts of energy and momentum at different places of the original system.

The interference of bremsstrahlung photons in collisions at high energies is difficult to observe because of large background due to decays of neutral pions and other particles. If, however, these problems could be solved, the HBT interference of soft photons could bring a new information about the space-time evolution of hadronic collisions since low transverse momentum photons are not bound to a particular value of rapidity as hadrons emitted by a thermalized matter.

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Figure captions

Fig.1

Amplitudes for radiating two photons from two charged particles.

Fig.2

Notation of kinematics variables for bremsstrahlung radiation of two photons by two charged particles.

Fig.3

Production of the pair of photons with the direction of the vector $\vec{K}$ given by the angle ϑ .

Fig.4

The dependence of Δz_{eff} on the angle ϑ for $\Delta z = 0$ fm and $\omega = 10$ MeV.

Fig.5

The situation where charged particles 'a' and 'b' are produced in different directions, given by the angle φ . The photons are produced perpendicular to the plane given by vectors $\vec{v}_a$ and $\vec{v}_b$.

Fig.6

The dependence of Δz_{eff} on the angle φ . The dashed curve is the simplest expectation described in the text.

Fig.7

Charged particles 'a' and 'b' are emitted at $t = 0$ and absorbed (stopped) by absorbers A_a and A_b at a short distance from their sources.

Fig.8

- a) Correlation functions for $\Delta z = 2$ fm, $\omega = 10$ MeV and some values of the distance d from sources to absorbers.
b) The dependence of $\Delta z_{\text{eff}}(d)$ for $\Delta z = 2$ fm and $\omega = 10$ MeV.

Fig.9

Simple interpretation of Eq.43.

Fig.10

Correlation functions of bremsstrahlung produced by 100 charged pions emerging in random directions out from the random points of the surface of a sphere. The momenta of the pions are 400 MeV/c and the radius of the sphere is 3 fm. Correlation functions for five values of ω are plotted.

Fig.11

The dependence $r_{\text{eff}}(\omega)$ calculated in situation, where bremsstrahlung is produced by 100 pions with momentum 400 MeV/c. Pions are emitted in random directions from random points on the surface of the sphere with radius 3 fm. The values calculated from the correlation functions are fitted by the function $r_{\text{eff}} = \sqrt{R^2 + \frac{a}{\omega^2}}$, $a = 2.91(\hbar c)^2$.

Fig.12

Correlation functions for some values of energy of the bremsstrahlung photons. The photons are produced by 100 charged pions emitted from random points on the surface of the sphere in directions perpendicular to the surface. The momenta of pions are 400 MeV/c, the radius of the sphere is 3 fm.

Fig.13

The dependence $r_{\text{eff}}(\omega)$ as given by correlation functions calculated in situation, where the charged pions are emitted from the surface of the sphere with radius 3 fm in directions perpendicular to the surface. There are 100 pions with momentum 400 MeV/c. The values calculated from the correlation functions is fitted by the dependence $r_{\text{eff}} = \sqrt{R^2 + \frac{a}{\omega^2}}$, $a = 2.41(\hbar c)^2$.

Fig.1

Fig.2

Fig.3

Fig.4

Fig.5

Fig.6

Fig.7

a)

b)

Fig.8

$$\begin{aligned}
& \sum_{j \neq 1} \left| \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right|^2 \\
& + \sum_j \left| \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right|^2
\end{aligned}$$

The diagrams are Feynman diagrams with two vertices, x_j and x_l .
 - **Diagram 1** (for $j \neq 1$): A wavy line from x_j to x_l with momentum p_j and energy k_1, ϵ_1 . A wavy line from x_l to x_l with momentum p_l and energy k_2, ϵ_2 .
 - **Diagram 2** (for $j \neq 1$): A wavy line from x_j to x_l with momentum p_j and energy k_2, ϵ_2 . A wavy line from x_l to x_l with momentum p_l and energy k_1, ϵ_1 .
 - **Diagram 3** (for j): A wavy line from x_j to x_j with momentum p_j and energy k_1, ϵ_1 . A wavy line from x_j to x_j with momentum p_j and energy k_2, ϵ_2 .
 - **Diagram 4** (for j): A wavy line from x_j to x_j with momentum p_j and energy k_2, ϵ_2 . A wavy line from x_j to x_j with momentum p_j and energy k_1, ϵ_1 .

Fig.9

Fig.10

Fig.11

Fig.12

Fig.13