Abstract

We consider a supersymmetric hypercolor gauge theory with six flavors of quarks interacting strongly at the grand unification scale. Dynamical breaking of the grand unified $SU(5)_{\text{GUT}}$ produces a massless pair of composite color-triplet states. A use of the missing partner mechanism yields eventually a pair of massless Higgs doublets, giving large masses to the color-triplet partners. We prove that this pair of massless Higgs doublets survives quantum corrections and remains in the low-energy spectrum as far as the supersymmetry is unbroken. Hence this solves the most serious problem – doublet-triplet splitting in the grand unified theories. We also show that the dangerous dimension-five operators for nucleon decays are suppressed simultaneously, which makes our model yet more attractive.
1. Introduction

The grand unified theory (GUT) has attracted us for a long time since it was constructed in 1974 [1], because of its various interesting and promising features. In particular, the recent high-precision measurements on the Weinberg angle have shown a remarkable agreement [2] with the prediction of a supersymmetric (SUSY) extension [3] of the GUT. The success of the SUSY-GUT has, thus, led us to a serious reconsideration of SUSY-GUT models.

In the minimum SUSY-GUT, one must require an extremely precise adjustment of parameters in order to obtain a pair of light Higgs doublets [3]. This pair of light Higgses is an inevitable ingredient in the SUSY standard model. Therefore, the necessity of the fine-tuning of parameters seems to be a crucial drawback in the minimum SUSY-GUT.

There have been, in fact, several attempts [4, 5, 6, 7, 8] to have the light Higgs doublets without requiring the fine-tuning of parameters. In Ref.[8] it has been shown that the dynamics of SUSY QCD-like theory at the GUT scale naturally generates pairs of Higgs doublets at low energies. However, quantum effects have not been fully investigated in Ref.[8] and seven flavors of (hyper)quarks have been assumed in order to have a manifest symmetry to protect massless Higgs doublets from quantum corrections. Although this model is very interesting due to its natural accommodation of the Peccei-Quinn symmetry, the low-energy spectrum is rather involved with two pairs of light Higgs doublets.

In this paper we investigate the quantum dynamics of this QCD-like theory and show that a pair of massless Higgs doublets in the minimal model with six (hyper)quarks is indeed stable quantum mechanically. This makes the dynamical approach proposed in Ref.[8] yet more attractive. We also stress that the dangerous dimension-five operators for the nucleon decays are suppressed simultaneously in the present model.
2. Classical vacua and massless states

Our model is based on a supersymmetric hypercolor \(SU(3)_H \times U(1)_H\) gauge theory [8] with \(N_f\) flavors of quarks \(Q^A_\alpha\) in the 3 representation and antiquarks \(\bar{Q}^A_\alpha\) in the \(3^*\) representation of \(SU(3)_H\), where \(\alpha = 1, \cdots, 3\) and \(A = 1, \cdots, N_f\). The chiral multiplets \(Q^A_\alpha\) and \(\bar{Q}^A_\alpha\) have \(U(1)_H\) charges +1 and −1, respectively.

Before investigating the realistic case of \(N_f = 6\), we first consider the basic case of \(N_f = 5\). The anomaly-free flavor symmetry is then given by

\[
SU(5)_L \times SU(5)_R \times U(1)_V \times U(1)_R, \tag{1}
\]

under which the quark multiplets transform as

\[
Q^A_\alpha : (5, 1, +1, \frac{2}{5}) \\
\bar{Q}^A_\alpha : (1, 5, -1, \frac{2}{5}) \tag{2}
\]

(\(\alpha = 1, \cdots, 3; A = 1, \cdots, 5\)).

We note that \(U(1)_V\) is nothing other than \(U(1)_H\), but we regard it as a flavor group. The GUT gauge group \(SU(5)_{GUT}\) is also embedded in a part of the flavor group \(SU(5)_L \times SU(5)_R\). Namely, the quarks \(Q^A_\alpha\) and \(\bar{Q}^A_\alpha\) transform as \(5^*\) and 5 under \(SU(5)_{GUT}\), respectively.

When \(SU(5)_{GUT}\) is spontaneously broken down to the standard gauge group, there appear unwanted Nambu-Goldstone multiplets. To avoid them, we introduce a chiral multiplet \(\Sigma^A_B\) in the adjoint (24) representation of \(SU(5)_{GUT}\) [8]. Then, we have a superpotential

\[
W = \bar{Q}^A_\alpha (m_\delta^A_B + \lambda \Sigma^A_B)Q^B_\alpha + \frac{1}{2} m_\Sigma Tr(\Sigma^2), \tag{3}
\]

where \(m\) and \(m_\Sigma\) denote mass parameters. Here, we have omitted a \(Tr(\Sigma^3)\) term in the superpotential for simplicity, since the presence of this term does not change the conclusions in this paper.
The classical vacua satisfy the following equations:

\[
\hat{m}^A_B Q^B_\alpha = \bar{Q}^\alpha_B \hat{m}^B_A = 0, \\
m_{\Sigma} \Sigma^A_B = -\lambda \left\{ Q^A_\alpha \bar{Q}^{\alpha}_B - \frac{1}{5} \delta^A_B \text{Tr}(Q\bar{Q}) \right\},
\]

where

\[
\hat{m}^A_B = m \delta^A_B + \lambda \Sigma^A_B.
\]

Together with the $D$-term flatness condition for the gauge group $SU(5)_{GUT} \times SU(3)_H \times U(1)_H$, we find four distinct vacua. Since three of them are not interesting phenomenologically, we restrict our discussion to one vacuum given by (up to gauge and global rotations)

\[
Q^A_\alpha = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & v \end{pmatrix}, \quad \bar{Q}^\alpha_A = \begin{pmatrix} 0 & 0 & v & 0 \\ 0 & 0 & 0 & v \\ v & 0 & 0 & 0 \\ 0 & 0 & 0 & v \end{pmatrix},
\]

\[
\Sigma^A_B = \frac{m}{\lambda} \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & -1 & -1 \\ \frac{3}{2} & \frac{3}{2} & -1 & -1 \\ -1 & -1 & \frac{5 \ m \ m_{\Sigma}}{2 \ \lambda^2} & \frac{5 \ m \ m_{\Sigma}}{2 \ \lambda^2} \\ -1 & -1 & \frac{5 \ m \ m_{\Sigma}}{2 \ \lambda^2} & \frac{5 \ m \ m_{\Sigma}}{2 \ \lambda^2} \end{pmatrix}; \quad v = \sqrt{\frac{5 \ m \ m_{\Sigma}}{2 \ \lambda^2}}.
\]

The vacuum-expectation values in Eq.(6) break the original gauge group down to the standard gauge group:

\[
SU(5)_{GUT} \times SU(3)_H \times U(1)_H \to SU(3)_C \times SU(2)_L \times U(1)_Y.
\]

Here, $U(1)_Y$ is a linear combination of $U(1)_H$ and a $U(1)$ subgroup of $SU(5)_{GUT}$. The GUT unification of three gauge coupling constants of $SU(3)_C \times SU(2)_L \times U(1)_Y$ is achieved in the limit $g_{1H} \to \infty$, where $g_{1H}$ is the gauge coupling constant of the hypercolor $U(1)_H$ [8]. If one requires the GUT unification by 2% accuracy, one
gets a constraint \( \alpha_{1H} \geq 0.06 \) for \( \alpha_5 \simeq 1/25 \) at the GUT scale (see Ref.[8] for the normalization of \( \alpha_{1H} \)) [9].

It is amusing that there is no flat direction and hence no massless state, which results from the fact that the superpotential (3) breaks explicitly the flavor symmetry (1) down to \( SU(5)_{GUT} \times U(1)_H \). Nambu-Goldstone multiplets transforming as \((3^*, 2)\) and \((3, 2)\) under \( SU(3)_C \times SU(2)_L \) are absorbed in the \( SU(5)_{GUT} \) gauge multiplets to form massive vector multiplets.

Let us now turn to the \( N_f = 6 \) case with an additional pair of quark \( Q^6_\alpha \) and antiquark \( \bar{Q}^6_\alpha \), whose mass term is forbidden by imposing an axial \( U(1)_A \) symmetry

\[
Q^6_\alpha \rightarrow e^{i\xi}Q^6_\alpha, \quad \bar{Q}^6_\alpha \rightarrow e^{i\xi}\bar{Q}^6_\alpha. \tag{8}
\]

These massless quarks have flat directions since they do not have a superpotential with \( \Sigma_{AB} \) and hence there are infinitely degenerate vacua. Around the vacuum in Eq.(6), the flat directions are given by

\[
Q^6_\alpha = (0, 0, w), \quad \bar{Q}^6_\alpha = \begin{pmatrix} 0 \\ 0 \\ we^{i\delta} \end{pmatrix}. \tag{9}
\]

When \( w = 0 \), the standard gauge group \( SU(3)_C \times SU(2)_L \times U(1)_Y \) remains unbroken. We assume that SUSY-breaking effects choose it as a true vacuum, avoiding \( w \neq 0 \) where the color \( SU(3)_C \) would be broken.

It is remarkable that there is a pair of massless color-triplets \( Q^6_\alpha \) and \( \bar{Q}^6_\alpha \) in the above vacuum at \( w = 0 \). Notice that the unbroken color \( SU(3)_C \) is a diagonal subgroup of the hypercolor \( SU(3)_H \) and an \( SU(3) \) subgroup of \( SU(5)_{GUT} \) and the original \( SU(3)_H \) quarks \( Q^6_\alpha \) and \( \bar{Q}^6_\alpha \) transform as \( 3 \) and \( 3^* \) under the color \( SU(3)_C \). The presence of massless color-triplets \( Q^6_\alpha \) and \( \bar{Q}^6_\alpha \) is a crucial point for obtaining a pair of light Higgs doublets as seen in section 4.

Before proceeding to the quantum analysis of the vacuum chosen above, two remarks are in order here:
i) The classical moduli space of vacua has a hypercolor-gauge invariant description in terms of the observable “meson” $M$ and “baryons” $B$ and $\bar{B}$:

$$M_{AB}^A = Q_{\alpha}^{A} \bar{Q}_{\beta}^{A},$$

$$B_{[ABC]} = \frac{1}{3!} \epsilon_{\alpha \beta \gamma} Q_{\alpha}^{A} Q_{\beta}^{B} Q_{\gamma}^{C},$$

$$\bar{B}_{[ABC]} = \frac{1}{3!} \epsilon_{\alpha \beta \gamma} \bar{Q}_{\alpha}^{A} \bar{Q}_{\beta}^{B} \bar{Q}_{\gamma}^{C}. $$

(10)

The vacua corresponding to Eqs.(6) and (9) are given by

$$M_{AB}^A = \begin{pmatrix}
0 & v^2 & v^2 \\
0 & v^2 & v^2 \\
v^2 & v^2 & v w e^{i\delta} \\
v w & v w & w^2 e^{i\delta}
\end{pmatrix},$$

$$B_{[3,4,5]}^{[3,4,5]} = \bar{B}_{[3,4,5]} = v^3,$$

$$B_{[3,4,6]} = v^2 w, \quad \bar{B}_{[3,4,6]} = v^2 w e^{i\delta}$$

(11)

with all the other components of $B$ and $\bar{B}$ vanishing.

It is clear that the flavor gauge group $SU(5)_{GUT} \times U(1)_H$ is spontaneously broken down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ in the $w = 0$ vacuum. Then we have a pair of massless composite states

$$M_6^a = Q_6^a \bar{Q}_a^6, \quad M_6^a = Q_6^a \bar{Q}_6^a \quad (a = 3, \cdots, 5),$$

(12)

which are $3$ and $3^*$ of the color $SU(3)_C$, respectively.
ii) We note that there is another interesting vacuum in the present model:

\[
Q^A_\alpha = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
v & 0 & 0 \\
0 & 0 & v
\end{pmatrix}, \quad \bar{Q}^\alpha_\alpha = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & v & 0 & 0 \\
0 & 0 & 0 & v & 0
\end{pmatrix},
\]

\[
\Sigma^A_B = \frac{m}{\lambda} \begin{pmatrix}
\frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\
\frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\
-1 & -1 & -1
\end{pmatrix}; \quad v = \sqrt{\frac{5 m m_{\Sigma}}{3 \lambda^2}}.
\]

In this vacuum $U(1)_Y$ is a diagonal subgroup of $U(1)'s$ in $SU(5)_{GUT}$ and $SU(3)_H$ and hence the additional $U(1)_H$ is not necessary. However, unwanted massless states $Q^6_1$ and $\bar{Q}^1_6$ exist in addition to a pair of $SU(2)_L$ doublets $Q^6_\alpha$ and $\bar{Q}^\alpha_6$ ($\alpha = 2, 3$) which may be identified with the Higgs multiplets in the standard model. These massless multiplets $Q^6_1$ and $\bar{Q}^1_6$ have $U(1)_Y$ charges $-1$ and $+1$, respectively, and give an additional contribution to the renormalization-group equations for gauge coupling constants. This destroys the success of gauge coupling unification in the SUSY-GUT. Thus we do not investigate this vacuum in this paper [10].
3. Quantum vacua and an effective superpotential

Let us analyze quantum effects on the classical vacua given in Eqs.(6) and (9). We see that the effective mass matrix for quarks has four zero eigenvalues in our vacua:

\[
\hat{m}' = \begin{pmatrix}
\hat{m} & \hat{m} \\
0 & 0
\end{pmatrix} = \begin{pmatrix}
\frac{5}{2} m & \frac{5}{2} m \\
0 & 0 \\
0 & 0
\end{pmatrix}.
\]

(14)

Therefore, our model becomes a SUSY QCD-like theory with the effective \( N_f = 4 \) at low energies.

Expanding the \( \Sigma \) fields around the values \( \langle \Sigma \rangle \) given in Eq.(6), we write the tree-level superpotential as

\[
W = \bar{Q}_i \left( \frac{5}{2} m \delta^j_i + \lambda \sigma^j_i \right) Q^i_a + \lambda \bar{Q}^a_i (\sigma^a_i) Q^b_a + \lambda \bar{Q}^a_i (\sigma^a_i) Q^b_a \\
+ \frac{\lambda}{\sqrt{30}} \sigma_0 \left( 3 \bar{Q}^a_i Q^i_a - 2 \bar{Q}^a_i Q^a_i \right) + \frac{1}{2} m_\Sigma \left( Tr(\sigma^2) + \sigma^2_0 \right) + \frac{\sqrt{30}}{2} \frac{m m_\Sigma}{\lambda} \sigma_0 \\
\quad (i, j = 1, 2; \ a, b = 3, \cdots, 5),
\]

(15)

where \( \sigma \) and \( \sigma_0 \) are defined by

\[
\Sigma^A_B = \langle \Sigma^A_B \rangle + \sigma^A_B + \frac{1}{\sqrt{30}} \begin{pmatrix}
3 & 3 \\
-2 & -2 \\
-2 & -2
\end{pmatrix} \sigma_0 ,
\]

(16)

\[
sigma^A_B = \begin{pmatrix}
\sigma^i_j & \sigma^i_b \\
\sigma^b_j & \sigma^b_b
\end{pmatrix} ; \quad Tr \sigma^i_j = Tr \sigma^a_b = 0 \\
\quad (A, B = 1, \cdots, 5).
\]
Notice that there is a tadpole term for $\sigma_0$ in Eq.(15), which is canceled out by the $\bar{Q}_a^\alpha Q_a^\alpha$ condensation.

We now think of our model as the QCD-like theory with two massive ($Q_i^\alpha$ and $\bar{Q}_i^\alpha$) and four massless ($Q_a^\alpha$ and $\bar{Q}_a^\alpha$; $a = 3, \cdots, 6$) quarks interacting with the $\sigma$ fields. Then we can integrate out the massive quarks and irrelevant $\sigma$ fields [11] to get the low-energy effective theory described by four massless quarks with the superpotential

$$W_{\text{low}} = \lambda \bar{Q}_a^\alpha \left( \sigma_{ab}^\alpha - \frac{2}{\sqrt{30}} \sigma_0 \delta_{ab}^\alpha \right) Q_b^\alpha$$

$$+ \frac{1}{2} m_{\Sigma} \left\{ Tr(\sigma^2) + \sigma_0^2 \right\} + \frac{\sqrt{30} \ m \ m_{\Sigma}}{2 \lambda} \sigma_0$$

$$(a, b = 3, \cdots, 5).$$

(17)

Here we have eliminated the Nambu-Goldstone modes stemming from breakdown of the flavor symmetry, since they will be absorbed in the gauge multiplets of $SU(5)_{\text{GUT}}$ to form massive vector multiplets.

We next proceed to find out the effective superpotential described by the composite meson $M$ and baryons $B$ and $\bar{B}$ interacting with the $\sigma$ fields. It is highly nontrivial to obtain the superpotentials dynamically generated by strong interactions. However, it has become clear recently that the effective superpotentials can be exactly determined for certain classes of SUSY nonabelian gauge theories [14, 15]. In particular, for the QCD-like theory with $N_f = N_c + 1$ flavors of quarks where $N_c$ is the number of colors, the quantum moduli space of vacua is the same as the classical one and the effective superpotential at low energies is uniquely determined [15].

According to Ref.[14, 15], we obtain the effective superpotential for our model from Eq.(17) as

$$W_{\text{eff}} = W_{\text{dyn}} + \lambda Tr(M\bar{\sigma}) + \frac{1}{2} m_{\Sigma} \left( Tr(\sigma^2) + \sigma_0^2 \right) + \frac{\sqrt{30} \ m \ m_{\Sigma}}{2 \lambda} \sigma_0;$$

$$W_{\text{dyn}} = \Lambda^{-5}(B_a M_{ab} \bar{B}^b - \det M),$$

(18)
where $M^a_b$, $B_a$ and $\bar{B}^a$ are composite meson and baryon states in the $N_f = 4$ case:

\begin{align}
M^a_b & \sim Q^a_{\alpha} \bar{Q}^a_{\beta}, \\
B_a & \sim \left(\frac{1}{3!}\right)^2 \epsilon^{\alpha\beta\gamma} \epsilon_{abcd} Q^b_{\alpha} Q^c_{\beta} Q^d_{\gamma}, \\
\bar{B}^a & \sim \left(\frac{1}{3!}\right)^2 \epsilon_{\alpha\beta\gamma} \epsilon^{abcd} \bar{Q}^a_{\alpha} \bar{Q}^c_{\beta} \bar{Q}^d_{\gamma}
\end{align}

(a, b = 3, \ldots, 6). \tag{19}

$\Lambda$ denotes the dynamical scale of $SU(3)_H$, and $\tilde{\sigma}$ is defined by

\begin{equation}
\tilde{\sigma} = \begin{pmatrix}
\sigma_{ab}^a \\
0
\end{pmatrix} - \frac{2}{\sqrt{30}} \begin{pmatrix}
1 & 1 \\
1 & 0
\end{pmatrix} \sigma_0. \tag{20}
\end{equation}

From Eq.(18) we find that the quantum vacua satisfy

\begin{align}
M^a_b \bar{B}^b = B_a M^a_b = 0, \\
-\frac{\partial \det M}{\partial M^a_b} + B_a \bar{B}^b + \Lambda^5 \lambda \tilde{\sigma}^a_b = 0,
\end{align}

(21)

for $a, b = 3, \ldots, 6$;

\begin{align}
m_\Sigma \sigma_{ab}^a + \lambda \{ M^a_b - \delta^a_b Tr(M^a_b) \} &= 0, \\
m_\Sigma \sigma_0 + \frac{\sqrt{30}}{2} \frac{m_m}{\lambda} - \frac{2}{\sqrt{30}} \lambda Tr(M^a_b) &= 0,
\end{align}

(22)

for $a, b = 3, \ldots, 5$. Using the $D$-term flatness condition for the flavor gauge group $SU(5)_{GUT} \times U(1)_H$ together with Eqs.(21) and (22), we obtain the quantum vacua which are exactly the same as those given in Eq.(11) for the classical theory. We take, as before, the $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant vacuum at $w = 0$.

In this vacuum, the mass matrix for color-triplet states $M$, $B$ and $\bar{B}$ is given by

\begin{equation}
(M^6_a \ B_a) \left(\begin{array}{cc}
\frac{v^4}{\Lambda^5} & -\frac{v^3}{\Lambda^5} \\
\frac{v^3}{\Lambda^5} & \frac{v^2}{\Lambda^5} \\
-\frac{v^2}{\Lambda^5} & \frac{v^1}{\Lambda^5}
\end{array}\right) \left(\begin{array}{c}
M^a_6 \\
\bar{B}^a
\end{array}\right).
\end{equation}

(23)
Notice that $M$ and $B$ ($\bar{B}$) have canonical dimensions two and three, respectively. A pair of massless color-triplet states is now a mixture of $M, B$ and $\bar{B}$ with its orthogonal states having GUT scale masses. We will see later that the structure of the mass matrix for these color-triplet states is crucial for suppression of the dangerous dimension-five operators for nucleon decays.

The Nambu-Goldstone modes in $M_{ab}^{a} \ (a, b = 3, \cdots, 5)$ get tree-level masses through the interactions (the $\lambda$ coupling term) with $\sigma$ and $\sigma_{0}$ given in Eq.(18). Thus there are no other massless composite states besides the $SU(3)_{C}$ triplets observed in the mass matrix (23).

4. **Missing partner mechanism for light Higgs doublets**

We introduce a standard Higgs multiplets, $H_{A}$ and $\bar{H}^{A}$ (with $A = 1, \cdots, 5$), in the fundamental representations 5 and $5^{\ast}$ of $SU(5)_{GUT}$. The mass term for them is forbidden by the axial $U(1)_{A}$ symmetry given in Eq.(8). Assuming their transformation property under $U(1)_{A}$ as

$$H_{A} \rightarrow e^{-i\xi}H_{A}, \ \bar{H}^{A} \rightarrow e^{-i\xi}\bar{H}^{A},$$

we have a superpotential

$$W_{H} = hH_{A}Q_{A}^{\alpha}\bar{Q}_{6}^{\alpha} + h'\bar{H}^{A}\bar{Q}_{A}^{\alpha}Q_{6}^{\alpha}.$$  

These Yukawa interactions induce the following mass terms at low energies:

$$W_{mass} = hH_{a}M_{a}^{6} + h'\bar{H}^{a}M_{a}^{6}$$  

for $a = 3, \cdots, 5$. Thus the color-triplet components of $H_{A}$ and $\bar{H}^{A}$ denoted by $H_{a}$ and $\bar{H}^{a}$ ($a = 3, \cdots, 5$) become massive together with the massless composites in $M_{a}^{6}, M_{a}^{6}, B_{a}$ and $\bar{B}^{a}$. On the other hand, the doublet components of $H_{A}$ and $\bar{H}^{A}$ denoted by $H_{i}$ and $\bar{H}^{i}$ ($i = 1, 2$) remain massless.
This missing partner mechanism is easily understood in terms of the Higgs phase, where $Q_a^\alpha$ and $\bar{Q}_a^\alpha$ have vacuum-expectation values for $a = 3, \ldots, 5$ as shown in Eq.(6). Then, it is clear from Eq.(25) that the color-triplets $H_a$ and $\bar{H}_a$ get masses together with $\bar{Q}_6^\alpha$ and $Q_6^\alpha$, respectively. On the other hand, the $SU(2)_L$-doublet Higgses $H_i$ and $\bar{H}^i$ have no partners to form masses and the $H_i\bar{H}^i$ mass term itself is forbidden by the axial $U(1)_A$ in Eq.(24). However, we need a more careful analysis at the quantum level, since the axial $U(1)_A$ is broken by the hypercolor $SU(3)_H$ instantons.

We now prove that the Higgs doublets $H_i$ and $\bar{H}^i$ are exactly massless even at the quantum level in the limit of SUSY being exact. We first integrate out the massive quarks $Q_i^\alpha$ and $\bar{Q}_i^\alpha$ to get the low-energy superpotential (neglecting the irrelevant $\sigma$ fields)

$$W'_\text{low} = W_\text{low} + h H_a Q_a^\alpha \bar{Q}_6^\alpha + h' \bar{H}^\alpha Q^6_a Q^\alpha + \frac{2hh'}{5m} H_i \bar{H}^i Q^\alpha_6 \bar{Q}_6^\alpha ,$$  \hspace{1cm} (27)

where $W_\text{low}$ is given in Eq.(17). Using the methods proposed in Ref.[14, 15], we obtain the effective superpotential

$$W'_\text{eff} = W_\text{eff} + h H_a M^\alpha_a \bar{Q}_6^\alpha + h' \bar{H}^\alpha M^6_a Q^\alpha + \frac{2hh'}{5m} H_i \bar{H}^i M^6_6 ,$$ \hspace{1cm} (28)

where $W_\text{eff}$ is given in Eq.(18). Since $M^6_6$ vanishes in the present vacuum, no mass term for the Higgs doublets $H_i$ and $\bar{H}^i$ is generated.

This important conclusion is also understood if one notices that the hypercolor-anomaly (instanton) effects are independent of any Yukawa coupling constants $\lambda$, $h$ and $h'$, and hence they are present only in the dynamical part $W_\text{dyn}$ in $W'_\text{eff}$. Therefore, the Yukawa-coupling dependent parts of $W'_\text{eff}$ must be $U(1)_A$-invariant, which shows that the $H_i\bar{H}^i$ terms are always accompanied by $M^6_6$ as seen in Eq.(28). On the other hand, the dynamical part $W_\text{dyn}$ does not have the $H_i\bar{H}^i$ term. This is easily proved by means of a higher symmetry in the limit of $h = h' = 0$ (e.g. $H_i \to e^{i\alpha} H_i$, $\bar{H}^i \to e^{i\beta} \bar{H}^i$ and all the other fields intact).
Note that the sixth quarks will become massive when the Higgs doublets $H_i$ and $\bar{H}^i$ acquire the vacuum expectation values at the electroweak scale, which is consistent with the choice $w = 0$ in Eq.(11).

5. Discussions

We have shown in this paper that our QCD-like theory with six quark flavors generates one pair of massless Higgs doublets naturally. These Higgs doublets survive all the quantum corrections and remain in the massless spectrum. The masslessness of the original Higgs multiplets, $H_A$ and $\bar{H}^A$ ($A = 1, \cdots, 5$), is understood by the axial $U(1)_A$ symmetry. Although this axial $U(1)_A$ is broken by the hypercolor instantons, the presence of massless Higgs doublets has been proved by means of the nonrenormalization theorem in SUSY theories [14, 15].

It is remarkable that nucleon decays due to the dangerous dimension-five operators are suppressed in our model. These operators are induced by exchanges of $SU(3)_C$ triplet Higgs multiplets $H_a$ and $\bar{H}^a$ [16]. Owing to (23) and (26), the mass matrix for the color-triplet sector is given by

$$
(H_a \ M^6_a \ B_a) \hat{m}_C \begin{pmatrix} \bar{H}^a \\ M^6_a \\ B^a \end{pmatrix},
$$

(29)

where

$$
\hat{m}_C = \begin{pmatrix} 0 & h & 0 \\ h' & v^4 & -v^3 \\ 0 & v^3 & -v^2 \\ 0 & v^2 & -v^3 \end{pmatrix},
$$

(30)

The dimension-five operators are proportional to $(\hat{m}_C^{-1})_{11}$, which is none other than zero. This conclusion can also be understood in terms of the Higgs phase: the mass for $Q^6_\alpha$ and $\bar{Q}^6_\alpha$ is not generated even in the presence of instanton effects and hence
there is no transition matrix between $H_a$ and $\bar{H}^a$. We note that this is consistent with the result by Affleck, Dine and Seiberg [17].

The GUT unification of three gauge coupling constants of $SU(3)_C \times SU(2)_L \times U(1)_Y$ is achieved in the strong coupling limit of $U(1)_H$. The necessity of the strong $U(1)_H$ is a possible drawback in the present model. However, it is very hard to distinguish phenomenologically the standard SUSY-GUT and our model, since the $SU(3)_C$ gauge coupling constant has experimental errors of several percent (see Ref.[8]). Moreover, some threshold corrections from GUT-scale particles also give a few percent ambiguity to the renormalization-group equations [18].

From a theoretical point of view, it may be a problem that the electromagnetic charge quantization is not an automatic consequence in our model. However, this can be solved by assuming the hypercolor $SU(3)_H \times U(1)_H$ to be embedded in, for example, $SU(4)_H$ at some higher scale. This extension of the hypercolor group also solves the problem that the gauge coupling constant of $U(1)_H$ blows up below the Planck scale. This possibility together with a phenomenological analysis will be considered in a future communication [19].

In the present paper we have assumed the global SUSY theory. In the framework of supergravity, the flat directions ($w$ and $we^{i\delta}$) in Eq.(9) may be lifted substantially when they reach the Planck scale. However, effects from such vacuum shifts are negligibly small near the origin ($w = 0$) of the flat directions.

In our model the global $U(1)_A$ plays a central role for having a pair of massless Higgs doublets. Such a global symmetry may be, in general, broken by topology-changing wormhole effects [20]. If it is the case, the pair of Higgs doublets is no longer massless. The magnitudes of these induced operators are, however, not known and hence we assume that these effects are sufficiently suppressed.

Finally we make a comment on the basic structure of our model. We have assumed a mass term for the first five quarks as seen in Eq.(3). However, we can obtain the same result by introducing a coupling with a singlet field $\phi$ instead of the mass term. In such a model the vacuum-expectation value of $\phi$ plays a role
of the mass $m$. This model seems very intriguing, since it may be regarded as a massless QCD-like theory with nonrenormalizable interactions if one integrates out the $\phi$ and $\Sigma$ multiplets.
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[9] The color $SU(3)_C$ is also a linear combination of the hypercolor $SU(3)_H$ and an
$SU(3)$ subgroup of $SU(5)_{GUT}$. Thus a similar consideration applies for $SU(3)_H$. 
whose coupling is strong enough under our setting of the confining $SU(3)_H$ at the GUT scale.

[10] If one introduces a pair of $10$ and $10^*$ of $SU(5)_{GUT}$ and assumes a superpotential $W = \frac{\lambda'}{M} \epsilon^{\alpha\beta\gamma} Q^A_{\alpha} Q^B_{\beta} Q^6_{\gamma} \cdot 10_{AB} + \frac{\lambda''}{M} \epsilon^{\alpha\beta\gamma} \bar{Q}^\alpha_{A} \bar{Q}^\beta_{B} \bar{Q}^\gamma_{6} \cdot 10^{*AB}$, the unwanted singlets $Q^6_1$ and $\bar{Q}^1_{6}$ may have large masses in the vacuum Eq.(13). However, we do not pursue such a possibility here.

[11] The equations of motion for massive quarks lead to $Q^i_\alpha = \bar{Q}^\alpha_i = 0 \quad (i = 1, 2)$. From the Konishi-anomaly relation [12], we see that the massive quarks have vanishing condensations, i.e. $\bar{Q}^\alpha_i Q^i_\alpha = 0$ even at the quantum level, which are consistent with the results implied by the equations of motion. We note that they are also obtained by means of the dual theory [13] to our model.


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