THE QED RUNNING COUPLING AND DAΦNE

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2. The Muon Anomalous Magnetic Moment
3. The Hadronic Contribution to $\alpha(m_Z)$

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1. Introduction.

The KLOE experiment at DAΦNE can be very useful in reducing the experimental errors on the hadronic cross section in e^+e^- annihilation at energies below 1.4 GeV. This in turn will allow a sizeable reduction of the theoretical errors associated with the hadronic contribution to the photon vacuum polarisation. This progress will be of great importance for reducing the theoretical error for the muon g-2, a_μ [1], and, to a lesser extent, for the determination of α(m^2), which is very important for precision tests of the Standard Model [2]. In the following I will describe the status and the relevance of these problems.

2. The Muon Anomalous Magnetic Moment

The muon anomalous magnetic moment provides a very precise QED test. The present experimental value is [3]:

\[ a_μ^{\text{exp}} = (11659230 \pm 85) \times 10^{-10} \]  

(1)

The comparison with the theoretical prediction gives [4]:

\[ 10Xp_{\text{ai}} = (11659230 \pm 85) \times 10^{-10} \]
The uncertainty is related to the parameters.

\[ \Delta a^\text{exp}_\mu = \pm 4 \times 10^{-10} \]  

A forthcoming Brookhaven experiment [5] aims at bringing the experimental accuracy down to

\[ a^\mu_{\text{had}} \approx 20 \times 10^{-10} \]  

The theoretical prediction can be decomposed as follows

\[ a^\mu_{\text{had}} = a^\mu_{\text{QED}} + a^\mu_{\text{had}} + a^\mu_{\text{weak}} + ? \]  

where the dots stand for possible small exotic contributions. The weak interaction term is estimated to be [6]:

\[ a^\mu_{\text{weak}} \approx 20 \times 10^{-10} \]  

The problem we address here is on the magnitude and the error associated to \( a^\mu_{\text{had}} \). In fig.1a-b two typical diagrams contributing to \( a^\mu_{\text{had}} \) are displayed: the hadronic vacuum

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Fig. 1: Diagrams contributing to \( a^\mu_\mu \): a) photon vacuum polarisation, b) light by light scattering. The uncertainty is related to the corresponding hadronic contributions.
polarisation insertion (a) (multiple exchanges of the dressed photon line have also been considered) and the light by light scattering contribution (b). The largest term arises from the contribution in (a) which can be cast into the form [7]

\[
a_{\mu} = \left( \frac{\alpha m_{B}}{3 \pi} \right)^{2} \int_{4m_{\pi}^{2}}^{\infty} \frac{d s}{s^{2} R(s) K(s)}
\]

(6)

The kernel \( K(s) \) is a smooth function that varies monotonically from 0.63 to 1 in the integration range. \( R(s) \) is the ratio between the total hadronic cross section in \( e^+e^- \) annihilation and the pointlike \( \mu^+\mu^- \) cross section. The \( s^2 \) denominator strongly emphasises the low energy part of the integration range. The integral can be evaluated starting from the existing data on \( e^+e^- \) annihilation into hadronic final states. The evolution of this procedure over the times is displayed in table 1, taken from ref. 7.

**Table 1**

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year and Ref.</th>
<th>( a_{\mu} \times 10^{10} ) had</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barger et al</td>
<td>1975 [ref. 16 in ref. 7]</td>
<td>663 ± 85</td>
</tr>
<tr>
<td>Barkov et al</td>
<td>1985 [ref. 17 in ref. 7]</td>
<td>684 ± 11</td>
</tr>
<tr>
<td>Kinoshita et al</td>
<td>1985 [ref. 18 in ref. 7]</td>
<td>707 ± 17</td>
</tr>
<tr>
<td>Casas et al</td>
<td>1985 [ref. 19 in ref. 7]</td>
<td>710 ± 11</td>
</tr>
<tr>
<td>Dubnicka et al</td>
<td>1990 [ref. 20 in ref. 7]</td>
<td>705 ± 8</td>
</tr>
<tr>
<td>Jegerlehner et al</td>
<td>1991 [ref. 21 in ref. 7]</td>
<td>724 ± 27</td>
</tr>
<tr>
<td>Dubnickova et al</td>
<td>1992 [ref. 22 in ref. 7]</td>
<td>699 ± 5</td>
</tr>
</tbody>
</table>

The present best estimate is given by:

\[
a_{\mu} = (725 \pm 16) \times 10^{-10}
\]

(7)
which corresponds to the last line of table 1. The improvement from the Renormalisation Group consists in performing the Dyson resummation of the iterated vacuum polarisation bubbles leading to the replacement

$$\alpha \left( \frac{1}{k^2} + \frac{1}{k^2} \frac{1}{\Pi} \frac{1}{k^2} + \ldots \right) = \frac{1}{k^2} \frac{\alpha}{1-\Pi} = \frac{\alpha(k^2)}{k^2}$$

(8)

where \( \Pi \) is the vacuum polarisation function and \( \alpha(k^2) \) is the running QED coupling.

A conspicuous part of the error arises from the integration in the energy range \( 2m_\pi \) - 1.4 GeV that can be scanned at DA\( \Phi \)NE and also at VEPP-2M. This can be appreciated from table 2, also taken from ref. 7.

<table>
<thead>
<tr>
<th>final state</th>
<th>energy range (GeV)</th>
<th>contribution (stat) (syst)</th>
<th>rel. err.</th>
<th>abs. err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>(0.28, 0.81)</td>
<td>426.66 (5.61) (10.62)</td>
<td>2.8%</td>
<td>1.7%</td>
</tr>
<tr>
<td>( \omega )</td>
<td>(0.42, 0.81)</td>
<td>37.76 (0.45) (1.02)</td>
<td>3.0%</td>
<td>0.2%</td>
</tr>
<tr>
<td>( \phi )</td>
<td>(1.00, 1.04)</td>
<td>38.55 (0.54) (0.89)</td>
<td>2.7%</td>
<td>0.1%</td>
</tr>
<tr>
<td>( J/\psi )</td>
<td></td>
<td>8.60 (0.41) (0.40)</td>
<td>6.7%</td>
<td>0.1%</td>
</tr>
<tr>
<td>( \Upsilon )</td>
<td></td>
<td>0.10 (0.00) (0.01)</td>
<td>6.7%</td>
<td>0.0%</td>
</tr>
<tr>
<td>hadrons</td>
<td>(0.81, 1.40)</td>
<td>112.85 (1.33) (5.49)</td>
<td>5.0%</td>
<td>0.8%</td>
</tr>
<tr>
<td>hadrons</td>
<td>(1.40, 3.10)</td>
<td>56.43 (0.45) (7.22)</td>
<td>12.8%</td>
<td>1.0%</td>
</tr>
<tr>
<td>hadrons</td>
<td>(3.10, 3.60)</td>
<td>4.47 (0.23) (0.86)</td>
<td>19.9%</td>
<td>0.1%</td>
</tr>
<tr>
<td>hadrons</td>
<td>(3.60, 9.46)</td>
<td>14.06 (0.07) (0.90)</td>
<td>6.5%</td>
<td>0.1%</td>
</tr>
<tr>
<td>hadrons</td>
<td>(9.46, 40.0)</td>
<td>2.70 (0.03) (0.13)</td>
<td>4.9%</td>
<td>0.0%</td>
</tr>
<tr>
<td>perturbative</td>
<td>(40.0, ( \infty ))</td>
<td>0.16 (0.00) (0.00)</td>
<td>0.2%</td>
<td>0.0%</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td>702.35 (5.85) (14.09)</td>
<td>2.2%</td>
<td>2.2%</td>
</tr>
</tbody>
</table>

Table 2: Contributions to \( a_\mu \cdot 10^{10} \)
In particular a large uncertainty comes from the $\rho$ region where some discrepancies among the existing data are present [7]. If DAΦNE succeeds to reduce to a negligible level the error from the energy range below 1.4 GeV, then the remaining uncertainty will be:

$$\Delta a_\mu = \pm 8 \times 10^{-10}$$

(9)

The light by light contribution shown in fig.1b is more model dependent. It cannot be determined directly starting from the data because there are no data. A theoretical prediction is difficult because it involves QCD in a non perturbative domain at small values of the energy scale. Back in 1985 Kinoshita et al [8] attempted to evaluate the contribution of the light by light contribution to $a_\mu$ from $\pi^\pm$ and $\pi^0$ exchange, while $K$ and $\eta$ exchange were neglected. Consistency with the alternative approach in terms of quark exchange was claimed. The result of ref.8 was:

$$a_\mu = (4.9 \pm 0.5) \times 10^{-10}$$

(10)

Recently, at the DAΦNE Workshop in 1992, the quoted accuracy was questioned by Barbieri and Remiddi [9]. They argued that, on the one hand, large cancellations take place among (external $\gamma$) gauge invariant terms in the evaluation in terms of $\pi^\pm$ and $\pi^0$ exchange. On the other hand the quark exchange determination is very much dependent of the quark mass values that are assumed. They concluded that the error in eq.10 is underestimated to such an extent as to possibly by far exceed the accuracy of the Brookhaven experiment. This important issue was reanalysed by Einhorn [10] and, independently, by de Rafael [11] in 1994. They agree that the error given by Kinoshita is underestimated but argue that the ambiguity is not as large as claimed in ref.9. In fact Einhorn points out that the large cancelling terms are gauge invariant for a gauge change on the external photon only. They are not gauge invariant under the separate transformations in all the four indices of the light by light scattering amplitude. In other words, the cancelling terms do not satisfy Bose statistics in the external photons. Thus the individual terms do not possess a definite physical significance. Einhorn considers the $\pi^\pm$ exchange amplitude more reliably computed than the one from $\pi^0$ exchange, because the latter is divergent in the point-like limit. But still there are residual sizeable cancellations between $\pi^\pm$ and $\pi^0$ exchange. The estimate of Einhorn for $a_\mu$ is around $a_\mu \sim 1.8 \times 10^{-10}$, de Rafael uses the chiral perturbation theory formalism. He also agrees that the terms in the
expansion corresponding to the $\pi^\pm$ exchange amplitude are more reliably computed than the ones from $\pi^0$ exchange. However de Rafael goes beyond the pion exchange approximation by including higher order terms in the chiral expansion. He ends up with a value of $a_\mu$ around $a_\mu \approx 7.4 \times 10^{-10}$.

Very recently, after my talk, two new studies have appeared, one by M. Hayakawa et al [12] and one by J. Bijnens et al [13]. These papers are based on two similar effective low energy QCD theories from an extended Nambu-Jona Lasinio model. Their results are $a_\mu \approx -3.6 \times 10^{-10}$ and $a_\mu \approx 1 \times 10^{-10}$.

In conclusion, the difference of the estimates in the second generation studies in refs. 10-13 is significant for guessing at what the error can be. Certainly, at the beginning, Kinoshita was too optimistic. However probably Barbieri and Remiddi exceeded in the opposite direction. From the combined analysis of refs. 10-13 something like $a_\mu = (2 \pm 5) \times 10^{-10}$ emerges. Clearly further studies could very much help in better clarifying this situation.

3. - The Hadronic Contribution to $\alpha(m_Z)$

The light quark masses are among the input parameters that must be fixed in order to compute the electroweak radiative corrections and compare the predictions with the data on precision tests of the Standard Model performed at LEP [2]. The only practically relevant terms where precise values of the light quark masses are needed are those related to the hadronic contribution to the photon vacuum polarisation diagrams. This correction is of order 6%, much larger than the accuracy of a few per mille of the precision tests. But, fortunately, one can use the actual data to almost completely solve the related ambiguity. But we shall see that the left over uncertainty is still one of the main sources of theoretical error.

According to eq. 8 the QED running coupling is given by:

$$\alpha(s) = \frac{\alpha}{1 - \Delta \alpha(s)}$$

(11)
\[ \Delta \alpha(s) = \Pi(s) - \Pi(0) - \text{Re} \, \Pi'(s) \]  

In perturbation theory \( \Delta \alpha(s) \) is given by [7]:

\[
\Delta \alpha(s) = \frac{\alpha}{3\pi} \sum_i Q_i^2 N_{\text{CF}} \left( \log \frac{s}{m_i^2} - \frac{5}{3} \right)
\]

where \( N_{\text{CF}} = 3 \) for quarks and 1 for leptons. However the perturbative formula is only reliable for leptons, not for quarks (because of the unknown values of the effective quark masses). Separating the leptonic, the light quark and the top quark contributions to \( \Delta \alpha(s) \) we have:

\[
\Delta \alpha(s) = \Delta \alpha(s)_l + \Delta \alpha(s)_h + \Delta \alpha(s)_t
\]

with [7]:

\[
\Delta \alpha(s)_l = 0.031421; \quad \Delta \alpha(s)_h = - \frac{\alpha}{3\pi} \frac{4}{15} \frac{m_Z^2}{m_t^2} = - 0.000061
\]

Note that in QED there is decoupling so that the top quark contribution approaches zero in the large \( m_t \) limit. For \( \Delta \alpha(s)_h \) one can use eq.12 and the Cauchy theorem to obtain the representation [7]:

\[
\Delta \alpha(m_Z^2)_h = - \frac{\alpha m_Z^2}{3\pi} \text{Re} \int \frac{d s}{s} \frac{R(s)}{s-m_Z^2 - i\epsilon}
\]

where \( R(s) \) has the same meaning as in eq.6. At \( s \) large one can use the perturbative expansion for \( R(s) \) while at small \( s \) one can use the actual data.
Recently there has been a lot of activity on this subject and a number of independent new estimates of $\Delta \alpha(m_Z^2)_h$ have appeared in the literature. In table 3 we report the results of these new computations together with the most significant earlier determinations (previously the generally accepted value was that of Jegerlehner in 1991 [17]).

Table 3

<table>
<thead>
<tr>
<th>Author</th>
<th>Year and ref.</th>
<th>$\Delta \alpha(m_Z^2)_h$</th>
<th>$\alpha(m_Z^2)_{1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jegerlehner</td>
<td>1986 [14]</td>
<td>0.0285± 0.0007</td>
<td>128.83± 0.09</td>
</tr>
<tr>
<td>Lynn et al</td>
<td>1987 [15]</td>
<td>0.0283± 0.0012</td>
<td>128.86± 0.16</td>
</tr>
<tr>
<td>Burkhardt et al</td>
<td>1989[16]</td>
<td>0.0287± 0.0009</td>
<td>128.80± 0.12</td>
</tr>
<tr>
<td>Jegerlehner</td>
<td>1991 [17]</td>
<td>0.0282± 0.0009</td>
<td>128.87± 0.12</td>
</tr>
<tr>
<td>Swartz</td>
<td>1994 [18]</td>
<td>0.02666± 0.00075</td>
<td>129.08± 0.10</td>
</tr>
<tr>
<td>Swartz (rev.)</td>
<td>1995 [19]</td>
<td>0.0276± 0.0004</td>
<td>128.96± 0.06</td>
</tr>
<tr>
<td>Martin et al</td>
<td>1994 [20]</td>
<td>0.02732± 0.00042</td>
<td>128.99± 0.06</td>
</tr>
<tr>
<td>Nevzorov et al</td>
<td>1994 [21]</td>
<td>0.0280± 0.0004</td>
<td>128.90± 0.06</td>
</tr>
<tr>
<td>Krasnikov</td>
<td>1994 [22]</td>
<td>0.0275 ± 0.0009</td>
<td>128.97 ± 0.13</td>
</tr>
<tr>
<td>Eidelman et al</td>
<td>1995 [7]</td>
<td>0.0280± 0.0007</td>
<td>128.90± 0.09</td>
</tr>
<tr>
<td>Burkhardt et al</td>
<td>1995 [23]</td>
<td>0.0280± 0.0007</td>
<td>128.89± 0.09</td>
</tr>
</tbody>
</table>

The differences among the recent determinations are due to the procedures adopted for fitting the data and treating the errors, for performing the numerical integration etc. The differences are also due to the threshold chosen to start the application of perturbative QCD at large $s$ and to the value adopted for $\alpha_s(m_Z^2)$. For example, in its first version Swartz [18] used parametric forms to fit the data, while most of the other determinations use a trapezoidal rule to integrate across the data points. It was observed that the parametric fitting introduces a definite bias [24]. In fact Swartz gets systematically lower results for all ranges of $s$. In its revised version Swartz improves his numerical procedure. Martin et al [20] use perturbative QCD down to $\sqrt{s} = 3$ GeV (except in the upsilon region) with $\alpha_s(m_Z^2) = 0.118 ± 0.007$. Eidelman et al [7] only use perturbative QCD for $\sqrt{s} > 40$ GeV and with $\alpha_s(m_Z^2) = 0.126 ± 0.005$, i.e. the value found at LEP.
[2]. They use the trapezoidal rule. Nevzorov et al. [21] make a rather crude model with one resonance per channel plus perturbative QCD with \( \alpha_s(m_Z) = 0.125 \pm 0.005 \). Burkhardt et al. [23] use perturbative QCD for \( \sqrt{s} > 12 \text{ GeV} \) but with a very conservative error on \( \alpha_s(m_Z) = 0.124 \pm 0.021 \). This value was determined in ref.25 from e\(^+\)e\(^-\) data below LEP energies. My impression is that the excitement produced by the original claim by Swartz [18] of a relatively large discrepancy with respect to the value obtained by Jegerlehner [17] resulted in a useful debate. As a conclusion of this reevaluation of the problem the method of Jegerlehner has proven its solidity. As a consequence I think that the recent update by Eidelman and Jegerlehner [7] gives a quite reliable result. Also I do not think that a smaller error than quoted by these authors can be justified.

A partition of the magnitude of the \( \Delta \alpha(m_Z)_h \) correction and of the related error is given in graphic form in fig.2 taken from ref.23. We see that the possible impact of

Fig. 2: Relative contributions to \( \Delta \alpha(m_Z) \) in magnitude and uncertainty.
DAΦNE is less important than for the muon g-2. Even if the error from the region below 1.4 GeV is totally eliminated the advantage is not very large, given that the errors from the different regions are added in quadrature.

Finally, to give an idea of the impact of the ambiguity on the \( \Delta \alpha(m_Z)_H \) correction on the theoretical errors for precision tests of the Standard Model, I report in table 4 a comparison of different sources of error for the most relevant observables. We see that the error from \( \Delta \alpha(m_Z)_H \) is especially important for \( \sin^2 \theta_W \) (this is the effective one determined from \( g_V/g_A \) for \( Z \rightarrow l^+l^- \) [2]), which is very precisely measured by a combination of asymmetries, and also, to a lesser extent for \( \Gamma_Z \) and \( \varepsilon_3 \). A plot of the effect of a shift by \( \Delta \alpha^{-1} = \pm 0.09 \) in the \( \varepsilon_3-\varepsilon_1 \) plane is shown in fig.3. The resulting ambiguity is of the order of (but somewhat smaller than) the LEP-SLC discrepancy.

Table 4: Errors from different sources: \( \Delta_{\text{now}}^\text{exp} \) is the present experimental error; \( \Delta_{\infty} \) is the error at the end of LEP; \( \Delta \alpha^{-1} \) is the impact of \( \Delta \alpha^{-1} = \pm 0.09 \); \( \Delta_{\text{th}} \) is the estimated theoretical error from higher orders; \( \Delta m_t \) is from \( \Delta m_t = \pm 12 \) GeV; \( \Delta m_H \) is from \( \Delta m_H = 60-1000 \) GeV; \( \Delta \alpha_s \) corresponds to \( \Delta \alpha_s = \pm 0.006 \). The epsilon parameters are defined in ref.2.

<table>
<thead>
<tr>
<th></th>
<th>( \Delta_{\text{now}}^\text{exp} )</th>
<th>( \Delta_{\infty} )</th>
<th>( \Delta \alpha^{-1} )</th>
<th>( \Delta_{\text{th}} )</th>
<th>( \Delta m_t )</th>
<th>( \Delta m_H )</th>
<th>( \Delta \alpha_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma_Z(\text{MeV}) )</td>
<td>( ^{+3.3}_{-2} )</td>
<td>( ^{+0.7}_{-0.8} )</td>
<td>( ^{+2.6}_{-4.5} )</td>
<td>( ^{+4}_{-4} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_{\text{th}}(\text{pb}) )</td>
<td>81</td>
<td>60</td>
<td>4.3</td>
<td>6.4</td>
<td>4</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td>( R_b,10^3 )</td>
<td>35</td>
<td>30</td>
<td>4.3</td>
<td>5</td>
<td>3.2</td>
<td>15</td>
<td>42</td>
</tr>
<tr>
<td>( A_{FB},10^4 )</td>
<td>13</td>
<td>10</td>
<td>4.2</td>
<td>1.3</td>
<td>7.2</td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td>( \sin^2 \theta,10^4 )</td>
<td>4</td>
<td>3</td>
<td>2.5</td>
<td>0.8</td>
<td>3.5</td>
<td>8</td>
<td>0.6</td>
</tr>
<tr>
<td>( m_W(\text{MeV}) )</td>
<td>160</td>
<td>40</td>
<td>12</td>
<td>9</td>
<td>70</td>
<td>102</td>
<td>13</td>
</tr>
<tr>
<td>( R_{bh},10^4 )</td>
<td>19</td>
<td>15</td>
<td>0.1</td>
<td>1</td>
<td>4.4</td>
<td>0.35</td>
<td>0.6</td>
</tr>
<tr>
<td>( \varepsilon_1,10^3 )</td>
<td>1.6</td>
<td>1.2</td>
<td>0</td>
<td>(-0.1)</td>
<td>0.35</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_3,10^3 )</td>
<td>1.6</td>
<td>1.2</td>
<td>0.6</td>
<td>(-0.1)</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_b,10^3 )</td>
<td>3.9</td>
<td>3.5</td>
<td>0</td>
<td>(-0.1)</td>
<td>2.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 3: The impact on the $\varepsilon_3,\varepsilon_1$ plane of an uncertainty $\Delta\alpha(m_Z)^{-1} = \pm 0.09$. The solid ellipse is the result of fitting the LEP data using the central value of $\alpha(m_Z)$, while the dotted ellipses correspond to 1σ shifts.

References


