**CHERENKOV RADIATION OF e^+e^- -PAIRS AND ITS APPLICATION
FOR THE MEASUREMENT OF γ -QUANTA BEAM POLARIZATION**

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Abstract

The dependence of the Cherenkov radiation intensity of e^+e^- pairs produced by high energy γ -quanta on the distance z from the pair production point, the radiator thickness L , and the azimuthal angle φ of the observation point from the γ -quanta linear polarization direction, is calculated, taking into account the multiple scattering. It is shown that the measurement of the γ -quanta polarization by detection of the expected φ -asymmetry with the help of a RICH detector is easier than with other existing or proposed methods.

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Due to the smallness of the angle $\theta_0 \approx 4mc^2/E$ between the components of the e^+e^- pairs produced by γ -quanta with energy E , the distance $l(z)$ between the components remains very small even at sufficiently large distances (z) from the production points. As a result of the interaction between the fields of the pair particles, certain effects can take place — in particular, the Chudakov effect, namely the decrease of the ionization losses calculated [1–4] and experimentally studied [5–7] as early as the 1950s. At $E \approx 100$ GeV the Chudakov effect appears to be detectable at distances of up to $z \approx 0.1$ cm, after which, when $l(z) > l_0 \approx 5 \cdot 10^{-7}$ cm, it disappears due to the multiple scattering. However, one can show that other effects also exist, even when $l(z) \gg l_0$. Indeed, as has been shown by I.M. Frank [8], the coherent effects of the Cherenkov radiation of an e^+e^- dipole, with constant moment $\vec{p} = e\vec{l}$ moving perpendicularly to $\vec{p}$, are noticeable at $l \leq \lambda$, where λ is the wavelength of the Cherenkov photons (see Refs. [9,10] as well). Similar effects are also expected for other types of radiation (transition, synchrotron, etc.). As a result of such interference phenomena, the radiation intensity $I_2(\omega, z)$ of an e^+e^- pair produced by a high-energy γ -quanta must have the following dependence upon z : in the beginning, for small z , the intensity must be suppressed due to the mutual screening of the particle field; for $l(z) \sim \lambda$ one will observe an interference variation of $I_2(\omega, z)$, and for $l(z) \gg \lambda$, $I_2(\omega, z) \approx 2I_1(\omega, z)$; where $I_1(\omega, z)$ is the radiation intensity of a relativistic particle.

In this work we shall calculate $I_2(\omega, z)$ for Cherenkov radiation without using the complicated expressions for the fields. Following this approach, [11] one can write the trajectories $r_j(t)$ of the pair components produced at $z = 0$ by a γ -quanta moving along the direction of z -axis as

$$r_j(t) = r_j(t + z_j/v_j) + \rho_j, \quad (1)$$

where v_j, z_j and ρ_j are the velocity ($v_j \approx v_z$), longitudinal and transversal coordinates of the j -th particle, respectively. Then, using the well-known expression (formula (122) [12]) for the spectral-angular distribution $dI_1/(d\omega d\Omega)$ for each particle, the following expression for the e^+e^- pair radiation of any type can be written down:

$$\frac{dI_2}{d\omega d\Omega} = \frac{dI_1}{d\omega d\Omega} F(\omega, z, \theta, \varphi), \quad (2)$$

where

$$F(\omega, z, \theta, \varphi) = \left| \sum_{j=1}^2 \exp[-i(\omega z_j/v_j + \kappa \rho_j)] \right|^2, \quad (3)$$

κ is the transversal component of the wave vector $\vec{k} = \omega \sqrt{\varepsilon} \vec{n}/c$, ε is the dielectric constant of the medium, θ , and φ are the polar and azimuthal (with respect to the e^+e^- pair dipole moment) angles of photons radiated in the direction $\vec{n}$.

It is easy to show that for an e^+e^- pair with momentum $\vec{p}(z) = e\vec{l}(z)$ perpendicular to the z -axis,

$$F(\omega, z, \theta, \varphi) = 4 \sin^2 \frac{\kappa l(z)}{2} = 4 \sin^2 \left(\frac{\pi \sqrt{\varepsilon} l(z)}{2\lambda} \sin \theta \cos \varphi \right), \quad (4)$$

The above described expectations follow from expression (4): I_2 equals its maximal value, and 0 at $\varphi = 0$ and $\pi/2$, respectively; $I_2 = 0$, for $\theta = 0$ and at certain values of λ and $l(z)$, when the sine argument becomes equal to $k\pi$ ($k = 0, 1, 2, \dots$). $I_2 \rightarrow 0$ when $l(z) \rightarrow 0$, just as in the case of Chudakov effect.

Limiting ourselves to the Cherenkov radiation of an e^+e^- pair in a homogeneous medium with refraction index $n = \sqrt{\varepsilon}$, we can write its intensity in the form:

$$\frac{dI_2}{dzd\omega d\varphi} = \frac{dI_1}{dzd\omega} F(\omega, z, \varphi), \quad (5)$$

where

$$\frac{dI_1}{dzd\omega} = \frac{e^2\omega}{c^2} \left(1 - \frac{1}{\beta^2 n^2} \right), \quad (6)$$

$$F(\omega, z, \varphi) = 4 \sin^2(A l(z) \cos \varphi), \quad (7)$$

$$A = \frac{\pi \sqrt{n^2 - 1}}{\lambda}. \quad (8)$$

For $l(z) \ll \lambda$, the expression (5) coincides with I.M. Frank's [8] formula (see also formula (324) in Ref. [10]), whilst after integration over φ it gives the result of our work [13] (see p. 302 of Ref. [10]). Incidentally, using the simple method [11], B.M. Bolotovskiy's formulae (see p. 298 of Ref. [10]) for Cherenkov radiation of constant dipoles of the same and opposite charges moving in parallel to their moments is easily derived.

In general, the pair or multiparticle production processes give a unique possibility for observing interference phenomena. However, in the case of Cherenkov radiation, the multiple scattering makes the simple dependences on z and φ more complicated. For this reason, in order to predict the observable magnitudes it is necessary to average $F(\omega, z, \varphi)$ over the multiple scattering azimuthal angles $\psi_{+,-}$ and transversal deviations $a_{+,-}$ from the direction of each pair particle. Similarly as with Refs. [1-3], we take the distance $c(z)$ between the components (without taking into account the multiple scattering) in the form $c(z) = \theta_0 z = 4mc^2 z / E$, and the normal distribution dispersion of $a_{+,-}$ equal to $\sigma_{+,-} = \beta mc^2 z^{3/2} X_0^{-1/2} / E$, where $\beta = 43.46$ and X_0 is the radiation length of the medium in cm. With the help of computers, after numerical integration of the four-fold or two-fold integrals with σ values twice differing from each other, we become convinced that they provide the same results. Here we omit the lengthy expression of these integrals, which cannot be integrated analytically. However, for small values of z , when the sine argument is less than 1, one can obtain the following simple expressions:

$$\langle F(z, \varphi) \rangle \approx 4A^2 \theta_0^2 z^2 \left(\frac{z}{z_0} + \cos^2 \varphi \right), \quad (9)$$

where $z_0 = 8X_0/\beta^2$ and

$$\Phi(L, \varphi) = \int_0^L \langle F(z, \varphi) \rangle dz \approx \frac{4}{3} A^2 \theta_0^2 L^3 \left(\frac{3}{4} \frac{L}{z_0} + \cos^2 \varphi \right), \quad (10)$$

which is proportional to the intensity of the Cherenkov radiation of an e^+e^- pair passing a distance, L , after pair production.

From the experimental point of view it is of greater interest to calculate the ratio $Q(L, \varphi)$ of the Cherenkov radiation mean intensities when a high energy γ -quantum and a single ultrarelativistic particle pass through a radiator with thickness L , taking into account the probability of the conversion of the photon into an e^+e^- pair:

$$Q(L, \varphi) = \frac{n_0 \sigma}{L} \int_0^L (L - z) \langle F(z, \varphi) \rangle dz \approx \frac{1}{3} A^2 \theta_0^2 L^4 \left[\frac{3}{5} \frac{L}{z_0} + \cos^2 \varphi \right], \quad (11)$$

where n_0 is the atomic density of the radiation, $\sigma(E)$ is the pair-production total cross-section, and $n_0\sigma \approx 7/(9X_0)$ for $L < X_0$.

The asymmetry is equal to

$$R(L) = \frac{Q(L, 0) - Q(L, \pi/2)}{Q(L, 0) + Q(L, \pi/2)} \approx \frac{1}{1 + \frac{6}{5} \frac{L}{z_0}} . \quad (12)$$

As an illustration, in Fig. 1 are given the z (or L) dependence of $\langle F(z, 0) \rangle$, $\langle F(z, \pi/2) \rangle$, $\Phi(L, 0)$, $Q(L, 0)$ and $R(L)$ (curves 1, 2, 3, 3' and 4), for a NaF radiator at $\lambda = 1.9 \times 10^{-5}$ cm ($n = 1.387$) and $E = 100$ GeV. Of course, the measurement of these, which recall the Chudakov effect curves, is of interest. However, as will be shown below, the fact that the predicted φ -asymmetry makes measurement of the γ -beam polarization at $E > 20$ GeV easier than by other methods (see, for instance, Ref. [14]) deserves especial attention.

Figure 1: The dependence of $\langle F(z, 0) \rangle$, $\langle F(z, \pi/2) \rangle$, $\Phi(l, 0)$, $Q(L, 0)$ and $R(L)$ (curves 1, 2, 3, 3' and 4) upon z or L (in mm) for a NaF radiator at $\lambda = 1.9 \times 10^{-5}$ cm, and $E = 100$ GeV, respectively. The magnitude $M(L)$ (curve 5) is proportional to the ‘figure of merit’ (see text).

Indeed, as is well known, pair production by polarized γ = quanta has its own φ -asymmetry for the produced-pair plane, with respect to the direction of the γ -beam vector of polarization. This asymmetry is equal to $R_{\gamma \rightarrow ee} = 0.14$, independent of E (see, for instance, Ref. [15]). Therefore, one can measure the γ -quanta beam polarization

$$P = \frac{1}{R_{\gamma \rightarrow ee} R} \frac{N(0) - N(\pi/2)}{N(0) + N(\pi/2)} \quad (13)$$

with an accuracy of

$$\left(\frac{\Delta P}{P}\right)^2 = \frac{1}{P^2 R_{\gamma \rightarrow ee}^2 R^2 [N(0) + N(\pi/2)]} , \quad (14)$$

where $N(0)$ and $N(\pi/2)$ are the numbers of detected Cherenkov photons at $\phi = 0$ and $\pi/2$, respectively. The product $F = R^2 [N(0) + N(\pi/2)]$ in the denominator of (??) is usually

called a ‘figure of merit’: its maximum allows a choice of the optimal radiator thickness L_{opt} requiring minimal measurement time for the given P and $\Delta P/P$. The curve ‘6’ in Fig. 1 is proportional to the ‘figure of merit’ and shows that for the parameters of this figure it is reasonable to take a radiator with thickness $L = 0.3$ cm, which, according to the curve ‘3’ provides $Q = 0.01$ times less Cherenkov photons for each γ -quanta passage through the radiator than for a single ultrarelativistic particle.

Figure 2 shows the energy dependence of L_{opt} , Q_{opt} and R_{opt} , obtained from curves similar to those of Fig. 1 for various E . Using this, one can easily determine the thickness relative to the Cherenkov radiation intensity, which allows an estimation of the absolute yield (without calculating various efficiencies if the radiation yield for a single charged particle is measured), and the necessary measurement time for measuring P .

Let us assume that it is necessary to measure the polarization P of a γ -beam produced by channelled 200 GeV electrons with 20% precision. In the region of $E = 100$ GeV such beams have $P \approx 50\%$ [14]. Supposing we use a RICH detector of the type described in Ref. [15], which has an $L = 1$ cm thick NaF radiator and detects 10 Cherenkov photons per relativistic particle, recording only 25% of the photons emitted around the direction $\varphi = 0; \pi/2; \pi$ and $3\pi/2$. With the help of Fig. 2, one can choose the radiator thickness $L = 0.3$ cm, which provides 0.01 detected Cherenkov photons per γ quantum. Using Eq. (??) one can show that the necessary statistics from $N(0) + N(\pi/2) \approx 1.4 \times 10^6$ detected Cherenkov photon pairs can be accumulated after $T \approx 1.1$ h, or 100 s in the case when the γ -beam intensity is equal to $2 \cdot 10^4$ or $2 \cdot 10^6$ photons per second.

Figure 2: The dependence of the optimal radiator length L_{opt} (in mm), corresponding asymmetry R_{opt} and relative Cherenkov radiation yield $Q(L, 0)$ upon the γ -quanta energy E (in GeV).

Thus, the interesting effects of the Cherenkov radiation of e^+e^- pairs produced by high energy γ -quanta can be detected and used for the measurement of photon polarization — without magnetic systems, single crystals and complicated detectors as with other methods [14], but nevertheless expending more time for collecting the necessary statistics.

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