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Topological Yang-Mills Theory with Scalar and Vector Fields in Two and Three Dimensions

Dae Sung Hwang^a and Chang-Yeong Lee^b

Department of Physics, Sejong University, Seoul 133-747, Korea

Abstract

We study the topological Yang-Mills theory of scalar and vector fields in two and three dimensions in the superconnection framework. We modify the horizontality condition in such a way that we can take care of the topological symmetry of the $\delta A_\mu = \beta_\mu$ type, as well as the usual gauge symmetry of the $\delta A_\mu = D_\mu c$ type. We then obtain a complete set of BRST and anti-BRST transformation rules of the component fields in a systematic way.

(a) e-mail: dshwang@phy.sejong.ac.kr

(b) e-mail: leecy@cs.sejong.ac.kr

1. Introduction

The classical action of the Witten's four dimensional topological Yang-Mills theory [1]

$$S_4 = \int d^4x \operatorname{tr}[\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}] \quad (1)$$

is invariant under the transformation

$$\delta_\beta A_\mu = \beta_\mu, \quad (2)$$

as well as under the usual gauge transformation

$$\delta_\alpha A_\mu = D_\mu c. \quad (3)$$

The transformation (2) is the characteristic symmetry of the action (1). Using (2) as well as (3) in the BRST quantization formalism, Baulieu and Singer, and Perry and Teo reproduced the Witten's quantum action which generates the Donaldson invariants [2, 3, 4].

On the other hand, Baulieu and Grossman [5] found that in three dimensions the classical action

$$S_3 = \int d^3x \operatorname{tr}[\epsilon^{\mu\nu\lambda} F_{\mu\nu} D_\lambda \varphi], \quad (4)$$

where $D_\lambda \varphi = \partial_\lambda \varphi + [A_\lambda, \varphi]$, is invariant under the following transformations.

$$\begin{aligned} \delta A_\mu &= D_\mu c + \beta_\mu, \\ \delta \varphi &= [\varphi, c] + \gamma. \end{aligned} \quad (5)$$

Then they studied the BRST symmetry of (4), and related its quantum action with the scattering of slowly moving magnetic monopoles. Chapline and Grossman [6] found that the classical action in two dimensions

$$S_2 = \int d^2x \operatorname{tr}[\epsilon^{\mu\nu} D_\mu \varphi D_\nu \varphi] \quad (6)$$

is also invariant under the transformations (5). They studied the BRST symmetry of (6), and its quantum action in connection with the theory of vortices and knots.

In this paper we study the BRST and anti-BRST symmetries of (5) in the superconnection framework. In section 2 we explain briefly the superconnection and the horizontality condition, which are essential tools in the present work. In section 3 we combine the scalar field φ and the vector field A_μ in a superconnection. We modify the horizontality condition such that it takes care of the symmetry (5) which contains the topological symmetry as well as the usual gauge symmetry. Then using this modified horizontality condition, we obtain the BRST and anti-BRST transformation rules of the component fields in the superconnection framework. In section 4, which constitutes discussions and conclusion, we interpret the three and two dimensional topological actions (4) and (6) in analogy with the correspondence between the four dimensional topological action (1) and the Chern-Simons action.

2. Superconnection and horizontality condition

In order to study the theory with scalar and vector gauge fields, it is much easier to work in the superconnection framework, since the theory contains both even and odd form fields. The superconnection framework has been developed by Ne'eman and Sternberg, and Coquereaux *et al.* [7, 8]. The superalgebra multiplication law is given as follows for general supermatrices $\mathcal{X}$ and $\mathcal{X}'$, which have arbitrary degree forms X and X' as their supermatrix elements [7, 8].

$$\mathcal{X}\mathcal{X}' = (a \otimes X)(a' \otimes X') = (-1)^{|X||a'|}(aa') \otimes (XX'), \quad (7)$$

where

$$\mathcal{X} = (a \otimes X) = \begin{pmatrix} A & C \\ D & B \end{pmatrix}, \quad \mathcal{X}' = (a' \otimes X') = \begin{pmatrix} P & R \\ S & Q \end{pmatrix}. \quad (8)$$

In (7), $|X|$ and $|a|$ are form grade and supermatrix grade respectively. $|X|$ is one for odd form, zero for even form. $|a|$ is zero for even (diagonal) supermatrix, and

one for odd (off-diagonal) supermatrix. (7) can be written more explicitly as

$$\mathcal{X}\mathcal{X}' = \begin{pmatrix} A & C \\ D & B \end{pmatrix} \begin{pmatrix} P & R \\ S & Q \end{pmatrix} = \begin{pmatrix} AP + (-1)^{|C|}|CS| & (-1)^{|A|}AR + CQ \\ DP + (-1)^{|B|}|BS| & (-1)^{|D|}DR + BQ \end{pmatrix}. \quad (9)$$

The exterior derivative which is consistent with the superalgebra multiplication law (7) is given by

$$d\mathcal{X} = \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} P & R \\ S & Q \end{pmatrix} = \begin{pmatrix} dP & -dR \\ -dS & dQ \end{pmatrix}, \quad (10)$$

for the general supermatrix $\mathcal{X}$ since d is an even supermatrix and $d \equiv dx^\mu \frac{\partial}{\partial x^\mu}$ is one form in the base manifold. This exterior derivative satisfies the following Leibniz rule.

$$d(\mathcal{X}\mathcal{X}') = d((a \otimes X)(a' \otimes X')) = (d\mathcal{X})\mathcal{X}' + (-1)^{|a|+|X|}\mathcal{X}(d\mathcal{X}') \quad (11)$$

for general supermatrices $\mathcal{X}$ and $\mathcal{X}'$.

The superconnection is defined as [7, 8]

$$\mathcal{A} = \begin{pmatrix} \omega_0 & L_{01} \\ L_{10} & \omega_1 \end{pmatrix} = \mathcal{A}_{ev} + \mathcal{A}_{od} = \begin{pmatrix} \omega_0 & 0 \\ 0 & \omega_1 \end{pmatrix} + \begin{pmatrix} 0 & L_{01} \\ L_{10} & 0 \end{pmatrix}, \quad (12)$$

where the even supermatrix elements ω_0, ω_1 are odd degree differential forms, and the odd elements L_{01}, L_{10} are even degree forms. Since the superconnection in (12) has odd and even degree forms for even and odd supermatrix elements respectively, the exterior derivative d satisfies the following Leibniz rule from (11).

$$d(\mathcal{A}\mathcal{X}) = (d\mathcal{A})\mathcal{X} - \mathcal{A}(d\mathcal{X}) \quad (13)$$

for the superconnection $\mathcal{A}$ and the general supermatrix $\mathcal{X}$. We note that (13) is the same Leibniz rule as that for the ordinary one form gauge field A which is given by $d(AX) = (dA)X - A(dX)$, where X is an arbitrary differential form. From the superconnection in (12) we have the supercurvature $\mathcal{F}$ as

$$\mathcal{F} = d\mathcal{A} + \mathcal{A}\mathcal{A} \quad (14)$$

$$= \begin{pmatrix} d\omega_0 + \omega_0\omega_0 + L_{01}L_{10} & -dL_{01} - \omega_0L_{01} + L_{01}\omega_1 \\ -dL_{10} + L_{10}\omega_0 - \omega_1L_{10} & d\omega_1 + L_{10}L_{01} + \omega_1\omega_1 \end{pmatrix}. \quad (15)$$

Now, we explain the horizontality condition by showing how it gives rise to the BRST transformation rules in the ordinary gauge theory. In the usual non-Abelian gauge theory we have a one form connection A

$$A = A_\mu dx^\mu, \quad (16)$$

where x^μ is the coordinate of the space-time base manifold. We adopt the differential form notation. The curvature is given by

$$F = dA + AA, \quad (17)$$

where $d = dx^\mu \frac{\partial}{\partial x^\mu}$ is the exterior derivative and $AA = A \wedge A$. We omit the wedge for the product of differential forms.

In order to interpret the BRST symmetry geometrically, Ne'eman and Thierry-Mieg introduced the gauge-fiber coordinate y^N in the fiber-bundle space, and extended A to the following connection $\tilde{A}$ in the fiber-bundle space [9].

$$\tilde{A} = A + c = A_\mu dx^\mu + c_N dy^N, \quad (18)$$

where $c = c_N dy^N$ is the gauge field in the gauge-fiber direction, which turns out to be the ghost field of the BRST symmetry. The curvature in the fiber-bundle space is given by

$$\tilde{F} = d\tilde{A} + \tilde{A}\tilde{A} = (d + s)(A + c) + (A + c)(A + c), \quad (19)$$

where $\tilde{d} = d + s$, and $s = dy^N \frac{\partial}{\partial y^N}$ is the exterior derivative in the gauge-fiber direction.

Then Ne'eman and Thierry-Mieg showed that the horizontality condition, which is the Maurer-Cartan equation,

$$\tilde{F} = F \quad (20)$$

gives rise to the BRST transformation rules which satisfy the nilpotency condition $s^2 = \bar{s}^2 = s\bar{s} + \bar{s}s = 0$. That is, when we expand (20) in terms of form degree, we

get the following well-known BRST transformation rules.

$$\begin{aligned} (dx)^1(dy)^1 &: sA = -dc - Ac - cA \\ (dy)^2 &: sc = -cc. \end{aligned} \quad (21)$$

The horizontality condition was first studied by Ne'eman and Thierry-Mieg [9], and it has played important roles in investigating the geometrical meaning of the BRST symmetry [9, 10, 11, 12].

3. Topological BRST and anti-BRST symmetries with scalar and vector fields

In this section we apply the horizontality condition in the superconnection framework, in order to study the BRST symmetry of a system which contains both scalar and vector gauge fields. Then we modify the horizontality condition in such a way that we can take care of the symmetry (5), which contains the topological symmetry of the type (2), as well as the usual gauge symmetry of the type (3). At the beginning we consider only the BRST symmetry, and then both the BRST and anti-BRST symmetries.

In order to study a system which contains both scalar and vector fields, we combine one form gauge field A and zero form scalar gauge field φ in the superconnection as

$$\mathcal{A} = \begin{pmatrix} A & \varphi \\ \varphi & A \end{pmatrix}, \quad (22)$$

where $A = A_\mu dx^\mu$ is one form and φ is zero form. Here A and φ are Lie algebra valued, that is, $A = A_a T_a$ and $\varphi = \varphi_a T_a$, where T_a are gauge group generators, for example, $SU(N)$ group generators. The ghost and auxiliary fields below are also Lie algebra valued in the same way. The superconnection in the fiber-bundle space is given by

$$\tilde{\mathcal{A}} = \begin{pmatrix} A + \alpha & \varphi \\ \varphi & A + \alpha \end{pmatrix}, \quad (23)$$

where

$$A + \alpha = A_\mu dx^\mu + \alpha_N dy^N, \quad \varphi = \varphi. \quad (24)$$

The supercurvature in the base manifold, $\mathcal{F}$, and that in the fiber-bundle space, $\tilde{\mathcal{F}}$, are given by

$$\mathcal{F} = d\mathcal{A} + \mathcal{A}\mathcal{A}, \quad (25)$$

$$\tilde{\mathcal{F}} = (\mathbf{s} + \mathbf{d})\tilde{\mathcal{A}} + \tilde{\mathcal{A}}\tilde{\mathcal{A}}. \quad (26)$$

Then the horizontality condition is given by

$$\tilde{\mathcal{F}} = \mathcal{F}. \quad (27)$$

From (27) we get the BRST transformation rule of $\tilde{\mathcal{A}}$,

$$\mathbf{s}\tilde{\mathcal{A}} = -\mathbf{d}\tilde{\mathcal{A}} - \tilde{\mathcal{A}}\tilde{\mathcal{A}} + \mathcal{F}. \quad (28)$$

However, in order to make sure that $\mathbf{s}$ in (28) is really a BRST transformation, we should check that $\mathbf{ss}\tilde{\mathcal{A}} = 0$ is satisfied. From (28) we have

$$\begin{aligned} \mathbf{ss}\tilde{\mathcal{A}} &= \mathbf{d}(\mathbf{s}\tilde{\mathcal{A}}) - (\mathbf{s}\tilde{\mathcal{A}})\tilde{\mathcal{A}} + \tilde{\mathcal{A}}(\mathbf{s}\tilde{\mathcal{A}}) + \mathbf{s}\mathcal{F} \\ &= \mathbf{d}\mathcal{F} + \tilde{\mathcal{A}}\mathcal{F} - \mathcal{F}\tilde{\mathcal{A}} + \mathbf{s}\mathcal{F}. \end{aligned} \quad (29)$$

In the right hand side of (29) there is no term with a ghost number greater than one in our case with $\mathcal{A}$ in (22) and $\tilde{\mathcal{A}}$ in (23). Therefore $\mathbf{ss}\tilde{\mathcal{A}} = 0$ is satisfied in the system composed of scalar and vector gauge fields with usual gauge symmetry. Expanding (28) in terms of form degree, we get the usual BRST transformation rules of both the gauge fields A , φ , and the ghost field α without topological symmetry;

$$\begin{aligned} (dx)^1(dy)^1 &: sA = -D\alpha \\ (dy)^1 &: s\varphi = -[\alpha, \varphi] \\ (dy)^2 &: s\alpha = -\alpha\alpha, \end{aligned} \quad (30)$$

where $[\ , \]$ is the graded commutator and $Df \equiv df + [A, f]$. By explicit application of $\mathbf{s}$ in (30) we can show that $\mathbf{s}$ is nilpotent, that is, $\mathbf{s}^2 = 0$.

In order to take care of the topological symmetry of (5), we extend $\tilde{\mathcal{F}}$ in (26) to

$$\tilde{\mathcal{F}}_T \equiv \tilde{\mathcal{F}} + \tilde{\mathcal{F}}'. \quad (31)$$

In our case where $\tilde{\mathcal{A}}$ is given by (23), $\tilde{\mathcal{F}}'$ is assigned as

$$\tilde{\mathcal{F}}' = \begin{pmatrix} \beta + \phi & \gamma \\ \gamma & \beta + \phi \end{pmatrix}, \quad (32)$$

where

$$\beta + \phi = \beta_\mu dx^\mu \wedge dy^N + \frac{1}{2}\phi_{MN} dy^M \wedge dy^N, \quad \gamma = \gamma_N dy^N. \quad (33)$$

Note that each component field of $\tilde{\mathcal{F}}'$ is assigned to have one more ghost number than the corresponding field of $\tilde{\mathcal{A}}$ in (23). Then we modify the horizontality condition (27) to

$$\tilde{\mathcal{F}}_T = \mathcal{F}. \quad (34)$$

From (34) we have the modified version of (28),

$$\mathbf{s}\tilde{\mathcal{A}} = -\mathbf{d}\tilde{\mathcal{A}} - \tilde{\mathcal{A}}\tilde{\mathcal{A}} + \mathcal{F} - \tilde{\mathcal{F}}'. \quad (35)$$

By applying $\mathbf{s}$ on the both sides of (35), and requiring the nilpotency for $\tilde{\mathcal{A}}$, $\mathbf{ss}\tilde{\mathcal{A}} = 0$, we get the BRST transformation rule of $\tilde{\mathcal{F}}'$,

$$\begin{aligned} \mathbf{s}\tilde{\mathcal{F}}' &= -\mathbf{d}\tilde{\mathcal{F}}' - \tilde{\mathcal{A}}\tilde{\mathcal{F}}' + \tilde{\mathcal{F}}'\tilde{\mathcal{A}} \\ &+ \mathbf{d}\mathcal{F} + \tilde{\mathcal{A}}\mathcal{F} - \mathcal{F}\tilde{\mathcal{A}} + \mathbf{s}\mathcal{F}. \end{aligned} \quad (36)$$

In reality four terms in the second line of the right hand side of (36) do not contribute to $\mathbf{s}\tilde{\mathcal{F}}'$ in the system composed of scalar and vector gauge fields, since none of them has a ghost number greater than one, whereas all the components of $\mathbf{s}\tilde{\mathcal{F}}'$ have ghost numbers greater than one. Again, we should check $\mathbf{ss}\tilde{\mathcal{F}}' = 0$. From (35) and (36) we have

$$\mathbf{ss}\tilde{\mathcal{F}}' = -\tilde{\mathcal{A}}(\mathbf{d}\mathcal{F}) - (\mathbf{d}\mathcal{F})\tilde{\mathcal{A}} - \tilde{\mathcal{A}}(\mathbf{s}\mathcal{F}) - (\mathbf{s}\mathcal{F})\tilde{\mathcal{A}} - \mathbf{d}\mathbf{s}\mathcal{F}. \quad (37)$$

There is no term with a ghost number greater than two in the right hand side of (37) in our case with $\mathcal{A}$ in (22), $\tilde{\mathcal{A}}$ in (23) and $\tilde{\mathcal{F}}'$ in (32), whereas all the components of $\text{ss}\tilde{\mathcal{F}}'$ have ghost numbers greater than two. Therefore $\text{ss}\tilde{\mathcal{F}}' = 0$ is satisfied in our case. Then it is confirmed that (35) and (36) give the BRST transformation rules of the topological Yang-Mills theory with scalar and vector gauge fields.

In order to obtain the BRST transformation rules of the component fields, we expand (35) and (36) in terms of form degree. Through this expansion we obtain

[even part]

$$\begin{aligned} (dx)^1(dy)^1 &: sA = -D\alpha - \beta \\ (dy)^2 &: s\alpha = -\alpha\alpha - \phi \\ (dx)^1(dy)^2 &: s\beta = -D\phi - [\alpha, \beta] \\ (dy)^3 &: s\phi = -[\alpha, \phi], \end{aligned} \quad (38)$$

[odd part]

$$\begin{aligned} (dy)^1 &: s\varphi = -[\alpha, \varphi] + \gamma \\ (dy)^2 &: s\gamma = -[\alpha, \gamma] - [\phi, \varphi]. \end{aligned} \quad (39)$$

where $[\ , \]$ is the graded commutator and $Df \equiv df + [A, f]$. (38) and (39) constitute a complete set of the BRST transformation rules of the component fields.

Now, let us consider both BRST and anti-BRST symmetries by assigning

$$\tilde{\mathcal{A}} = \begin{pmatrix} A + \alpha + \bar{\alpha} & \varphi \\ \varphi & A + \alpha + \bar{\alpha} \end{pmatrix}, \quad (40)$$

$$\tilde{\mathcal{F}}' = \begin{pmatrix} \beta + \bar{\beta} + \phi + \rho + \bar{\phi} & \gamma + \bar{\gamma} \\ \gamma + \bar{\gamma} & \beta + \bar{\beta} + \phi + \rho + \bar{\phi} \end{pmatrix}, \quad (41)$$

where

$$A + \alpha + \bar{\alpha} = A_\mu dx^\mu + \alpha_N dy^N + \bar{\alpha}_{\bar{N}} d\bar{y}^{\bar{N}},$$

$$\varphi = \varphi, \quad (42)$$

$$\begin{aligned} \beta + \bar{\beta} + \phi + \rho + \bar{\phi} &= \beta_{\mu N} dx^\mu \wedge dy^N + \bar{\beta}_{\mu \bar{N}} dx^\mu \wedge d\bar{y}^{\bar{N}} + \frac{1}{2} \phi_{MN} dy^M \wedge dy^N \\ &\quad + \rho_{M\bar{N}} dy^M \wedge d\bar{y}^{\bar{N}} + \frac{1}{2} \bar{\phi}_{\bar{M}\bar{N}} d\bar{y}^{\bar{M}} \wedge d\bar{y}^{\bar{N}}, \\ \gamma + \bar{\gamma} &= \gamma_N dy^N + \bar{\gamma}_{\bar{N}} d\bar{y}^{\bar{N}}. \end{aligned}$$

In the above ϕ, ρ and $\bar{\phi}$ fields are ghost for ghost fields which are necessary to fix the residual gauge symmetries of β and $\bar{\beta}$. The forms in (40) and (41) are defined on the tangent and cotangent manifolds of the fiber as well as on the space-time base manifold, that is, they are defined in the fiber-bundle space. The component fields with odd forms in dy and $d\bar{y}$ directions have wrong spin-statistics and reduce the physical degrees of freedom. Here, $\alpha, \bar{\alpha}, \beta$ and $\bar{\beta}$ are such fields.

In analogy with (35) and (36), the BRST and anti-BRST transformation rules are given by

$$(s + \bar{s})\tilde{\mathcal{A}} = -d\tilde{\mathcal{A}} - \tilde{\mathcal{A}}\tilde{\mathcal{A}} + \mathcal{F} - \tilde{\mathcal{F}}', \quad (43)$$

$$\begin{aligned} (s + \bar{s})\tilde{\mathcal{F}}' &= -d\tilde{\mathcal{F}}' - \tilde{\mathcal{A}}\tilde{\mathcal{F}}' + \tilde{\mathcal{F}}'\tilde{\mathcal{A}} \\ &\quad + d\mathcal{F} + \tilde{\mathcal{A}}\mathcal{F} - \mathcal{F}\tilde{\mathcal{A}} + (s + \bar{s})\mathcal{F}. \end{aligned} \quad (44)$$

We obtain the following BRST and anti-BRST transformation rules of the component fields by expanding (43) and (44) in terms of form degree.

[even part]

$$\begin{aligned} (dx)^1(dy)^1 &: sA = -D\alpha - \beta \\ (dx)^1(d\bar{y})^1 &: \bar{s}A = -D\bar{\alpha} - \bar{\beta} \\ (dy)^2 &: s\alpha = -\alpha\alpha - \phi \\ (d\bar{y})^2 &: \bar{s}\bar{\alpha} = -\bar{\alpha}\bar{\alpha} - \bar{\phi} \\ (dy)^1(d\bar{y})^1 &: s\bar{\alpha} + \bar{s}\alpha = -[\alpha, \bar{\alpha}] - \rho \\ (dx)^1(dy)^2 &: s\beta = -D\phi - [\alpha, \beta] \end{aligned} \quad (45)$$

$$\begin{aligned}
(dx)^1(dy)^2 &: \bar{s}\bar{\beta} = -D\bar{\phi} - [\bar{\alpha}, \bar{\beta}] \\
(dx)^1(dy)^1(dy)^1 &: s\bar{\beta} + \bar{s}\beta = -[\alpha, \bar{\beta}] - [\bar{\alpha}, \beta] - D\rho \\
(dy)^3 &: s\phi = -[\alpha, \phi] \\
(dy)^3 &: \bar{s}\bar{\phi} = -[\bar{\alpha}, \bar{\phi}] \\
(dy)^2(dy)^1 &: \bar{s}\phi + s\rho = -[\alpha, \rho] - [\bar{\alpha}, \phi] \\
(dy)^1(dy)^2 &: s\bar{\phi} + \bar{s}\rho = -[\bar{\alpha}, \rho] - [\alpha, \bar{\phi}],
\end{aligned}$$

[odd part]

$$\begin{aligned}
(dy)^1 &: s\varphi = -[\alpha, \varphi] + \gamma \\
(dy)^1 &: \bar{s}\varphi = -[\bar{\alpha}, \varphi] + \bar{\gamma} \\
(dy)^2 &: s\gamma = -[\alpha, \gamma] - [\phi, \varphi] \\
(dy)^2 &: \bar{s}\bar{\gamma} = -[\bar{\alpha}, \bar{\gamma}] - [\bar{\phi}, \varphi] \\
(dy)^1(dy)^1 &: s\bar{\gamma} + \bar{s}\gamma = -[\alpha, \bar{\gamma}] - [\bar{\alpha}, \gamma] - [\rho, \varphi].
\end{aligned} \tag{46}$$

The equations of $(dy)^1(dy)^1$, $(dx)^1(dy)^1(dy)^1$, $(dy)^2(dy)^1$ and $(dy)^1(dy)^2$ in (45) and that of $(dy)^1(dy)^1$ in (46) do not completely specify the transformation rules of the component fields. Therefore we need to introduce the auxiliary fields t , m , n , $\bar{n}$ and u given below to solve these equations.

$$\begin{aligned}
s\bar{\alpha} &\equiv t, & \bar{s}\alpha &= -t - [\alpha, \bar{\alpha}] - \rho \\
s\bar{\beta} &\equiv m, & \bar{s}\beta &= -m - [\alpha, \bar{\beta}] - [\bar{\alpha}, \beta] - D\rho \\
s\rho &\equiv n, & \bar{s}\phi &= -n - [\alpha, \rho] - [\bar{\alpha}, \phi] \\
s\bar{\phi} &\equiv \bar{n}, & \bar{s}\rho &= -\bar{n} - [\bar{\alpha}, \rho] - [\alpha, \bar{\phi}] \\
s\bar{\gamma} &\equiv u, & \bar{s}\gamma &= -u - [\alpha, \bar{\gamma}] - [\bar{\alpha}, \gamma] - [\rho, \varphi].
\end{aligned} \tag{47}$$

From (47) we get the following transformation rules of the auxiliary fields.

$$st = sm = sn = s\bar{n} = su = 0$$

$$\begin{aligned}
\bar{s}t &= -[\bar{\alpha}, t] + \bar{n} \\
\bar{s}m &= -[D\alpha, \bar{\phi}] - D\bar{n} - [\beta, \bar{\phi}] - [\bar{\beta}, t] - [\bar{\alpha}, m] \\
\bar{s}n &= -[\alpha\alpha, \bar{\phi}] - [\alpha, \bar{n}] - [\bar{\alpha}, n] - [\phi, \bar{\phi}] - [\rho, t] \\
\bar{s}\bar{n} &= -[\bar{\alpha}, \bar{n}] - [\bar{\phi}, t] \\
\bar{s}u &= -[\bar{\alpha}, u] - [\bar{\gamma}, t] - [\varphi, \bar{n}] - [\gamma, \bar{\phi}] + [[\alpha, \varphi], \bar{\phi}].
\end{aligned} \tag{48}$$

The equations (45), (46), (47) and (48) constitute a complete set of the BRST and anti-BRST transformation rules of the component fields of the topological Yang-Mills theory with scalar and vector gauge fields.

4. Discussions and conclusion

It is well-known that $\mathcal{S}_4$ in (1) can be expressed as

$$\mathcal{S}_4 = \text{tr} \int_{M_4} FF = \text{tr} \int_{M_4} d(AdA + \frac{2}{3}AAA). \tag{49}$$

From this property we get the Chern-Simons action in three dimensions

$$\mathcal{S}_3^{CS} = \text{tr} \int_{M_3} (AdA + \frac{2}{3}AAA), \tag{50}$$

which is invariant under the usual gauge transformation $sA = -D\alpha$. At the same time, the fact that $\mathcal{S}_4$ is a total derivative of something as in (49) explains why $\mathcal{S}_4$ is invariant under the topological symmetry $sA = -\beta$ which is equivalent to (2). This is true because $sd = -ds$, and β is set to be zero at the boundary.

$\mathcal{S}_3$ in (4) and $\mathcal{S}_2$ in (6) are also invariant under the topological symmetry (5). Then we expect that $\mathcal{S}_3$ and $\mathcal{S}_2$ can also be expressed as total derivatives of something. This is also true as we can see below.

$$\mathcal{S}_3 = \text{tr} \int_{M_3} FD\varphi = \text{tr} \int_{M_3} d(\varphi F), \tag{51}$$

$$\mathcal{S}_2 = \text{tr} \int_{M_2} D\varphi D\varphi = \text{tr} \int_{M_2} d(\varphi d\varphi). \tag{52}$$

Actually, we can say that for some action, having a topological symmetry and being able to be expressed as a total derivative of something are equivalent. From (51) and (52) we have the following two and one dimensional Chern-Simons actions [13].

$$\mathcal{S}_2^{CS} = \text{tr} \int_{M_2} \varphi F, \quad (53)$$

$$\mathcal{S}_1^{CS} = \text{tr} \int_{M_1} \varphi d\varphi, \quad (54)$$

which are invariant under the usual gauge transformation of $sA = -D\alpha$ and $s\varphi = -[\alpha, \varphi]$.

In this paper we studied the topological Yang-Mills theory of a system composed of scalar and vector gauge fields in two and three dimensions. Since the system contains both even and odd form gauge fields, we worked in the superconnection framework. Furthermore, in order to take care of the topological symmetry of the $\delta A_\mu = \beta_\mu$ type as well as the usual gauge symmetry of the $\delta A_\mu = D_\mu c$ type, we modified the horizontality condition. In this scheme it was possible to understand the BRST and anti-BRST symmetries geometrically. That is, by applying the modified horizontality condition in the superconnection framework, we could obtain a complete set of the BRST and anti-BRST transformation rules of the component fields in a systematic way.

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